INFORMAL WORKSHOP ON GEOMETRIC AND TOPOLOGICAL ANALYSIS

Friday, April 1. 2022, Matematicum, room 134, building 303B

Time	Speaker
10:00 – 10:45	Pat McDonald
11:00 – 11:45	Martin Raussen
12:00 – 13:00	Lunch
13:00 – 13:45	Kim Knudsen
14:00 – 14:45	Hjørdis Amanda Schlüter
15:00 – 15:45	Andreas Bærentzen
18:00 –	Dinner

TITLES AND ABSTRACTS:

- Pat McDonald, New College of Florida:
  Heat content, geometric analysis and inverse problems.
  Heat content is an invariant of piecewise smoothly bounded Riemannian domains. Starting with definitions we will survey results which address the relationship between the heat content of a domain and its geometry. The material will involve both forward results (in which constraints on geometry imply constraints on heat content) and inverse results (in which constraints on heat content imply constraints on geometry). In particular, we compare and contrast the behavior of heat content and Dirichlet spectrum in the context of inverse problems.

- Martin Raussen, Department of Mathematical Sciences, Aalorg University:
  Spare capacity as a measure in concurrent computation: Simple topological techniques applied to a combinatorial/geometric model.
  Higher Dimensional Automata (HDA) are higher dimensional relatives to transition systems in concurrency theory taking into account to which degree various actions commute. Mathematically, they take the form of labelled cubical complexes. It is important to know, and challenging from a geometric/topological perspective, whether the space of directed paths (corresponding to executions) between two vertices (states) is connected; more generally, to estimate higher connectivity of these path spaces. The talk will present an approach for such an estimation for particularly simple HDA taking into

account the access of a number of processors to a number of resources with given limited capacity each. It defines the spare capacity *of a concurrent program with prescribed periods of access of the processors to the resources using only the syntax of individual programs and the capacities of shared resources. It shows that the connectivity of spaces of directed paths can be estimated (from above) by spare capacities. The key theoretical ingredient is a transition from the calculation of local connectivity bounds (of the upper links of vertices of an HDA) to global ones by applying a version of the combinatorial/topological nerve lemma due to Anders Björner.*

- Kim Knudsen, DTU Compute, Scientific Computing:
  Electromagnetic imaging: Inverse problems, mathematical analysis and computations.
  The inverse problem in Electrical Impedance Tomography (EIT), mathematically known as the Calderón problem, is known to be extremely ill-posed, and hence any reconstruction from noisy data suffers from low resolution and low contrast. Recently new ideas have emerged that appear to resolve the issues by utilizing interior information that in principle can be computed using so-called hybrid data from other imaging devices. One such combined tomographic modality is known as Acousto-Electric Tomography (AET) and makes use of both ultrasonic waves and EIT simultaneously. Another modality relies on the joint use of Magnetic Resonance and EIT (MREIT). The combination of the physical waves gives rise to new and challenging mathematical questions of both theoretical and computational nature. In this talk the basic difficulties in EIT will be discussed and the mathematical problem of AET and MREIT will be introduced. The fundamental questions will be posed and (partially) answered. In particular we will focus on the anisotropic problem and geometric nature of this.

- Hjørdis Amanda Schlüter, DTU Compute, Scientific Computing:
  Reconstructing conductivities on conductive two-dimensional Riemannian manifolds.
  Reconstructing the electrical conductivity from internal data is an inverse problem typically studied in Euclidean domains. We lift the Euclidean two-dimensional setting to compact 2D Riemmanian manifolds with a smooth boundary. This setting defines a natural conductive Laplacian on the manifold and hence also voltage potentials, current fields and corresponding power densities arising from suitable boundary conditions. Motivated by Acousto-Electric Tomography we show that if the manifold has genus zero and the metric is known, then the anisotropic conductivity can be recovered uniquely and constructively from knowledge of a few power densities. We illustrate the reconstruction procedure numerically by an example of a conductivity on a non-simply connected surface in three-space.

- Andreas Bærentzen, DTU Compute, Visual Computing:
  Four pathways to the Laplacian on a triangular net.
  The Laplacian has been formulated in a number of ways for triangle meshes. The simplest approach is the so-called Umbrella Operator which is identical to the Graph Laplacian. Originally introduced to computer graphics by Gabriel Taubin as a tool for smoothing, the Umbrella Operator can indeed be used for effective mesh smoothing, but while it tends to improve mesh quality, the smoothing of geometry depends on properties of the local triangulation. In order to smooth geometry (almost) independently of the triangulation, a discrete Laplace-Beltrami Operator was introduced. We will discuss how to go from the continuous formula to a discrete LBO. Specifically, we will discuss the Finite Element discretization and the approach based the fact that the LBO applied to the coordinate functions is proportional to mean curvature. From a mathematical point of view, the FE discretization is arguably more elegant, but the mean curvature connection leads to a simpler presentation which is advantageous in the context of teaching. The talk closes with a number of examples of applications of the LBO to geometry processing ranging from feature descriptors over smoothing to maps between (nearly) isometric surfaces.