Cryptanalysis of FORK-256 and some comments on the state of hash functions research

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Talk overview

Part I: Cryptanalysis of FORK-256

- Short description of FORK-256
- Micro-collisions in the step transformation
- Simple differential path for the compression function
- General method of finding differential paths
- Collisions for the compression function
- Some improvements

Part II: Some comments on the current hash functions research

- Current situation in the world of hash functions
- NIST call for new hash functions
- Do we know what we want?
- How to deal with the situation?

PART I: Cryptanalysis of FORK-256

Joint work with Thomas Peyrin[1], Olivier Billet[1], Scott Contini[2] and Josef Pieprzyk[2].

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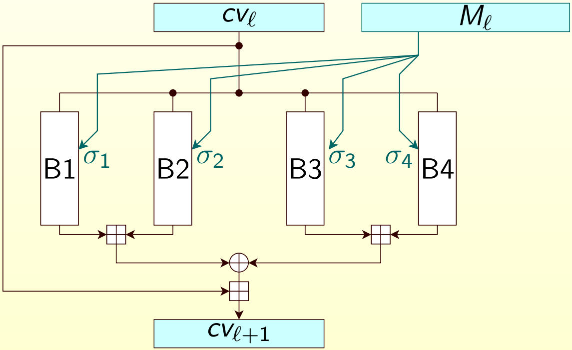
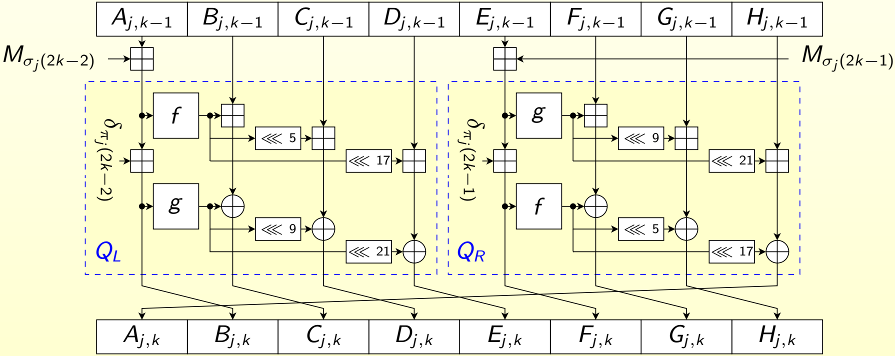
Structure of FORK-256 :: four parallel branches



- 256 bits of chaining variable *cv*
- 512 bits of message *M*
- each branch B1, B2, B3, B4 consists of **8 steps**
- each branch uses a different permutation $(\sigma_1, \sigma_2, \sigma_3, \sigma_4)$ of message words $M_0, \ldots, M_{15}$

Structure of FORK-256 :: step transformation



- there are 8 steps in each branch
- step transformation – composition of 3 simple operations
  - addition of two different message words
  - two parallel **Q-structures**
  - rotation of registers

- Short description of FORK-256
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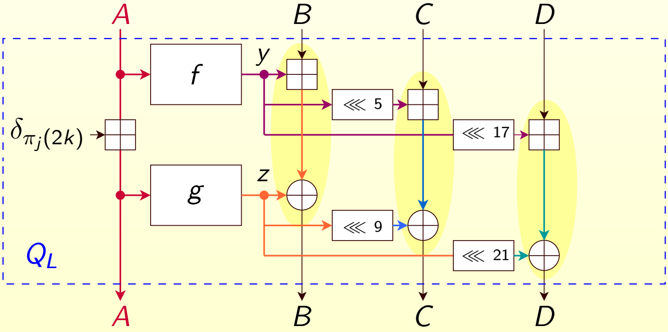
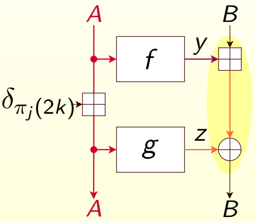
What is a “micro-collision”?



Micro-collision: a difference in register A does not propagate to the selected register (B,C,D).

If it does not propagate to more than one other register we have *simultaneous micro-collisions*.



Let us denote

$$y = f(x), \quad y' = f(x') \qquad z = g(x \boxplus \delta), \quad z' = g(x' \boxplus \delta).$$

We have a micro-collision in the first line if the equation

$$(y \boxplus B) \oplus z = (y' \boxplus B) \oplus z' \tag{1}$$

is satisfied for given y, y', z, z' and some constant B.

Our aim is to find the set of all constants B for which (1) is satisfied.

Three representations of a difference

- usual XOR difference:

$$\Delta^{\oplus}(z, z') = (z_0 \oplus z'_0, \ldots, z_{31} \oplus z'_{31}) \quad \in \{0,1\}^{32}$$

- integer difference:

$$\partial y = y' - y \quad \in \{-2^{32}+1, \ldots, 2^{32}-1\}$$

- singed binary difference:

$$\Delta^{\pm}(y, y') = (y_0 - y'_0, \ldots, y_{31} - y'_{31}) \quad \in \{-1,0,1\}^{32},$$

Two useful relationships between different representations

- If $\Delta^{\pm}(y, y') = (r_0, r_1, \ldots, r_{31})$ is a signed binary difference, then the corresponding XOR difference is $(|r_0|, |r_1|, \ldots, |r_{31}|)$.

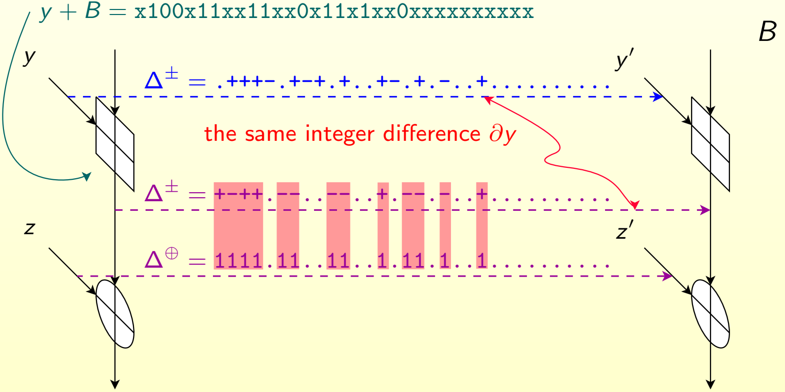
- Having a signed binary difference we can easily recover the (unique) corresponding integer difference:

$$\partial y = \sum_{i=0}^{31} 2^i \cdot \Delta^{\pm}(y, y')_i \quad .$$

Finding micro-collisions

- We can rewrite $(y \boxplus B) \oplus z = (y' \boxplus B) \oplus z'$ as $(y \boxplus B) \oplus (y' \boxplus B) = z \oplus z'$
- This means that the signed difference $\Delta^{\pm}(y \boxplus B, y' \boxplus B)$ has to have non-zero digits in those places where $\Delta^{\oplus}(z, z')$ has ones.
- There are $2^{h_w(\Delta^{\oplus}(z,z'))}$ such signed differences that "fit" into the XOR difference.
- They correspond to $2^{h_w(\Delta^{\oplus}(z,z'))}$ integer differences that may yield a micro-collision
- Integer difference is not changed by adding the constant B !

Finding micro-collisions



XOR difference $\rightarrow 2^{h_w}$ signed binary diffs $\rightarrow\ 2^{h_w}$ integer diffs $\rightarrow$ one of them must be $\partial y = y - y'$

Finding micro-collisions: Necessary condition

To test whether the quadruple (y, y', z, z') may yield a micro-collision we have to check whether there exist a signed binary representation corresponding to $\partial y = y - y'$ that "fits" into XOR difference $\Delta^{\oplus}(z, z')$.

This problem can be reduced to an easy (superincreasing) knapsack problem:

Having a set of positions $I = \{k_0, k_1, \ldots, k_m\}$ (determined by non-zero bits of $\Delta^{\oplus}(z, z')$), decide whether it is possible to find a binary signed representation $r = (r_0, \ldots, r_{31})$ corresponding to ∂y s.t.:

$$\partial y = \sum_{i=0}^{m} 2^{k_i} \cdot r_{k_i} \quad \textit{where } r_{k_i} \in \{-1, 1\} \ .$$

This test can be implemented very efficiently!

```
int micro_possible(WRD y1, WRD y2, WRD dz) {
    WRD tmp, delta_y, sum;
    if ( y2 > y1 ) {
        tmp = y2; y2 = y1; y1 = tmp;
    }
    delta_y = y1 - y2;
    sum = delta_y;
    sum += dz;
    if ( sum < delta_y ) {
        if ( (dz>>31)==0 )
            return 0;
    }
    dz <<= 1;
    return ( (dz|sum) == dz );
}
```

Finding micro-collisions: Also a sufficient condition

In fact we can prove that this condition is also sufficient: if we can find such a representation, we can always find constants B that make the difference "fit" into the prescribed XOR pattern.

Moreover, the analysis shows that the size of the set of good constants B is equal to

$$2^{32-h_w(z\oplus z')+1} \quad ,$$

with the grey one added if the MSB of $\Delta^{\oplus}(z,z')$ is one.

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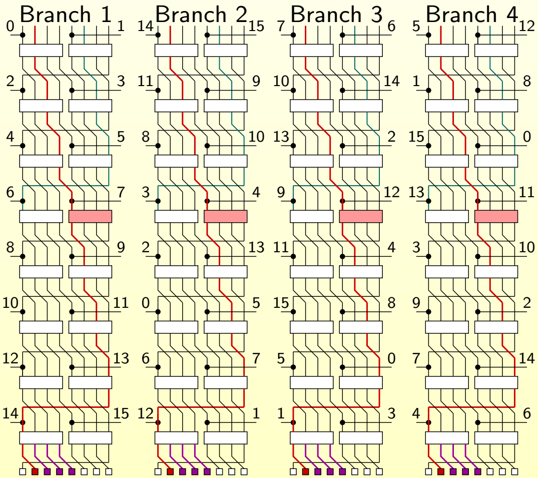
Simple differential path using micro-collisions



By introducing differences in B_0 and finding simultaneous microcollisions in four Q-structures in step 4 we obtain a differential restricted to 4 registers.

Simple path: complexity analysis

- Once we pass through step 4, we can generate 2^{32} pairs,
- To pass step 4 we have to make a few simple checks for 2^{32} values, altogether equivalent to $2^{32}/4$ of FORK evaluations, we succeed with probability P_d^6, where P_d depends on the difference, for $d = \texttt{0x00000404}$ we have $P_d \approx 2^{-3}$.
- the average cost of a single solution $\approx 1/4 \cdot P_d^{-6} \approx 2^{16}$.
- an example of a pair with output difference of weight 22:

cv_n	8406e290	5988c6af	76a1d478	0eb60cea	f5c5d865	458b2dd1	528590bf	c3bf98a1
cv'_n	8406e290	5988cab3	76a1d478	0eb60cea	f5c5d865	458b2dd1	528590bf	c3bf98a1
M	396eedd8	0e8c2a93	b961f8a4	f0a06fc6	9935952b	e01d16c9	ddc60aa4	0ac1d8df
	c6fef1d8	4c472ca6	58d9322d	2d087b65	7c8e1a26	71ba5da1	ba5d2bfc	1988f929
cv_{n+1}	9897c70a	4e18862d	b4725ac1	cfc9f92c	9aa0637d	ae772570	74dd4af1	cd444dd7
cv'_{n+1}	9897c70a	4e1880f9	1e677302	4c650966	f4792bf4	ae772570	74dd4af1	cd444dd7

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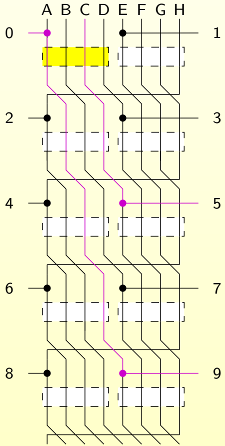
Finding high-level paths: idea and model

Let's be optimistic:

- Assume that we can always avoid mixing introduced by Q-structures (finding micro-collisions is always easy).
- Assume that any two differences cancel each other (i.e. we don't need to worry about many different values, either there is a difference or not and any two differences added together disappear).

So now we are in $\mathbb{F}_2$...

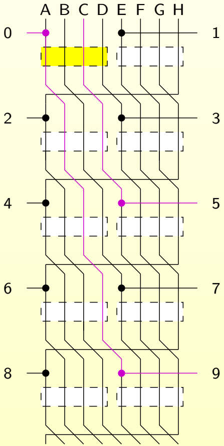
- The model is $\mathbb{F}_2$-linear function L_{out} that maps input differences in M and cv_n to output diffs.
- We can find the kernel of this map to get the set of all input differences that vanish at the output.



Finding high-level paths: example

Example

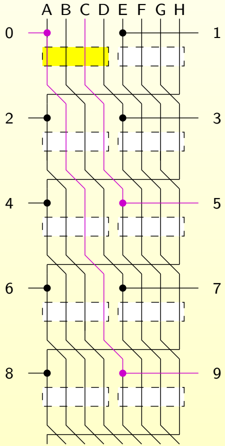
Input differences
$S = (A, B, C, D, E, F, G, H, M_0, \ldots, M_9)$.
For
$S = (0, 0, 1, 0, 0, 0, 0, 0,\ 1, 0, 0, 0, 0, 1, 0, 0, 0, 1)$ we have $L_{out}(S) = (0, 0, 0, 0, 0, 0, 0, 0)$.



Finding high-level paths: going back to reality

The more micro-collisions we have to find and the longer the path the smaller probability that differences in the original function will follow the path.

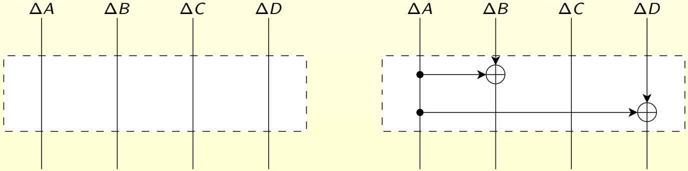
- We look for paths with as few micro-collisions as possible (a few differences in registers A and E)
- Generally, the shorter path the better.
- Let's look at the registers A and E and pick those input differences S that yield only a few non-zero differences in A and E.
- Optimal paths – minimum weight words in a linear code.



Finding high-level paths: more general model

So far we assumed that differences in A (or E) do not propagate to any other registers in the Q-structure. We can relax this condition.

$$(q_B, q_C, q_D) = (0, 0, 0) \qquad (q_B, q_C, q_D) = (1, 0, 1)$$

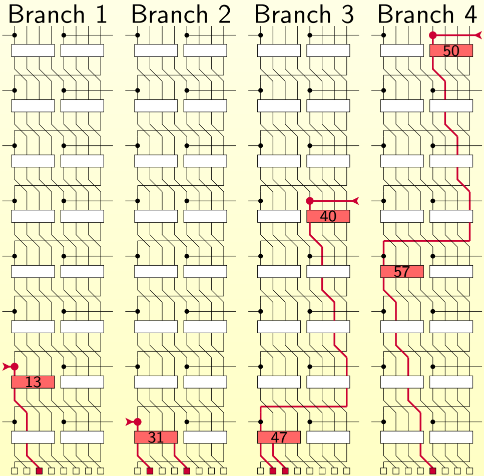


$$\begin{aligned}
\Delta A_{i+1} &= \Delta A_i \ , \\
\Delta B_{i+1} &= \Delta B_i + q_B \cdot \Delta A_i \ , \\
\Delta C_{i+1} &= \Delta C_i + q_C \cdot \Delta A_i \ , \\
\Delta D_{i+1} &= \Delta D_i + q_D \cdot \Delta A_i \ .
\end{aligned}$$

For each Q-structure we have 2^3 possible configurations. This gives 2^{3+64} different models for FORK-256 – more freedom to look for short differential paths.

Example of a path: Collisions for all branches

Differences in M_{12}. Configuration of Q-structures: 13:000, 31:001, 40:000, 47:100, 50:000, 57:000



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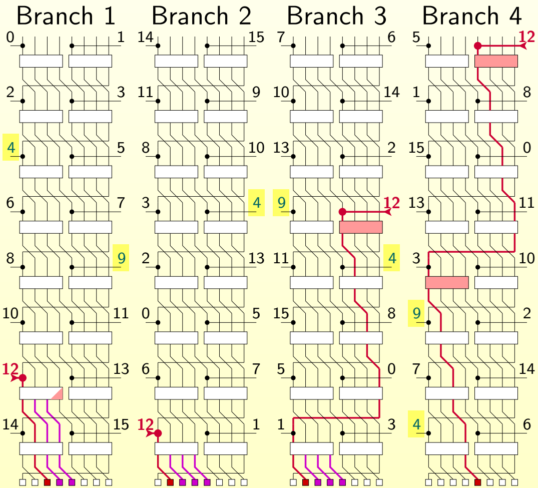
  - The differential path
  - Complexity analysis
- Some improvements
- Conclusions

Collisions: The differential path



$d =$ 0xdd080000 or $d =$ 0x22f80000

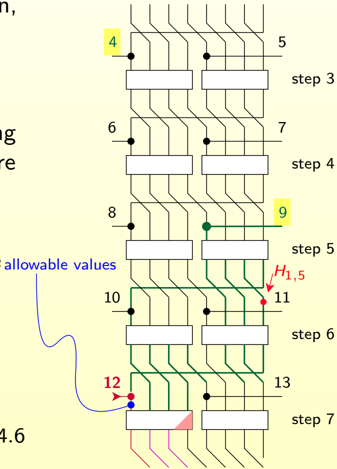
- ▶Using a modified path we need micro-colls in only $3\frac{1}{3}$ Q-structures.
- ▶Get 3 micro-collisions in branches 3 and 4 first.
- ▶Using different values of M_4 and M_9 compute branch 1 and hope there is a single micro-collision in Br. 1 step 7.
- ▶Using d with only 13 MSB set only 108 bits are affected.

Collisions: the principle of the attack

- Get three micro-collisions in branches 3 and 4.
  This leaves two message words M_4 and M_9 free, the rest is fixed
- Using different values of M_4 and M_9 compute branch 1 and hope that there is a single micro-collision in strand D in step 7.
- If a micro-collision there is found, compute the rest of the function and check the output difference.
  Note that the output differences have weights always ≤ 108

Collisions: complexity of getting close hashes

1. Compute internal registers up to 7th step. Then, for each allowable value x, set $A_{1,6} = x - M_{12}$, get the corresponding $H_{1,5}$ and store the result into a hash table T.
2. For each value of M_9 compute the corresponding value of $H_{1,5}$ and look for a match in T. If there is a match, go to point 3. When all M_9 are exhausted, increment M_4 and go to point 1.
3. *Check.* If current value of M_9 leads to a single micro-collision in the thread $D_{1,6} \to E_{1,7}$ then return (M_4, M_9), else continue point 2.



Point 1: $\eta/64 = 2^{15.7}$ FORK evaluations.
Point 2: $2^{32}/64 = 2^{26}$ FORK evaluations.
Since point 3 succeeds with probability $2^{-24.6}$ we get $2^{7.4}$ solutions for a work effort of 2^{26}. Per single solution: about $2^{18.6}$ FORK evaluations.

Collisions: the complexity of getting full collisions

- Complexity of finding a single solution: $2^{18.6}$.
- Now, if the distribution of outputs is close to uniform, we expect to find a collision after testing 2^{108} pairs.

Complexity of finding a collision: $2^{108} \cdot 2^{18.6} = 2^{126.6}$.

- *faster than by birthday paradox*
- *using only small memory (hash table + stored allowable values: 2^{23} 32-bit words in total)*
- *trivially parallelizable*
- *practical for obtaining near-collisions*

Example of a near-collision for the compression function with weight 28

IV	6a09e667	db1bb914	3c6ef372	a54ff53a	510e527f	767b0824	66410f7d	90f7ce64
M	85a83e55	91d3ca9d	a6c2facb	027afd32	000000cb	00000000	9d4a6aba	00000000
	e649c148	4606ae35	6efb18d8	2d6ade8f	1dcb6936	ec995db1	d2ad257b	730f5bb4
M'	85a83e55	91d3ca9d	a6c2facb	027afd32	000000cb	00000000	9d4a6aba	00000000
	e649c148	4606ae35	6efb18d8	2d6ade8f	40c36936	ec995db1	d2ad257b	730f5bb4
diff	**00000000**	**8c300000**	**1d010204**	**52520104**	**c0908122**	**00000000**	**00000000**	**00000000**

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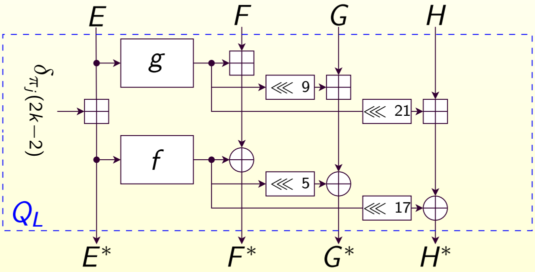
  - Improving efficiency using large memory
  - Achieving collisions for the hash function

Collisions: improving efficiency using large tables



Problem: To what extent can we influence the values of E^*, F^*, G^*, H^* changing only E ?

- We can set E^* to any value (bijective map),
- For any given pair (G, w) we can *very often* find such E that $G^* = w$.
- We can precompute a look-up table T that for any pair (G, G^*) returns the necessary value of E, $T(G, G^*) = E$.

Collisions: improving efficiency using large tables

- We can use such look-up tables to significantly reduce the time spent in branch 1
- Theoretical complexity of finding a single solution: $2^{1.6}$.

Complexity of finding a collision: $2^{108} \cdot 2^{1.6} = 2^{109.6}$.

- *we improved the speed by the factor of* 2^{17},
- *but we assume we can use huge, fast memory,*
- *we use around 512 tables (family parametrized by a), each one of size* 2^{64} *32-bit words, i.e.* 2^{73} *words of memory in total*

Collisions for the full hash function: principle

- We can avoid using B_0 in branch 3 either by using look-up tables or by a smarter scheduling in branch 3 we have to have only three IV words (F_0, G_0, H_0) set to one of the good constants to allow for micro-collisions in step 1 of branch 4.
- Probability that a random IV has all three values (F_0, G_0, H_0) acceptable to the algorithm is bigger than $2^{-3\cdot 32}$, in fact around 2^{-65} for differences `0xdd080000` and `0x22f80000`.
- At the cost of 2^{65} FORK evaluations we can find a prefix message block that after the first application of the compression function yields IV suitable for the main part of the attack.

Collisions for the full hash function: example

- For other modular differences this probability is much bigger.
- Using "easier" modular difference we've got near-collisions for the full hash function with Hamming weight 42.
  However, this modular difference is not as effective when it comes to solving branch 1.

Example of a near-collision for the full hash function with weight 42

M	2d4458a4	57976f57	3e44cfd9	1ab54cb2	7ec11870	173f6573	6141c261	7db20d3e
	2feeb74d	5fac87a6	61a73fa1	3454b23d	451d389b	78f061ec	7c32fb06	57ef1928
	79dcd071	39dc97f0	3a1bff42	031d364c	fef000e6	40873ef5	d0741256	649430cf
	97ef5538	3eab6a7e	b4f9cf72	9eba8257	4e84d457	5a6c49b6	ad1d9711	0f69afa2
M'	2d4458a4	57976f57	3e44cfd9	1ab54cb2	7ec11870	173f6573	6141c261	7db20d3e
	2feeb74d	5fac87a6	61a73fa1	3454b23d	451d389b	78f061ec	7c32fb06	57ef1928
	79dcd071	39dc97f0	3a1bff42	031d364c	fef000e6	40873ef5	d0741256	649430cf
	97ef5538	3eab6a7e	b4f9cf72	9eba8257	8df0c460	5a6c49b6	ad1d9711	0f69afa2
diff	**00000000**	**83480012**	**32b4070c**	**681a1279**	**648600ad**	**00000000**	**00000000**	**00000000**

Summary

We exploited a particular weakness of the step transformation of FORK-256 to cryptanalyse the function. We showed

- how to find micro-collisions efficiently,
- how to look for high-level differential paths,
- how to combine those two steps to produce near-collisions efficiently and evaluated the complexity of getting collisions at $2^{126.6}$ using small amount of memory
- that using large memory we can find collisions in $2^{109.6}$,
- how to extend the attack to the full hash function (with predefined IV),
- that using truncated versions of FORK is extremely risky.

You can download our program that finds near-collisions from:

http://www.ics.mq.edu.au/~kmatus/FORK

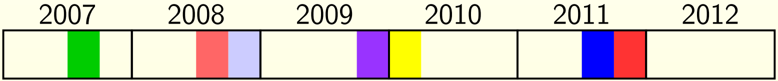
Part II: Some comments on the current state of hash functions research

- Current situation in the world of hash functions
- NIST call for new hash functions
- Do we know what we want?
- How to deal with the situation?

Current situation in the world of hash functions

- MD5 really should not be used for applications requiring collision resistance
  - Colliding certificates with identical MD5 hashes [Stevens, Lenstra, de Weger; EUROCRYPT'07]
  - Practical attacks on APOP protocol exploiting MD5 collisions [Sasaki, et al., Leurent; FSE'07]
- closer and closer to actual SHA-1 collisions
  - automatic path-finding [De Cannière, Rechberger; ASIACRYPT'06]
  - colliding pairs for 70 steps of SHA-1 [De Cannière, Mendel, Rechberger; Rump session FSE'07]
- SHA-256 seems to be OK so far
  - best attacks can break $\approx$ 20 rounds out of 64 [FSE'06]
  - but trust in such designs is undermined

NIST competition for new hash functions : 2007



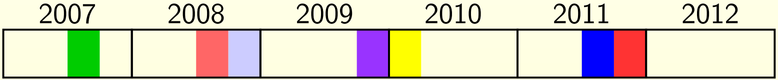
1Q Publish the preliminary minimum acceptability requirements, submission requirements, and evaluation criteria for public comments. Present the draft minimum acceptability requirements, submission requirements, and evaluation criteria for candidate hash functions at the RSA Conference and at FSE 2007.

4/27/07 Public comment period ends.

2Q Resolve comments.

3Q Finalize and publish the minimum acceptability requirements, submission requirements, and evaluation criteria for candidate hash functions. Request submissions for new hash functions.

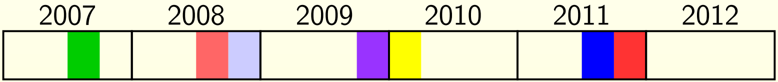
NIST competition for new hash functions : 2008



3Q Submission deadline for new hash functions.

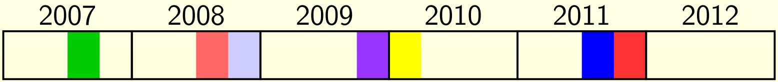
4Q Review submitted functions, and select candidates that meet basic submission requirements.
Host the First Hash Function Candidate Conference to announce first round candidates. Submitters present their functions at the workshop. Call for public comments on the first round candidates.

NIST competition for new hash functions : 2009



- 4Q Public comment period ends. Note: Depending on the number and quality of the submissions, NIST may either extend the length of the initial public comment period to allow sufficient time for the public analysis of the candidate algorithms, or may include additional rounds of analysis in order to successively reduce the number of candidate algorithms for further consideration as finalist algorithms.
- 4Q Hold the Second Hash Function Candidate Conference. Discuss the analysis results on the submitted candidates. Submitters may identify possible improvements for their algorithms

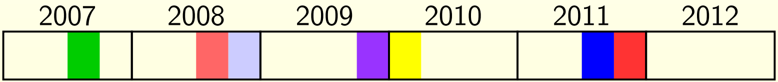
NIST competition for new hash functions : 2010



1Q Address the public comments on the submitted candidates; select the finalists. Prepare a report to explain the selection. Announce the finalists. Publish the selection report.

2Q Submitters of the finalist candidates announce any tweaks to their submissions. Final round begins.

NIST competition for new hash functions : 2011



2Q Public comment period for the final round ends.

3Q Host the Final Hash Function Candidate Conference.
Submitters of the finalist algorithms discuss the comments on their submissions.

4Q Address public comments, and select the winner. Prepare a report to describe the final selection(s).
Announce the new hash function(s).

Proposed Draft Minimum Acceptability Requirements for Candidate Algorithms

A.1 The algorithm must be publicly disclosed and available on a worldwide, non-exclusive, royalty-free basis.

A.2 The algorithm must be implementable in a wide range of hardware and software platforms.

A.3 The algorithm must support 224, 256, 384, and 512-bit message digests, and must support a maximum message length of at least 2^{64} bits

Proposed Draft Evaluation Criteria of Candidate Algorithms

- Security,
- Computational efficiency,
- Memory requirements,
- Hardware and software suitability,
- Simplicity,
- Flexibility, and
- Licensing requirements.

Evaluation Criteria : C.1 Security

- The actual security provided by the algorithm as compared to other submitted algorithms (of the same hash length), including (but not limited to) first and second preimage resistance, collision resistance, and resistance to generic attacks (e.g., length extension).
- The extent to which the algorithm output is indistinguishable from a random oracle.
- The soundness of the mathematical basis for the algorithms security.
- Other security factors raised by the public during the evaluation process ...

Do we really know what we want?

Typical definition:

> *Collision resistance – it must be computationally infeasible to find $x_1 \neq x_2$ such that $h(x_1) = h(x_2)$.*

In theory: we define it in complexity theory model using infinite families of functions [Damgård, EUROCRYPT'87] or concrete security setting using finite families and advantage of adversary in computational games [Rogaway, Shrimpton, FSE'04]

In practice: we usually have only a single fixed instance of the function and the only security notion we can speak of is "human ignorance" model [Rogaway, VIETCRYPT'06]

There is a serious gap between theory and practice

Do we really know what we want?

Federal register specs. C.1. :

> *The extent to which the algorithm output is indistinguishable from a random oracle.*

But what does it mean? Any fixed function is trivially distinguishable from a random oracle...

Intuitively, the output should "look random", but more precise definition is necessary.

How to deal with the situation : a suggestion

- Look at the applications and extract precise security requirements (there will be more than 3!)
  - password hashing
  - commitments
  - deterministic message hashing for signatures
  - randomized message hashing for signatures
  - HMAC
  - FIPS-186 PRG
  - ...
- Express those security requirements in a formal (but realistic) way (concrete security setting seem to be suitable)
- Design appropriate function families that satisfy required properties
  - Looks like the approach "one hash to fit them all" is unrealistic
  - Some protocols may need to be fixed

How to deal with the situation : discussion

Advantages:

- bridging the gap between theory and practice
- sound security notions in practice
- not relying on “human ignorance” only – we may prove security reductions

Problems:

- no single, simple solution “SHA replacement”
- in some situations more complex designs (keyed families)
- some protocols may need modification - high cost, compatibility issues

If we want to change our approach, this is the right time!

Thank you!

Additional slides

[just in case of questions about details]

Functions f and g

$$f(x) = x \boxplus (x^{\lll 7} \oplus x^{\lll 22}) \ ,$$
$$g(x) = x \oplus (x^{\lll 13} \boxplus x^{\lll 27})$$

Results of the search

Scenario	Branches	m	Differences in	active Q-structures
Pseudo-collisions	1,2,3,4	5	H_0, M_2, M_{11}	12:000, 25:000, 35:001, 41:001, 51:010
Collisions	1,2,3,4	6	M_{12}	13:000, 31:001, 40:000, 47:100, 50:000, 57:000
Pseudo-collisions	1,2,3	2	B_0, M_{12}	8:100, 24:000
	1,2,4	3	H_0, M_{11}	3:000, 51:010, 60:000
	1,3,4	3	H_0, M_2	35:001, 44:000, 51:000
	2,3,4	3	D_0, M_9	36:010, 43:000, 52:000
Collisions	1,2,3	3	M_0, M_3, M_9	1:001, 20:010, 39:100
	1,2,4	4	M_1, M_2	2:001, 9:000, 25:100, 51:000
	1,3,4	5	M_9	10:000, 39:001, 42:001 43:010, 59:000
	2,3,4	5	M_3, M_9	20:010, 27:000, 39:000 57:000, 59:010

Legend: 47:100 means that the 47-th Q-structure is modelled with coefficients $(q_B, q_C, q_D) = (1, 0, 0)$.

Collisions: improving efficiency using large tables

We can use such precomputed tables to speed up the algorithm.

- In branch 3 we can use one to control the thread $C_{3,1} \to D_{3,2}$ through M_{10}
- In branch 1 we use a family of such tables T_a for some (best) allowable values a. For a fixed a, $T_a(G_{1,4}, M_{11} + E_{1,5})$ returns the value of M_9 that gives us $A_{1,6} = a - M_{12}$
- For that allowable value a we get a micro-collision with probability $2^{-8} \sim 2^{-9}$. So after 512 lookups we expect to get a micro-collision.
- If 1 look-up = 1 op (e.g. ADD) then this takes 1/2 FORK and we have $\approx 3/2$ FORK per single solution.