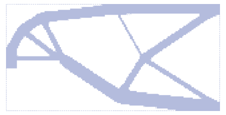
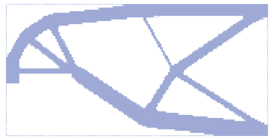
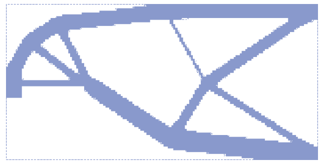
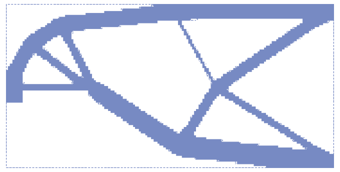
Topology optimization:

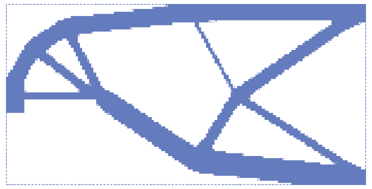
from airplanes to nano-optics

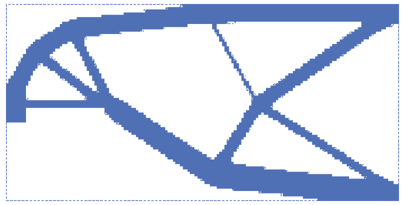


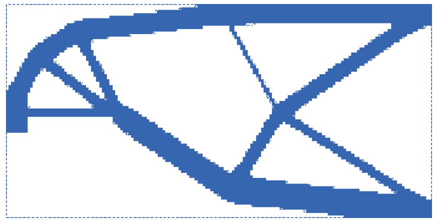


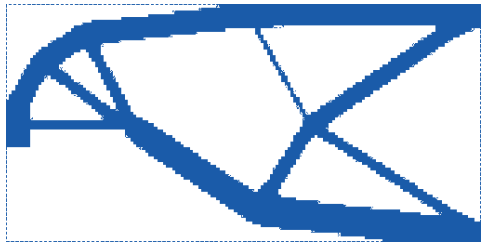












Ole Sigmund
& Martin P. Bendsøe

Imagine that you are supposed to construct a new type of gripper. Not the normal one used to turn the steaks at a garden barbecue but a small device that can be mounted on the head of a pin and that is operated not by direct application of force but by heating and application of electric current. The purpose would not be to move steaks, but perhaps to manipulate molecules, blood cells or other tiny objects. This type of engineering design is actually not very much unlike finding a good lightweight solution for a part of an aircraft wing, at least not if computers help to generate the answer. A computational method called "topology optimization" enables optimized layouts to be found for such diverse things as micro-robots, wing parts, sound barriers and crystals for managing light in nano-optical applications.

Hands-on

As an initial example let us consider the design of a hanging device for a lamp – a classical Danish solution is illustrated in Fig. 3.1. The lamp has to hang a few feet off a wall and the hanger can only occupy a certain amount of space in the room and on the wall, as we may want to place a table under the lamp and have bookshelves over the lamp. For our design we wish to use a limited amount of material, for example aluminum, and we prefer that the lamp does not deform the hanger too much (we here ignore the deformation due to the weight of the hanger itself). This goal is

Fig. 3.1. A hanging device for a lamp. An example of classical Danish design. Photograph courtesy of Severine Baillet.

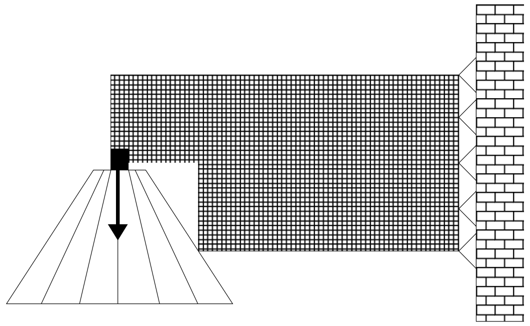


Fig. 3.2. For the design problem one needs to define the design domain, loads and possible supports, as shown here. The associated finite element mesh on which the computations are based is also shown.

formulated as minimizing the displacement at the tip of the hanger when the lamp is attached (Fig. 3.2).

The design problem is now seemingly well defined in terms of goal and constraints, and we are ready to use the computer. One may either use commercial software or go to www.topopt.dtu.dk and design the hanger using the graphical interface shown in Fig. 3.3. The design problem can be defined by specifying loads, supports, design domain and the amount of available material. Pressing the submit button produces a new window with a short movie that illustrates how the optimized hanger evolves; Fig. 3.3 shows a few frames of the movie. The calculations that the computer performs are based on ideas that have evolved since the late 1980s and that now constitute the foundation for a technique we call *topology optimization*. The cornerstones for this technique are how the hanger is described as an object and how to compute how well the hanger performs.

Thus, even though the design problem seems to be precisely formulated, the computer needs more information to improve the hanger automatically. We need to formulate the parameters that can be changed in designing the hanger and we need a method to compute the displacement of the hanger when we attach the lamp. The latter is typically expressed as "computing the structural response'".

The structural response is found by dividing the structure into numerous small elements called *finite elements* (Fig. 3.2). A standard analysis technique called *finite-element analysis* (FEA) can be used to determine structural responses such as the displacement at the tip of the hanger.

In order to change the design, we use a "trick'" that is crucial to success: the structure is visualized as black-and-white pixels on a television screen. Turning on and off material in each pixel can produce a picture of the optimal structure. In practice, gray pixels are allowed during the design process. This corresponds to elements with porous material. Thus, the variables that give us the structure describe the density of material in each finite element. However, only black and white pixels corresponding to material or no material elements are left in the final design; this is handled in the optimization procedure.

The hanger is now optimized by performing computations that successively improve the design. We compute the performance of the hanger for a given distribution of material and also compute the improvement that can be made by changing the gray-level of each pixel. Here we are talking about small variations of the material distribution, and we say that we perform a so-called *sensitivity analysis* (corresponding to calculating derivatives as it is taught in calculus classes in mathematics). Sensitivity analysis is not as time-consuming as performing full new analyses for all possible changes in design. This is crucial for the method, as the computations otherwise would take forever.

Enough information is now available to improve the design. The sensitivity information indicates in what direction the changes should be made. This results in an improved hanger, and the procedure can be repeated. This is then continued until no or very small improvements are possible. This is then the final design.

The procedure (or algorithm) is similar to mountain climbing in dense fog and without a map. The height of the mountain is a measure of how good the structure may be, and we wish to get to the summit. However, we cannot see the summit before we reach it. So instead of trying to go directly for the summit, we make a survey of the mountain in a small area around our present position. This we do by examining the slope of the mountain – we calculate the sensitivity to a change in position. Based on this survey, we make a decision on how to take a further step up the mountain. We proceed like this until no further ascent is possible and we have reached a summit. However, there may be a taller peak elsewhere. We say that the procedure gives us a *local extremum*. If we believe that there is a higher peak (there is no way to know, as the map is still not complete!) we can try from a new starting-point.

The principles of this iterative design procedure were used for making the frames illustrated in Fig. 3.3. Before the first iteration, the hanger is discretized by 1000 finite elements (pixels), so a map in this context is quite a challenge! Then the iterative procedure is started. It is seen that, after 10 steps, some material has been redistributed, and we already get an idea of what the hanger should look like. After 30 iterations, the optimum structure is roughly outlined, and after 50 iterations, the optimal structure has been found (we have reached a summit).

The resulting optimized hanger is quite different from the one in Fig. 3.1. This is because this design fulfills more requirements: it can also be stretched to provide light at different distances from the wall. This illustrates a crucial aspect of applying optimization — the result is governed by the selected design criteria. An important task for the engineer when using optimization in new settings is thus to formulate the "right" problem to be solved. This is often not

Design optimization – a Danish perspective

If you attend the biannual World Congress on Structural and Multidisciplinary Optimization (WCSMO), you will probably be surprised by the size of the Danish delegation and the number of Danish presentations. For its size, Denmark is somewhat of a "superpower" in the field. In science, such things often result from timing and the insight of individuals. The base for the Danish research community was created in the mid-1960s when Fridhiof Niordson began working in this field. He hired a number of very talented young faculty and created an internationally oriented research group in solid mechanics at the Technical University of Denmark.

Today, Denmark has two research groups in structural optimization, one at the Technical University of Denmark and one at Aalborg University (AAU). The latter was established when Niels Olhoff moved from the Technical University of Denmark in the mid-1980s, and the two groups have maintained close contact since then. Also, the international orientation has been maintained, enabling young researchers entering the area to easily establish collaboration with foreign research centers, which is a key to success in science.

The research groups based in Denmark have developed several fundamental concepts and results in this field, often in collaboration with visiting scientists. In the early 1970s, Pauli Pedersen did fundamental work in the topology design of trusses; later in the 1970s, Olhoff and Rasmussen discovered the emergence of multiple buckling and vibration modes in optimum structures. This and the later discovery by Cheng and Olhoff of multiple scales in problems related to plate design were central for understanding the fundamental nature of optimal design problems. General computer methods were also in focus, and Pauli Pedersen was one of the first to exploit the interaction of finite-element methods and mathematical programming. This computer systems approach was followed by one of the first programs for shape design based on computer-aided design by John Rasmussen from Aalborg University in the late 1980s. At that time, the Technical University of Denmark took the initial steps towards developing topology design. Topology optimization methods are now the major fields of interest for the group at the Technical University of Denmark and for numerous other research institutions around the world.

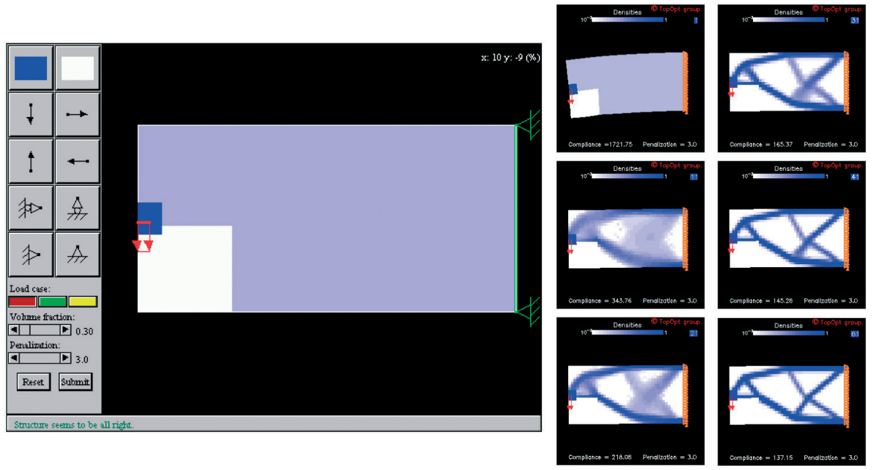


Fig. 3.3. Designing the lamp support using the topology design system at www.topopt.dtu.dk: the interface and a few frames from the evolution history of the optimized design.

Fig. 3.4. A refined version of the design shown in Fig. 3.3, using a resolution of 12,800 “pixels” – this demands more computer power, resulting in a more well-defined design.

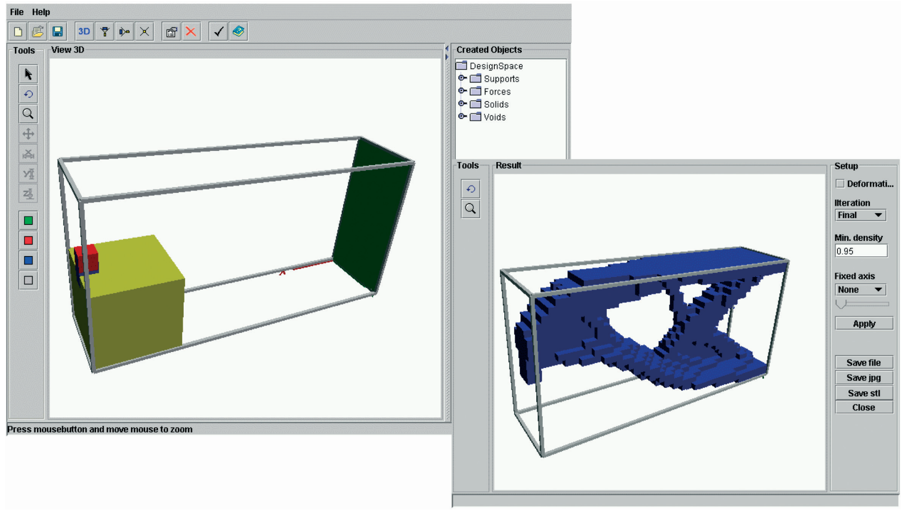


Fig. 3.5. Here the lamp-holder is designed using the more advanced 3D version of the interactive topology optimization program found at www.topopt.dtu.dk. This requires even more computational effort, even for a rather “rough” model as here, using 12,000 voxels.

achieved in one attempt but requires some experimenting to succeed.

The computer in the design process

Generating the hanger is an example of using the computer to assist in designing a structure. Many types of software are routinely used today for helping engineers in creating efficient products. For technical drawings, computer-aided design (CAD) software is used not only for easy storage and manipulation but also for such purposes as tracking parts and making production lists. In generating the composition of products, decision support systems are used to help the engineering in the choice of parts, materials, and perhaps standardized solutions. When trying to understand the functionality of the product prior to making any real physical device, various analysis tools may be used to conduct virtual tests on a computer. Software systems may calculate the displacement of a lamp hanger when the hanger is carrying a lamp, and the engineer may employ more advanced simulation tools that can model the air flow around an airplane wing, the crash of an automobile, the solidification of metal in a casting process, etc.

Applying optimization techniques adds an extra dimension to these tools. Improving a design is traditionally based solely on the ingenuity of the engineer and is often a cumbersome procedure when intuition is limited and improvements are found by trial and error. Analysis tools have made this approach much more efficient, as physical prototypes are not necessary for determining the effect of design changes. Still, the final product is typically tested in real laboratory tests to safeguard against flaws in the analysis tools. However, optimization techniques allow the procedure for generating good designs to be automated. This means *rational* procedures that employ the mathematics of optimization in order to obtain guaranteed improved designs. As this requires assigning numbers to goals and to the description of the design, not all challenges of design can yet be tackled. For example, as there is no well-defined quantification of a broad and subjective concept like “beauty”, one has to be satisfied with the *beauty of efficiency* in rather down-to-earth engineering terms.

Structural optimization

A computer program for the optimization of mechanical structures will typically, as outlined earlier, consist of a part that can do analysis and sensitivity analysis and a part that can perform optimization based on the sensitivity analysis. The development of efficient computational methods for the optimization of structures is a very active area of research called *structural optimization*. One main aspect that distinguishes structural optimization methods is how the design is described: the characteristics of the structures that can be changed by the optimization procedure. These have to be defined in terms of numbers: the *design variables* of the design problem.

In *sizing optimization*, the layout of the structure is prescribed: for example, a lamp-holder consisting of 8 elements as in Fig. 3.1. The cross-sectional area of each element is then a design variable. The structure is optimized by finding the areas of the individual bars that maximize the stiffness of the structure for a given total weight. Sizing optimization is the simplest way of doing structural optimization. *Material optimization* deals with the effect of changing the material used; for example, the hanger can be built using fiber composites instead of wooden bars. A goal here could be to find the material composition that optimizes the stiffness of the hanger by varying design variables that define the orientations of fibers in the composite.

An intuitive way to save weight in a structural component is to drill circular holes; this is seen in numerous applications. However, circular holes are not necessarily structurally efficient, and the structure may be improved by *shape optimization*. In this case, the design variables are parameters that change the shape of the holes. The number of holes remains unchanged. However, in *topology optimization*, as illustrated in Fig. 3.1–3.5, we not only find the optimal number of holes in a structure but also the optimal

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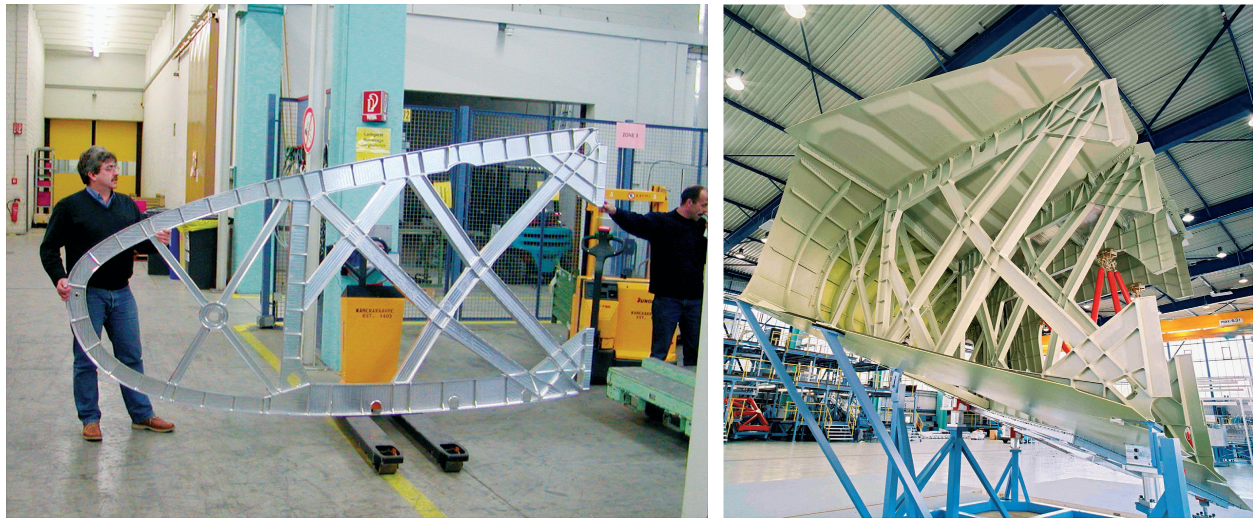
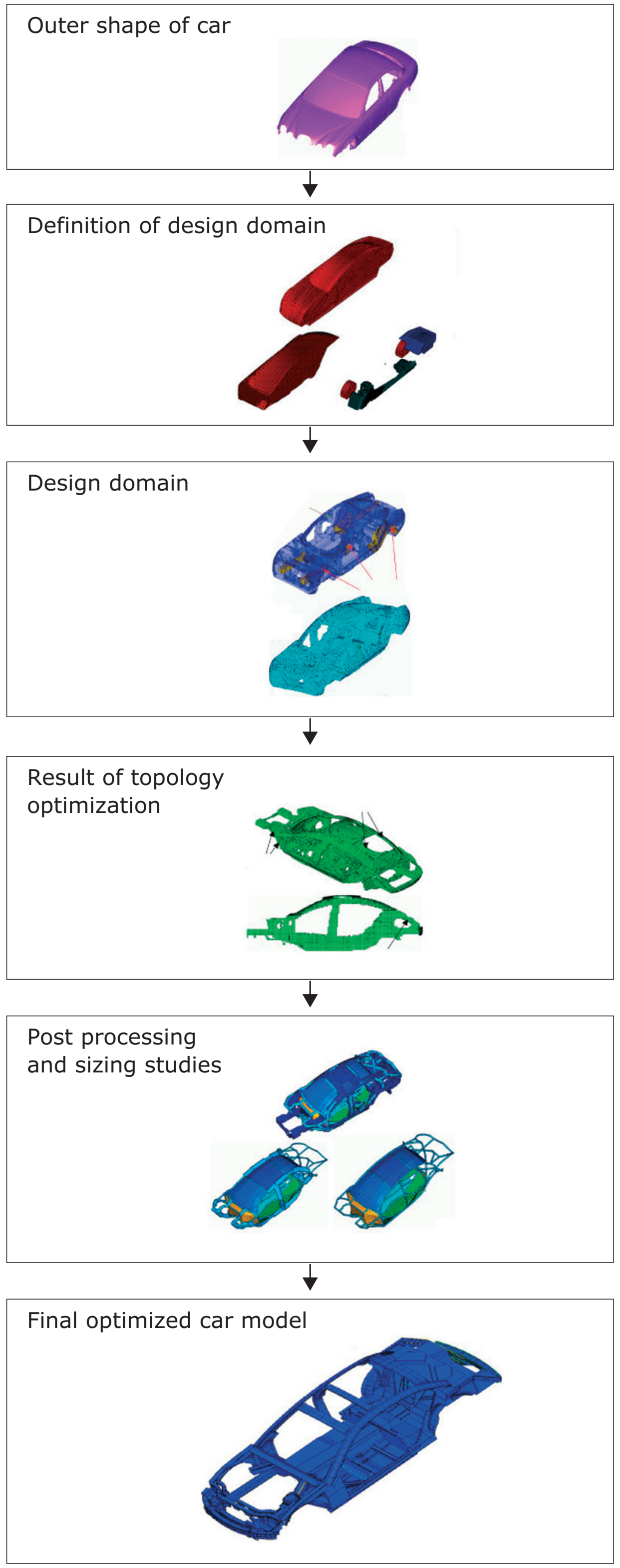


Fig. 3.6. Design of the leading edge of a wing. The new mega-airliner, the Airbus 380, has required much effort in weight reduction studies for making the aircraft viable. During the development of the aircraft, topology design has been tested for its applicability in the aerospace industry. One such study was the design of integrally stiffened machined ribs for the inboard inner fixed leading edge. Two types of software were applied, one which is similar to the method described here, and one that also includes information on the type of (eventually composite) material that is useful for the design. Based on these results and quite a bit of engineering interpretation, a new type of structure was devised for the ribs that gave a weight benefit against traditional (up to 40%) and competitive honeycomb/composite designs.
The process of generating this new type of rib is typical for applications of topology design in many mechanical engineering settings. The technique is not necessarily used for creating the final design, but rather to give inspiration to the experienced engineer, who can see new possibilities on the basis of the computational results. In other fields, however, one also sees that the results of the topology design are transferred directly to production (see the fig. 3.9 on the design of an optical waveguide – the Z-bend).

Fig. 3.7. Design of the load carrying frame of an automobile. An illustration of the design process using topology optimization at Jaguar Cars Ltd. The styling department determines the outer shape of the car. The structural design domain is determined as the entire car structure minus the areas reserved for cabin, engine, drive train etc. The final design domain is discretized by several hundred thousand elements. The result of the topology optimization indicates the optimal distribution of material and points out zones where reinforcements are especially important. After post-processing of the topology optimization results, actual sizes of frame, shell and other body parts are determined from a simplified optimization problem. The final optimized car model is further checked for crashworthiness, acoustic and other design criteria. Illustrations courtesy of Altair Engineering Ltd. and Jaguar Cars Ltd.



shape of the holes. In special cases, the orientations of fibers in a composite structure may also be found.

The word *topology* originates from the Greek word *topos*, which means landscape or place. In other words, topology optimization means optimizing the “landscape”, consisting of the number, shape and connectivity of the elements that make up a structure.

Issues of structural topology design

As described for the lamp-holder problem, the optimizing of topology sounds fairly simple. However, various computational and theoretical challenges have arisen during the process of developing the method into a reliable and useful computer tool.

As the pixel representation used for describing the structure in optimizing topology gets finer and finer, also more details appear in the optimum structure. The best structure will comprise an infinitely fine grid of closely spaced beams, similar to a fiber composite. This may be useful information but is typically impractical for manufacturing. Such situations are then handled by applying methods from image processing. The results in Fig. 3.3–3.5 are all based on the use of a filter to limit geometrical complexity.

Computational speed is another important issue. This is especially true for computations in three dimensions, where even modern high-speed computers have a hard time doing topology design, both in terms of computational time and in terms of RAM storage needed. The basic method is unchanged, and many of the voxels (cubes) now used instead of pixels are needed to suitably describe a structure. That more rough models nevertheless can provide useful information is illustrated in Fig. 3.5, where a lamp-holder has been generated using the 3D facility at www.topopt.dtu.dk.

Industrial applications

The computer-based topology optimization method was first introduced in 1988 for the minimum weight design of structural components. Since then, the topology optimization method has gained widespread popularity in academia and industry and is now being used to reduce weight and optimize the performance of automobiles, aircraft, space vehicles and many other structures (Fig. 3.6). Today, several commercial software systems provide topology optimization for industry. These programs are based on aca-

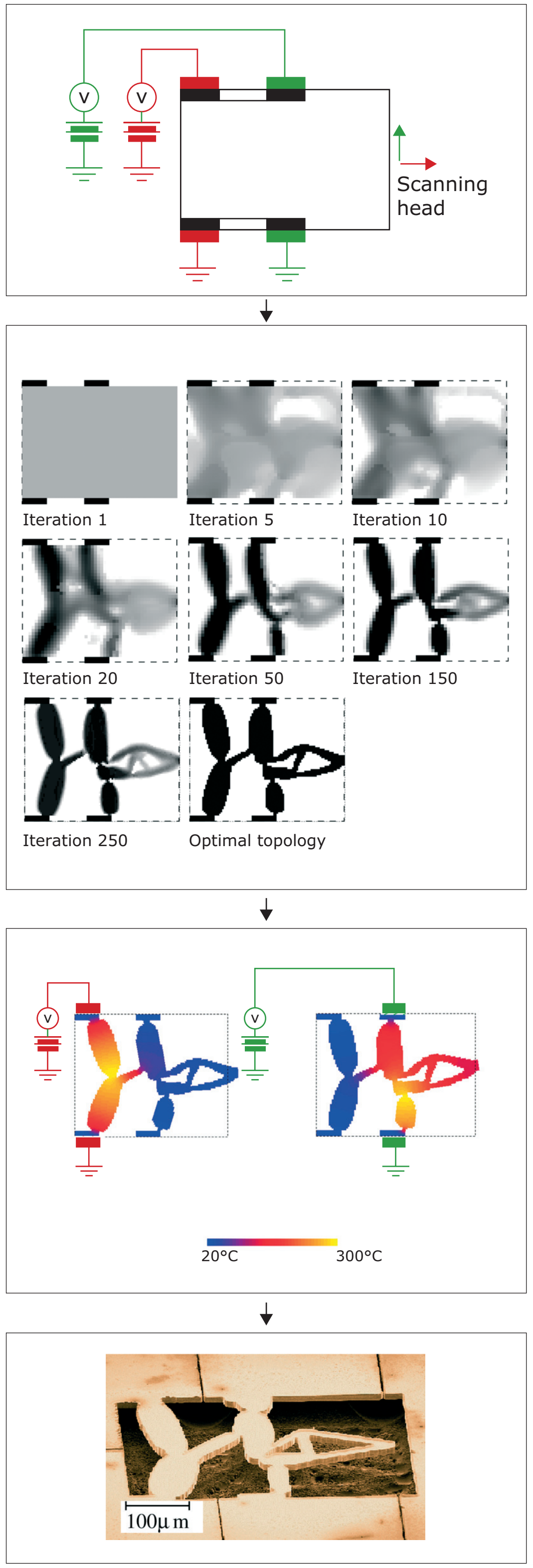


Fig. 3.8. The design procedure for a multiphysics device: a microscopic manipulator.The illustration shows snapshots from the topology optimization process, simulations of the optimized manipulator and an electron microscope picture of the manufactured device. The manipulator works via electro-thermo-mechanical actuation. Applying the red electric field to the structure heats the left part of the manipulator and pushes the whole device to the right. Applying the green electric field heats the right part of the manipulator. Because the two beams are offset, the heating causes rotation and thereby vertical displacement of the scanning head. The device was manufactured in nickel by surface etching and electroplating techniques at the Department of Micro and Nanotechnology (MIC) at the Technical University of Denmark. Note the length-scale – the whole device is less than half a millimeter in size – small enough to be mounted on the head of a pin.

demic research, where DTU has played a major role. A main user of topology design in the daily design efforts is the automobile industry, where most major manufacturers and their sub-suppliers now use the methodology (Fig. 3.7).

Topology optimization – an emerging technology in a broader context

The topology optimization method has recently been applied to several other design problems. Examples are the design of tailored "exotic" materials with counter-intuitive properties such as negative Poisson's ratios (materials that expand transversely when pulled) and negative thermal expansion coefficients (materials that shrink when heated). Other applications include the design of transducers for underwater sound detection and MicroElectroMechanical Systems for use in hearing aids, air-bag sensors and micro-robots.

These new challenges can be treated within the same basic format of the design parametrization, problem statement and computational procedure. Thus, as mentioned in the introduction, the design of a microscopic manipulator is closely related to the design of a hanger, and we can again use the topology optimization for the design process. However, several issues need to be addressed: how to formulate objective functions and constraints that result in useful engineering designs. Another central issue is how to relate gray-scale (density) to physical properties that allow these objective functions and constraints to be evaluated. Finally, a scheme should be imposed to obtain black-and-white designs.

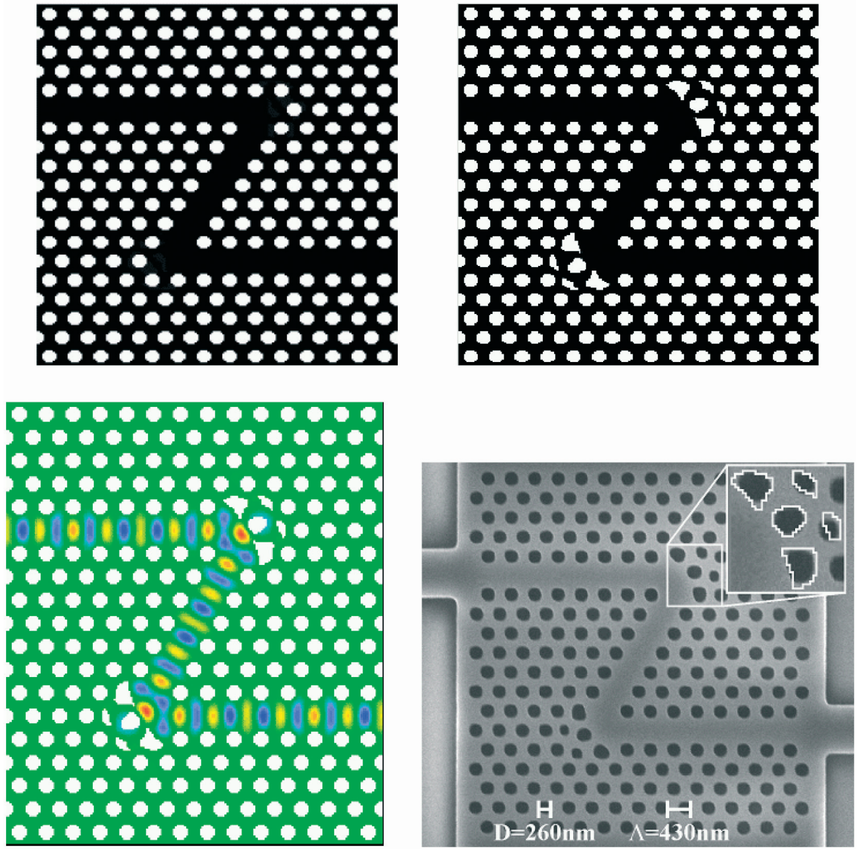


Fig. 3.9. Design of an optical waveguide – the Z-bend. The results of the design process for guidance of light around a Z-bend. If we had just removed some of the periodically distributed air-filled holes to obtain a Z-shaped bend, the light transmitted through the bend would have been less than 50% due to losses and reflections. After a topology optimization process where we optimized the shape and position and number of air holes in the corner regions, we obtained close to 100% transmission in a wide frequency range. The optimized Z-bend was manufactured by electron-beam lithography techniques at the Center for Optical Communication (COM) at the Technical University of Denmark. The manufactured device performs very well with record-breaking bandwidth and transmission properties.

Design of multiphysics devices

Imagine that we are interested in designing a microscopic (less than 1 mm²) scanning device that can be used to manipulate molecules on a surface or to work as a pickup arm in a microscopic hard disk. The goal of the design process is to come up with a structure that can move a scanning head in a rectangular area by controlling two different input voltages (Fig. 3.8). The actuation principle involves electric, thermal and mechanical modeling; thus, we call it a *multiphysics problem*. When we apply an electric field to a metal structure, the structure heats like the filament in a light bulb. The heating will cause the material to expand, and this expansion can be used to move the scanning head. The problem is to find the manipulator topology that gives the largest displacements for a given electric input and at the same time makes the scanning head move vertically when one electric field is turned on and move horizontally when another electric field is turned on. Since the modeling is more complicated than before and since there are several extra constraints to be satisfied, the topology optimization procedure is slower than for the purely mechanical problems discussed earlier. However, the basic idea of alternating between material redistribution steps and analysis steps is the same as before.

Design in nano-photonics

In principle, the topology optimization method may be applied to any design problem where the placement of material or holes determines the efficiency or functionality of a device. An interesting example of optimal hole placement is in photonic crystals based on the band gap effect.

The idea is the following. Light propagates as waves and, if transmitted through a transparent medium like glass, it will propagate essentially without losses. However, if the glass structure is perforated with a periodic arrangement of air holes with hole distances a little less than the wavelength of the light (smaller than micrometers: nano-scale), waves at certain frequencies will no longer propagate through the glass structure. This effect can be used to produce mirrors in nano-scale, or it can be used to guide light in optical chips. The latter can be obtained by filling some of the air holes in channel-like patterns, as seen for a Z-bend in Fig. 3.9. Since the light cannot travel through the perforated structure, it will stay within the channel and can be led around sharp corners and may be manipulated in other ways. Such photonic crystal structures may in the future provide the basic building blocks for optical devices and computers. At present, optical signals that are transmitted through optical fibers must be converted to electrical signals in order to be amplified, diverted, split or distributed to the end-users. Using photonic crystals may avoid the need for optical-to-electric conversion, and the signal-processing devices may become much smaller and less expensive. However, this requires optimizing the efficiency of photonic crystals.

The idea of loss-less transmission of optical waves through photonic crystals cannot be fully realized: transmission is less than 100% because of leaking waves (the amplitudes decay exponentially away from the channels) and reflections at channel corners. Changing the shape, number and position of the holes along the channels may optimize efficiency. The design of photonic crystals is therefore an interesting application for topology optimization.

Optimizing the topology of photonic crystals requires modeling the propagation of light through inhomogeneous media. The equations that describe this phenomenon are called Maxwell's equations. Although these equations are physically very different from the elasticity equations discussed previously, finite-element analysis may again be used to calculate the response. Instead of maximizing the stiffness as for the hanger, we now maximize the energy transported by the electromagnetic light wave. As before, the design variables are the material densities, now describing the interpolation of the dielectric properties of air and glass.

Perspectives

In the description of the topology optimization method, we have so far shown how movies of the appearance of a structure through the iterative optimization scheme can be useful for illustrating how the structure evolves from a uniform gray to a solution to the design problem. In the future this kind of illustration may be suppressed, and the designer instead sees how designs evolve and change as the data of the optimization problem is varied. The loads, the design domain and the support conditions can be changed in real time. Ultimately, a virtual reality setting can be established where the designer can move in the design domain and change the settings of the problem. For example, for architectural applications, one can experiment with shapes and designs that are always structurally optimal, but that can be manipulated to satisfy aesthetic considerations and requirements: what would happen if rooms are moved, windows added or wind and snow loads changed? There is thus much scope for developing how designers interface with topology design methods and how the techniques are used in the daily work in design offices.

Another perspective for the future of computer based design optimization is how to use the techniques for a broader range of multiphysics problems. There have already been initial studies of its use in many more areas than described here, including design of microfluidic systems, design of biomedical devices (such as implants), and design for crashworthiness. One of the key issues for other applications is how to manage situations where central physical properties are related to boundaries between materials. This could be chemical reactions at the surface of a catalyst. How this can be modeled in topology optimization where the gray-scale pictures do *not* have any explicit representation of boundaries is one of the great puzzles of the field. That people see boundaries in the pictures is one of the wonders of the brain – but the computer does not have that facility in the methodology as outlined here. One may argue that aspects of this problems have been successfully tackled in image processing (edge detection), but here we have to take care of physics and optimization as well, and this within reasonable computational resources (time and storage). We foresee many attempts in the next decade to tackle this and similar issues related to the pixel geometry representation of topology design.

Other outstanding challenges of more technical nature will also keep researchers busy for the coming years. This relates, for example, to understanding what design criteria should govern the optimization procedures in various fields involving new physics and it relates to the computational procedures used. Thus, results may be produced more efficiently by alternative approaches to the computational procedures, not only by improving individual parts of the code, but perhaps also by a complete change of computational philosophy.

Topology design is a thriving research area that has attracted much interest in the past decade. It is in a sense a rather mature practical tool, but this should not distract from the challenges that we face and the great potential that exists for turning this methodology into a general design tool for most areas of modern technology.

Acknowledgments

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More to explore

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Much material can also be found on the web. See www.topopt.dtu.dk for tutorials, programs and relevant links.