Extending FORK-256 Attack to the Full Hash Function

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Outline

Introduction

- FORK-256 is a dedicated cryptographic hash function designed by Hong et al. and presented during second NIST hash workshop and FSE 2006.
- Heuristic design, but with some unorthodox design choices.
- Meant as a possible replacement for SHA-256 (compatible interface, better speed).

History of cryptanalysis of FORK-256

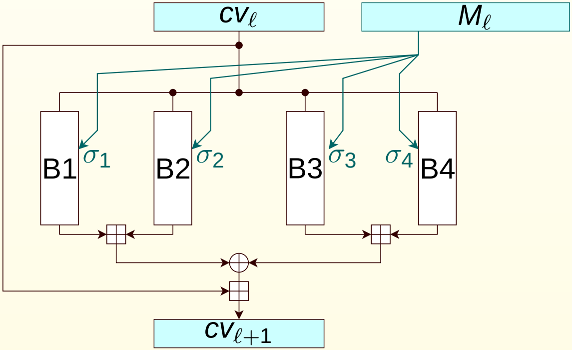
Received considerable cryptanalytic attention since it was proposed in 2006.

- Matusiewicz, Contini, Pieprzyk – IACR ePrint 2006/317
  cryptanalysis of reduced variants
- Mendel, Lano, Preneel – CT-RSA 2007
  cryptanalysis of reduced variants
- Matusiewicz, Peyrin, Billet, Contini, Pieprzyk – FSE 2007
  cryptanalysis of the full compression function

Our current contribution: Extending the attack to the full hash function (actually, with any predefined IV).

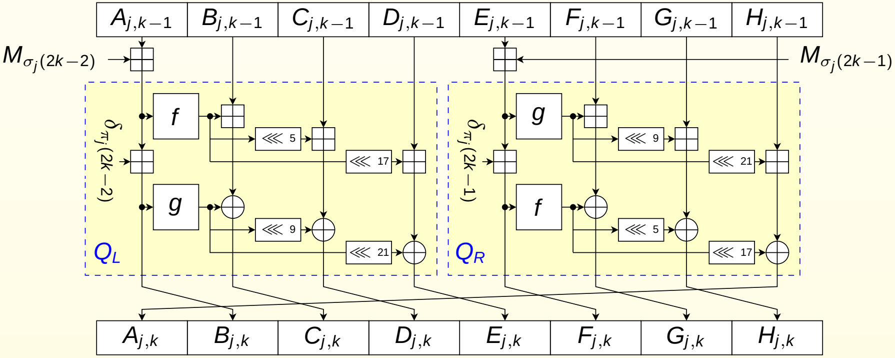
Outline

FORK-256



- 256 bits of chaining variable *cv*
- 512 bits of message *M*
- each branch uses a different permutation $(\sigma_1, \sigma_2, \sigma_3, \sigma_4)$ of message words $M_0, \ldots, M_{15}$
- each branch B1, B2, B3, B4 consists of **8 steps**

Structure of FORK-256 : step transformation



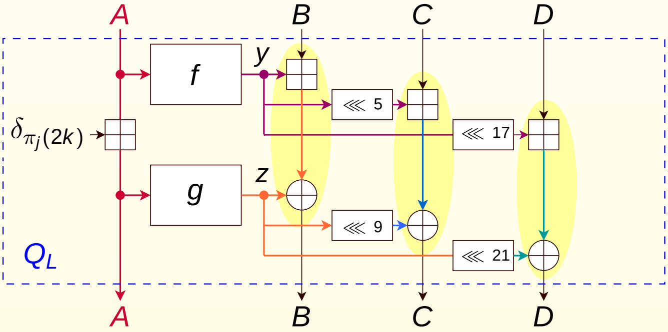
- there are 8 steps in each branch
- step transformation – composition of 3 simple operations
  - addition of two different message words
  - two parallel **Q-structures**
  - rotation of registers

Functions f and g

$$f(x) = x + \left(x^{\lll 7} \oplus x^{\lll 22}\right)$$
$$g(x) = x \oplus \left(x^{\lll 13} + x^{\lll 27}\right)$$

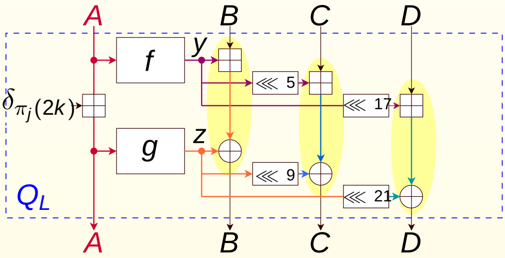
Outline

Micro-collisions in the step transformation



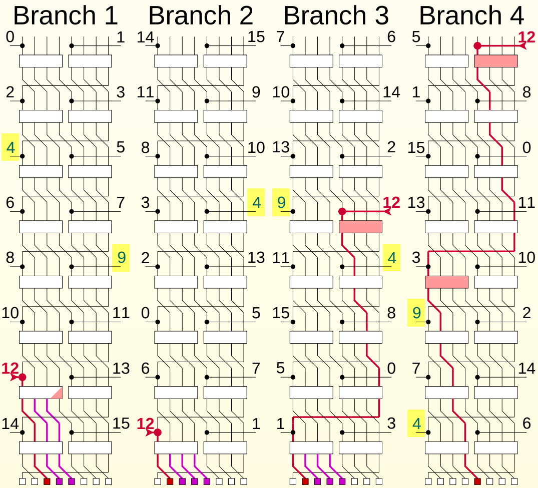
Micro-collision: a difference in register A does not propagate to the selected register (B,C,D).
If it does not propagate to more than one other register we have *simultaneous micro-collisions*.

Micro-collisions



- Let us fix a modular difference d. Having a value a of register A and $a' = a + d$, we can efficiently determine sets $\mathcal{B}_a$, $\mathcal{C}_a$, $\mathcal{D}_a$ of values of B, C, D such that simultaneous micro-collisions appear in all three lines.
- If sets $\mathcal{B}_a$, $\mathcal{C}_a$, $\mathcal{D}_a$ are non-empty, we call such a an allowable value (meaning we can achieve micro-collisions for that value of a)

Using micro-collisions in a differential path



We need microcollisions in only three and 1/3 Q-structures.

Only four output registers are influenced by the differential.

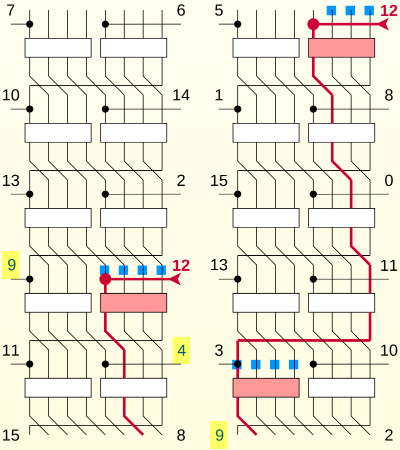
Using a difference with only 13 MSB set we reduce this to 108 bits.

$d = \texttt{0xdd080000}$ or $d = \texttt{0x22f80000}$

Collisions: the principle of the attack

- Get three micro-collisions in branches 3 and 4.
  This leaves two message words M_4 and M_9 free, the rest is fixed
- Using different values of M_4 and M_9 compute branch 1 and hope that there is a single micro-collision in strand D in step 7.
- If a micro-collision there is found, compute the rest of the function and check the output difference.
  Note that the output differences have weights always ≤ 108

Obtaining micro-collisions in branches 3 and 4



To deal with branches 3 and 4 we have to:

1) Set values of

$$F_0^{(4)}, G_0^{(4)}, H_0^{(4)}.$$

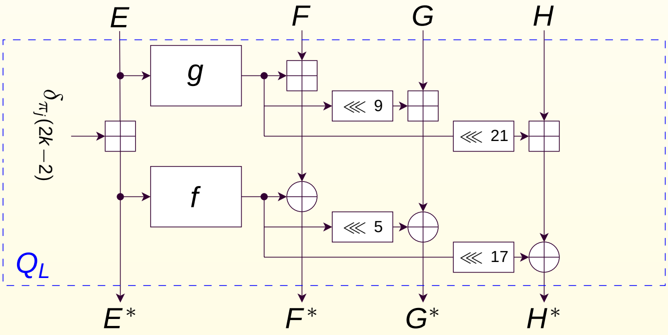
2) Set values of

$$A_4^{(4)}, B_4^{(4)}, C_4^{(4)}, D_4^{(4)}$$

3) Set values of

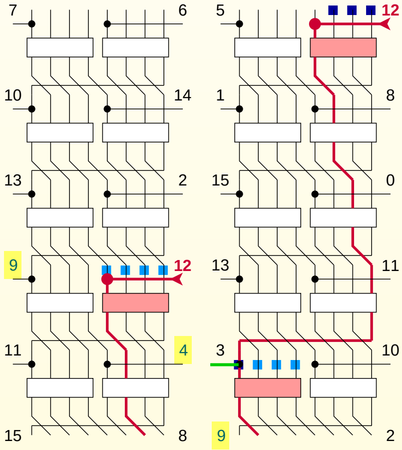
$$E_4^{(4)}, F_3^{(3)}, G_3^{(3)}, H_3^{(3)}$$

A property of Q-structures



- We can set E^* to any value by adjusting the value of E
- We can set F^* to any value by adjusting the value of F (true for G, H too).

Adjusting the values: branch 4 before step 5



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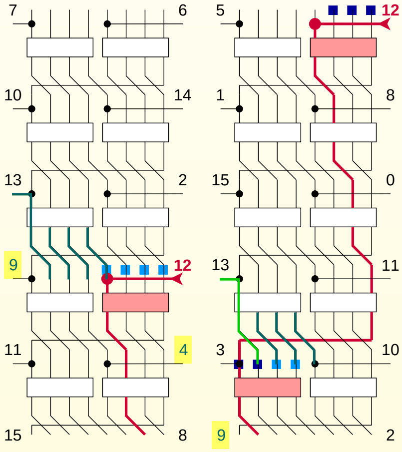

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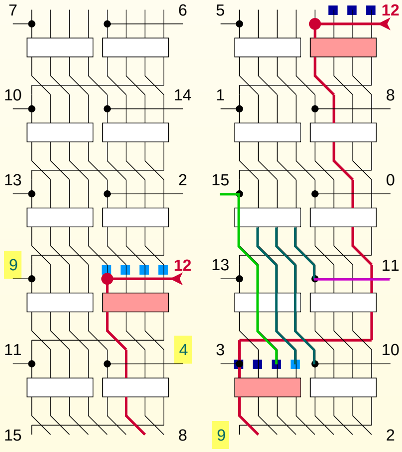

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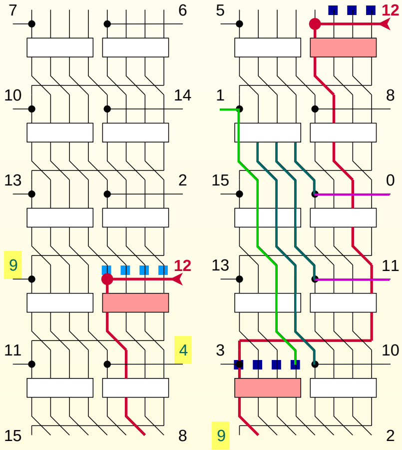
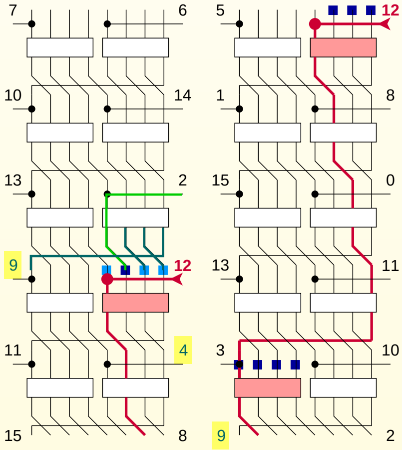


Adjusting the values: branch 3 before step 4



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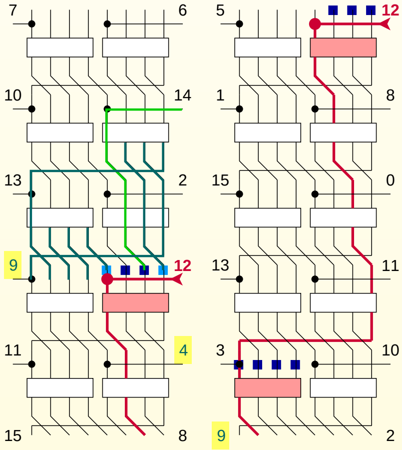

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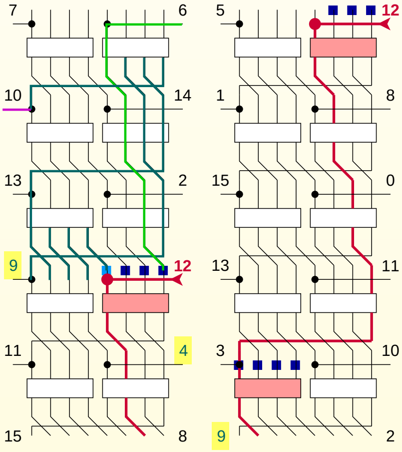
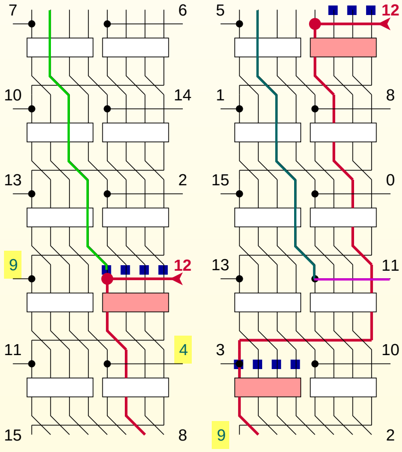
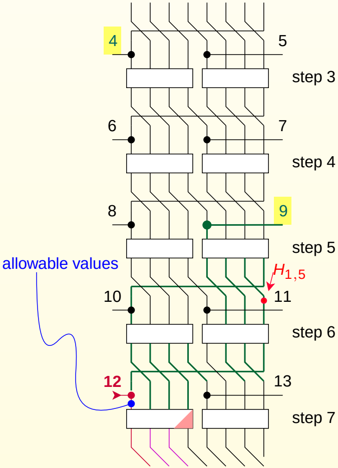


Adjusting the values: branch 3 before step 4



Passing branch 1



We use free words M_4 and M_9 to search for the values that yield a single micro-collision in step 7.

Outline

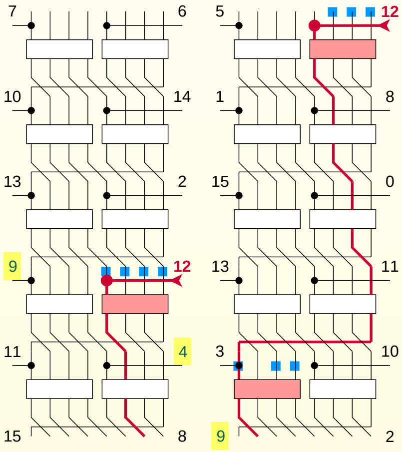
Why we are not happy?

Problem: During the attack, we have to adjust the value of B_0.

- This value depends on other values of message words so we cannot know it in advance or precompute it.
- We cannot use this method to obtain (near)collisions for the full hash function that needs a predefined IV.

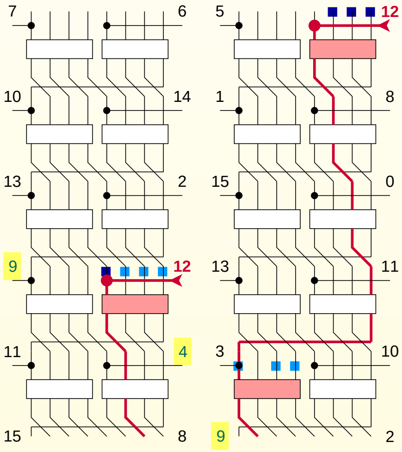
- We want to extend the attack to the full hash function.
- We want to avoid the need for modification of B_0.

Better message adjustment strategy



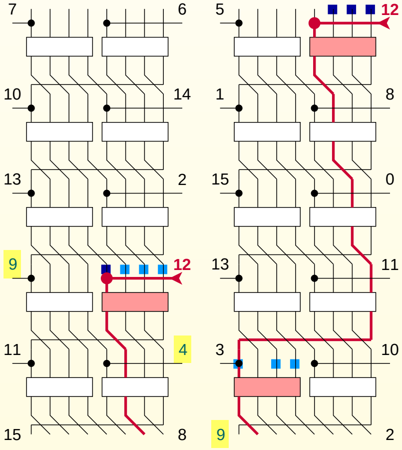
- Solve branch 4 step 1
- Deal with whole branch 3: use M_{13} to preserve the value of $E_3^{(3)}$
- Finish with branch 4
- In branch 4 leave a single micro-collision in $C_5^{(4)}$ to chance

New message adjustment: branch 4 step 1



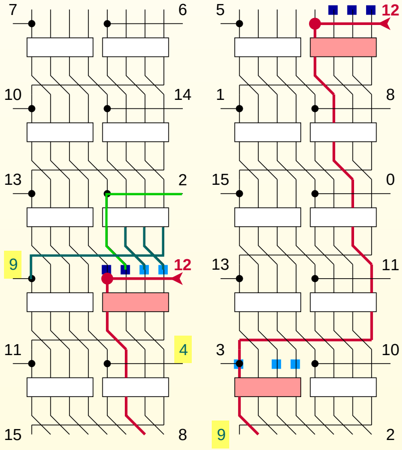
- Find a value x_1 that may lead to simultaneous microcollisions in Q_R in step 1 of branch 4
- Pick appropriate constants for F_0, G_0, H_0
- Set M_{12} to $E_0 - x_1$, $M'_{12} = M_{12} + d$

New strategy: branch 3 (1)

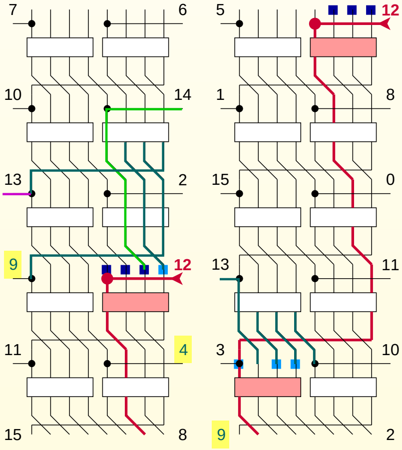


- Pick message words M_7, M_6, M_{10}, M_{14}, M_{13}, M_2 randomly and compute until step 4
- If $E_4^{(3)} + M_{12}$ does not allow for finding simultaneous micro-collisions, start over. [We need around 2^{23} trials]
- When it does, keep that value and later adjust values of $F_4^{(3)}$, $G_4^{(3)}$, $H_4^{(3)}$

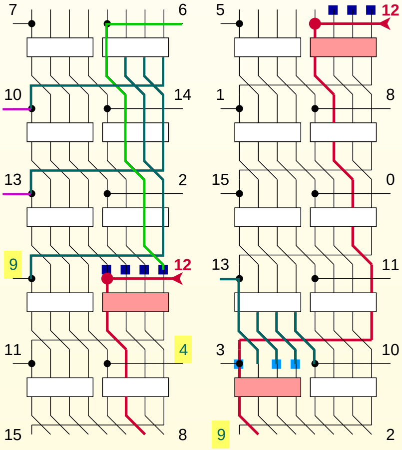
New strategy: branch 3, adjusting $F_3^{(3)}$



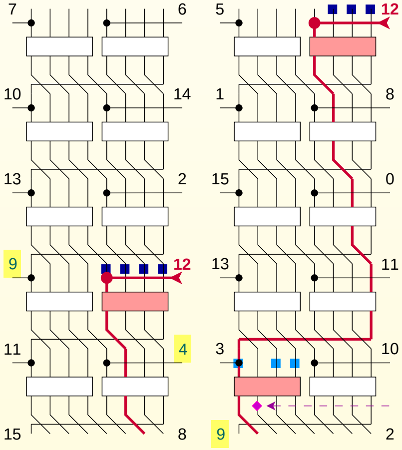
New strategy: branch 3, adjusting $G_3^{(3)}$



New strategy: branch 3, adjusting $H_3^{(3)}$

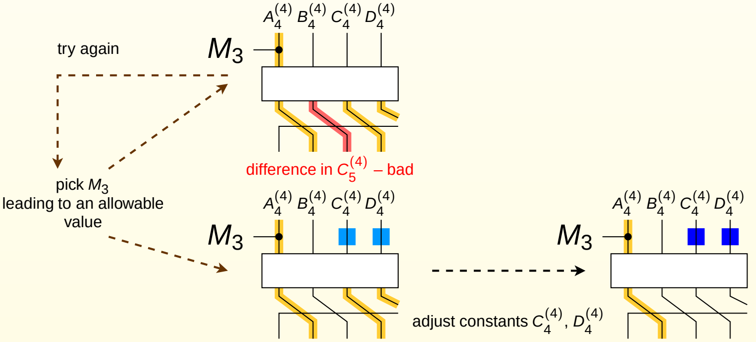


New strategy: branch 4



- Pick M_5, M_1, M_{15} randomly.
- Pick M_8, M_0, M_{11} that preserve the modular difference.
- Compute up to step 4.
- Keep picking allowable values of M_3 and testing if there is no difference in $C_5^{(4)}$
- Once we find a good value of M_3, we can adjust constants $C_4^{(4)}$ and $D_4^{(4)}$

Passing step 5 in branch 4



- We need around 2^{19} trials like that

New strategy: complexity of getting close hashes

- Work effort of passing branches 3 and 4 for using the difference $0x22f80000$:

$$< 2^{24} \cdot 2^{19} \cdot 2^{-3} = 2^{40}$$

FORK evaluations.
- The second phase of the attack (branches 1 and 2) dominates with the complexity of 2^{58}.
- Conclusion: The new strategy for dealing with branches 3 and 4 does not affect the total complexity of the attack.

Fixing appropriate chaining values F_0, G_0, H_0

- We removed the need for the fourth initial chaining value to be used.
- The three values F_0, G_0, H_0 have to be set to one of possible constants required by simultaneous micro-collisions in step 1 of branch 4.
- If we require three particular values, we can achieve this in 2^{96} FORK evaluations by hashing a random prepended message block
- In fact we can do much better: any of the set of good constants for each register will do.

Estimating the probability of getting good constants

- Let $\mathcal{F}_a$, $\mathcal{G}_a$, $\mathcal{H}_a$ denote sets of constants that yield a micro-collision in line F, G, H for an allowable value a.
- Probability that a random IV value will match one of those values for each register F, G, H is

$$P = 1 - \prod_{a \in \mathcal{A}} \left(1 - \frac{|\mathcal{F}_a| \cdot |\mathcal{G}_a| \cdot |\mathcal{H}_a|}{2^{96}} \right)$$

- For original differences $d = \texttt{0xdd080000}$ and $d = \texttt{0x22f80000}$ it is equal to $P = 2^{-64.8}$,
- for other differences it may be much bigger, e.g. for $d = \texttt{0x3f6bf009}$ we have $P = 2^{-21.7}$.

Outline

New FORK-256

Modified version designed to counter attacks exploiting micro-collisions.

- Different functions f, g
- Modified step transformation

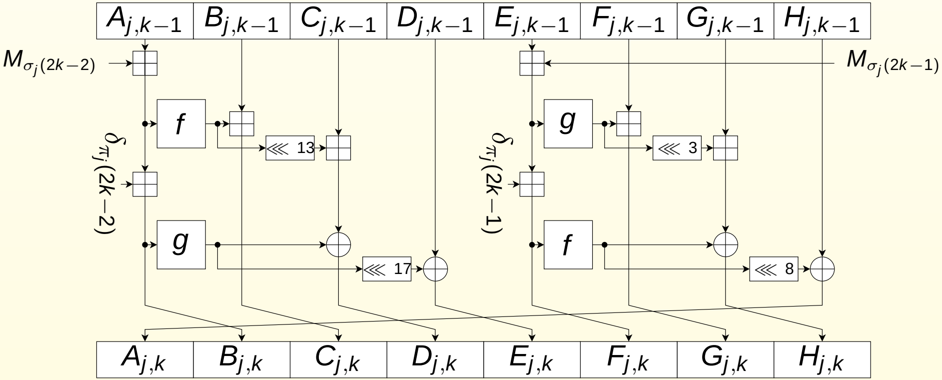
New FORK-256: Functions f and g

$$f(x) = x \oplus x^{\lll 15} \oplus x^{\lll 27}$$
$$g(x) = x \oplus \left(x^{\lll 7} \boxplus x^{\lll 25}\right)$$

- f is a bijection

New FORK-256: Step transformation

New step transformation:



New FORK-256: Rationale

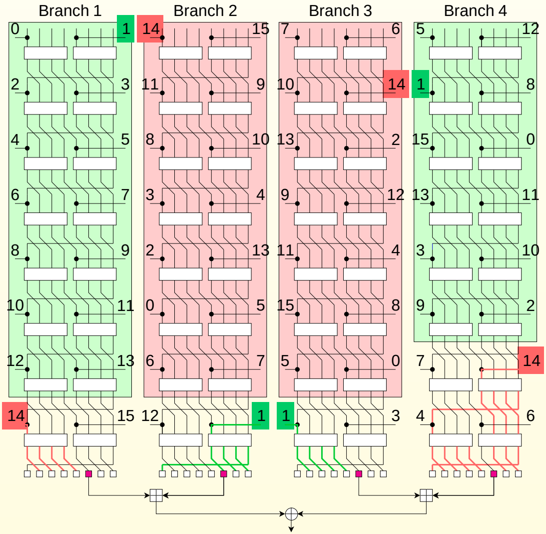
Impossible to get step differentials of the form

$$(\Delta A, 0, 0, 0, 0, 0, 0, 0) \rightarrow (0, \Delta B, 0, 0, 0, 0, 0, 0)$$

- not possible to get collisions in both f and g
- not possible to get micro-collisions: difference in A or$E$ propagates to at least two registers

Saarinen's meet-in-the-middle attack

- It is possible to produce hashes that have a fixed value of register F equal to the initial value of F_0.
- This effectively reduces the range of the function to 2^{224} possible outputs.
- If we can generate such hashes efficiently enough, we can mount birthday attack on the whole hash function.



Ouput F is equal to F_0 iff

$$F_8^{(1)} \boxplus F_8^{(2)} = F_8^{(3)} \boxplus F_8^{(4)}$$

Equivalently:

$$F_8^{(2)} \boxminus F_8^{(3)} = F_8^{(1)} \boxminus F_8^{(4)}$$

Note that:

- B.(1): $F_8^{(1)}$ doesn't depend on M_{14}
- B.(2): $F_8^{(2)}$ is a linear function of M_1
- B.(3): $F_8^{(3)}$ doesn't depend on M_1
- B.(4): $F_8^{(4)}$ doesn't depend on M_{14}

Outline of the attack

- Set random values of message words M_i, $i = 0, 2..13, 15$
- Set M_0 to zero
- For each value of $M_{14} = 0, \ldots, 2^{32} - 1$:
  - compute branches 2 and 3 to obtain

$$x = F_8^{(2)} \boxminus F_8^{(3)}$$

  - Add the pair (x, M_{14}) to a dictionary (hash table)
- For each value of $M_1 = 0, \ldots, 2^{32} - 1$:
  - compute branches 1 and 4 to get

$$y = F_8^{(1)} \boxminus F_8^{(4)} \boxplus M_1$$

  - if $x = y$, output the value M_1 with corresponding value(s) of M_{14}

Complexity of the attack

- For the effort of $3/2 \cdot 2^{32}$ we get around 2^{32} “restricted” hashes
- We need to repeat the procedure $\sqrt{\pi/2 \cdot 2^{224}} = \sqrt{\pi/2} \cdot 2^{112}$
- Total expected complexity of $\approx 3/2\sqrt{\pi/2} \cdot 2^{112} \approx 2^{112.9}$
- Memory requirements of the same order

Outline

Conclusions

- Presented an improved attack on FORK-256
  - finds collisions for any value of IV
  - breaks the full hash function
  - practical for finding near-collisions
- New FORK-256
- And new attacks...

Thank you!