

Projekt RBC

Røde blodlegemer

Geometrisk analyse

Optimering

v./ Steen Markvorsen, DTU Compute, Mathematics

Intro

- ⦿ Størrelse og form



Intro

Red Blood Cells; Wikipedia

Matematik 1; eNote 23;

Matematisk fortælling om middelkrumning

Krumme fladers krumning

Modelling

⊙ Den kubiske model

Tre design parametre a , b , og c for en profil-kurve:

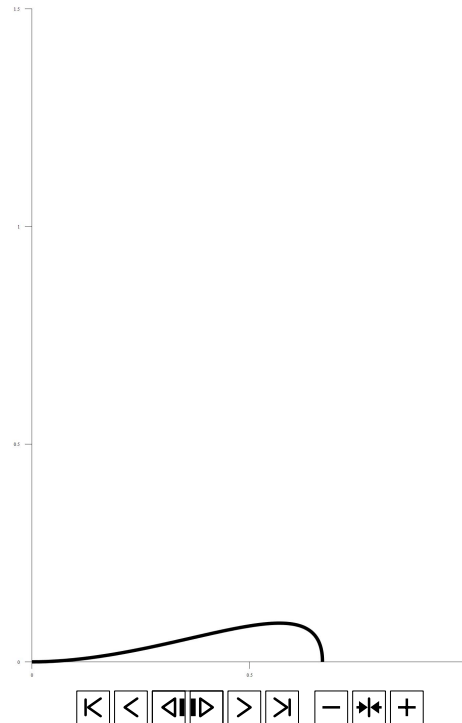
$$\gamma(u) = (g(u), 0, h(u)) \quad , \quad u \in [0, 1] \quad ,$$

$$g(u) = a \cdot (u - (1/3) \cdot u^3)$$

$$h(u) = b + c \cdot u^2 - (b + c) \cdot u^3$$

Modellering

- Nogle forskellige profilkurver:
Her for $a = 1$, variabel b , og $c = 0.6$:



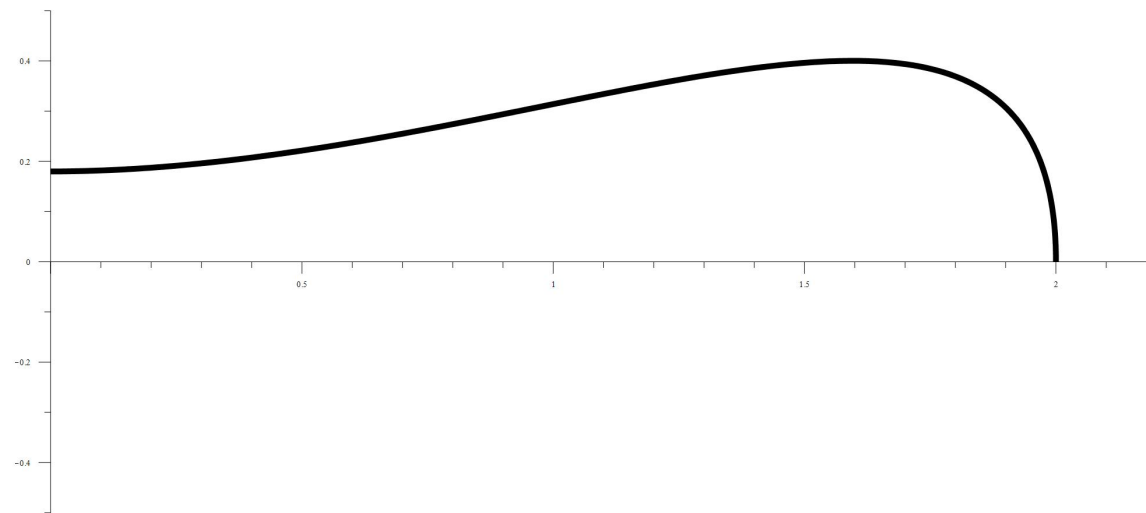
Modellering

- ⊙ Konstruktion af tilsvarende **omdrejnings-flader og -legemer**, se Matematik 1; eNote 23:

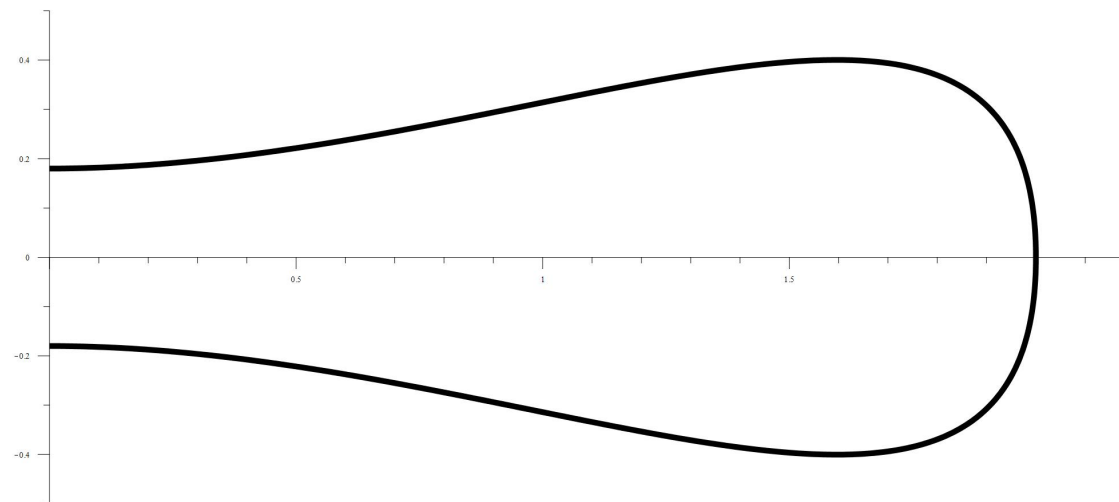
$$\begin{aligned}\mathbf{r}(u, v) &= (g(u) \cdot \cos(v), g(u) \cdot \sin(v), h(u)) \\ \mathbf{R}(u, v, w) &= (g(u) \cdot \cos(v), g(u) \cdot \sin(v), w \cdot h(u)) \quad ,\end{aligned}$$

hvor $u \in [0, 1]$, $v \in [-\pi, \pi]$ og $w \in [0, 1]$.

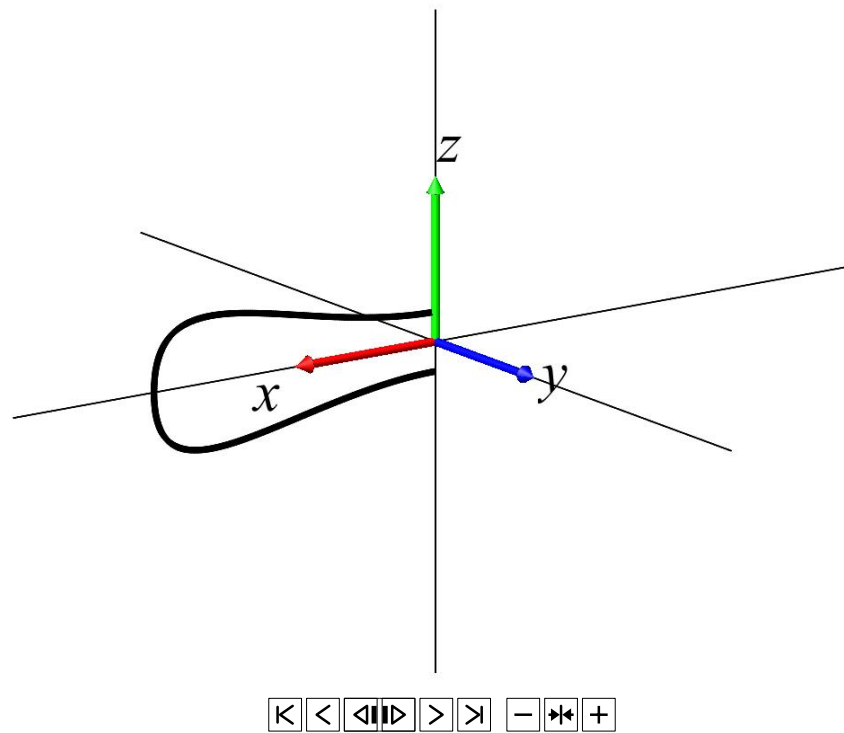
Modelling



Modelling

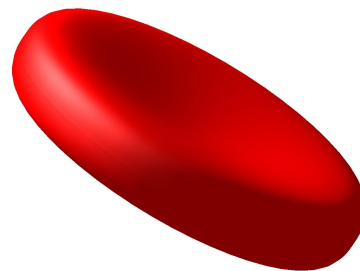
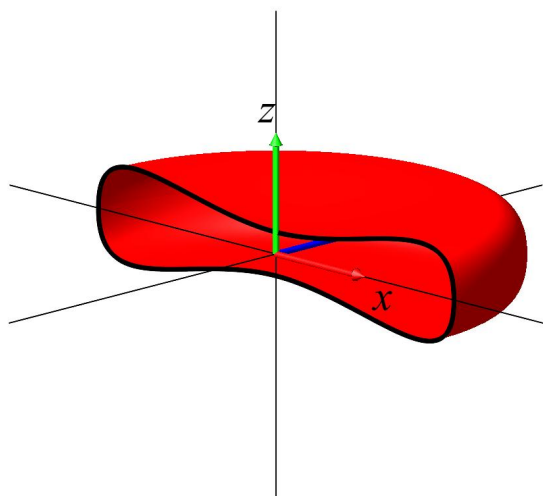


Modelling



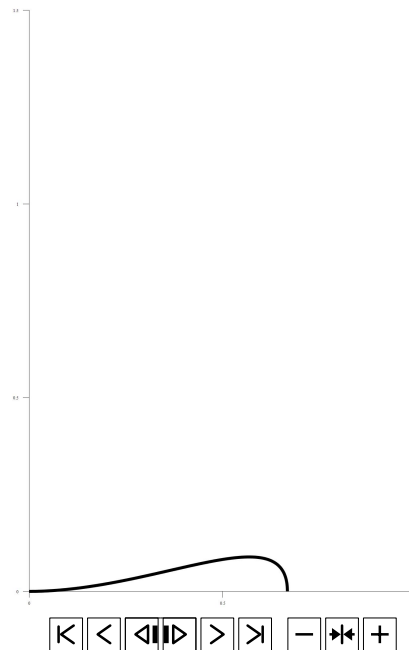
Modellering

- ⊙ Er det så et "korrekt" blodlegeme?



Modellering

- Nogle typiske profilkurver:
For typiske pattedyr, og for dværg-hjorte, lamaer, dromedarer og kameler – i den rækkefølge:



Modellering

⊙ For en given pattedyr-art betyder "korrekt":

1. Korrekt **overfladeareal** A – vi vælger $A = 20$
2. Korrekt **rumfang** V – vi vælger $V = 3$
3. Mindst mulig **overflade-energi** – vi vælger Canham-Helfrich-energi

Modelling

- ⊙ Overfladeareal: Matematik 1; eNote 23;

$$\begin{aligned} A &= \int_{F_{\mathbf{r}}} 1 \, d\mu \\ &= \int_{-\pi}^{\pi} \int_0^1 \text{Jacobi}_{\mathbf{r}}(u, v) \, du \, dv \end{aligned}$$

Modelling

- ⊙ Rumfang: Matematik 1; eNote 23;

$$\begin{aligned} V &= \int_{\Omega_{\mathbf{R}}} 1 \, d\mu \\ &= \int_{-\pi}^{\pi} \int_0^1 \int_0^1 \text{Jacobi}_{\mathbf{R}}(u, v, w) \, du \, dv \, dw \end{aligned}$$

Modelling

- ⊙ Canham-Helfrich energien

$$\begin{aligned} E_{CH} &= \int_{F_{\mathbf{r}}} (2 H_{\mathbf{r}})^2 d\mu \\ &= \int_{-\pi}^{\pi} \int_0^1 (2 H_{\mathbf{r}}(u, v))^2 \text{Jacobi}_{\mathbf{r}}(u, v) du dv \end{aligned}$$

Modellering

- ⊙ Hvordan beregnes middelkrumningen $H_{\mathbf{r}}(u, v)$ for fladen $\mathbf{r}(u, v)$?
- ⊙ Se Krumme fladers krumning
- ⊙ For omdrejningsflader afhænger H kun af u :

$$H_{\mathbf{r}}(u) = \frac{1}{2} \left(\frac{g'(u)h''(u) - g''(u)h'(u)}{(g'(u)^2 + h'(u)^2)^{3/2}} + \frac{h'(u) (g'(u)^2 + h'(u)^2)}{g(u) (g'(u)^2 + h'(u)^2)^{3/2}} \right)$$

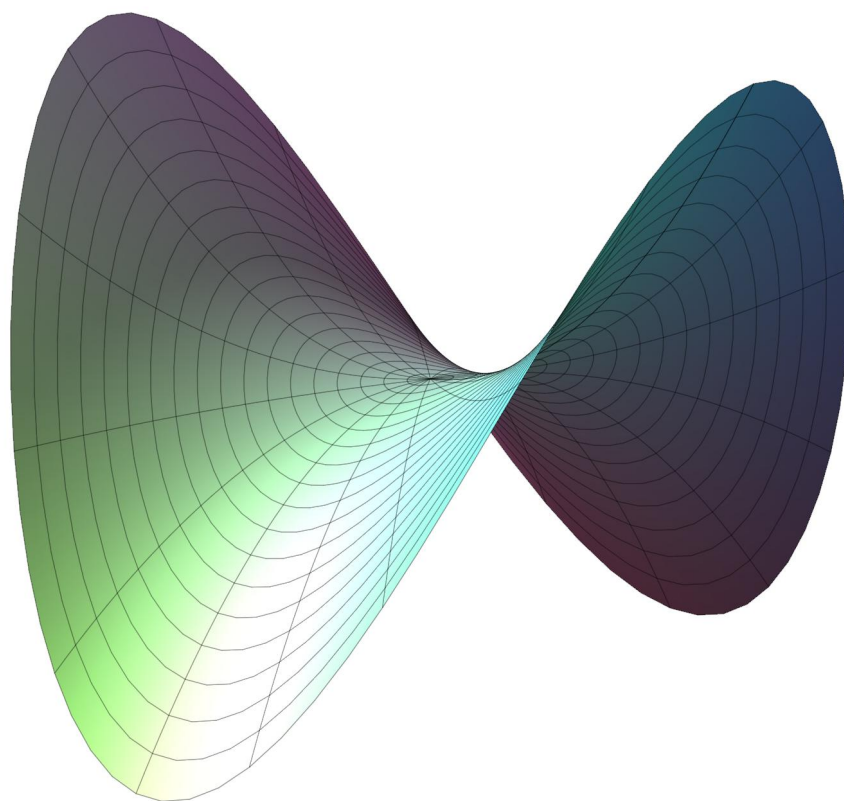
Modellering

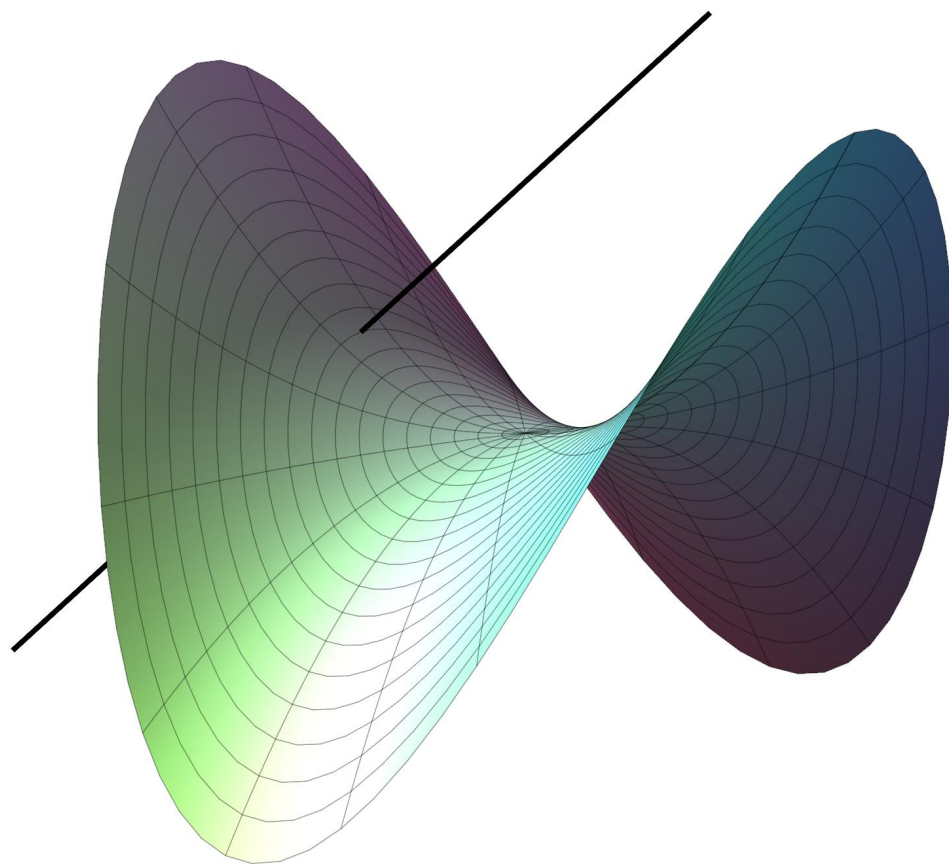
Husk, at profil-funktionerne $g(u)$ og $h(u)$ er forholdsvis simple:

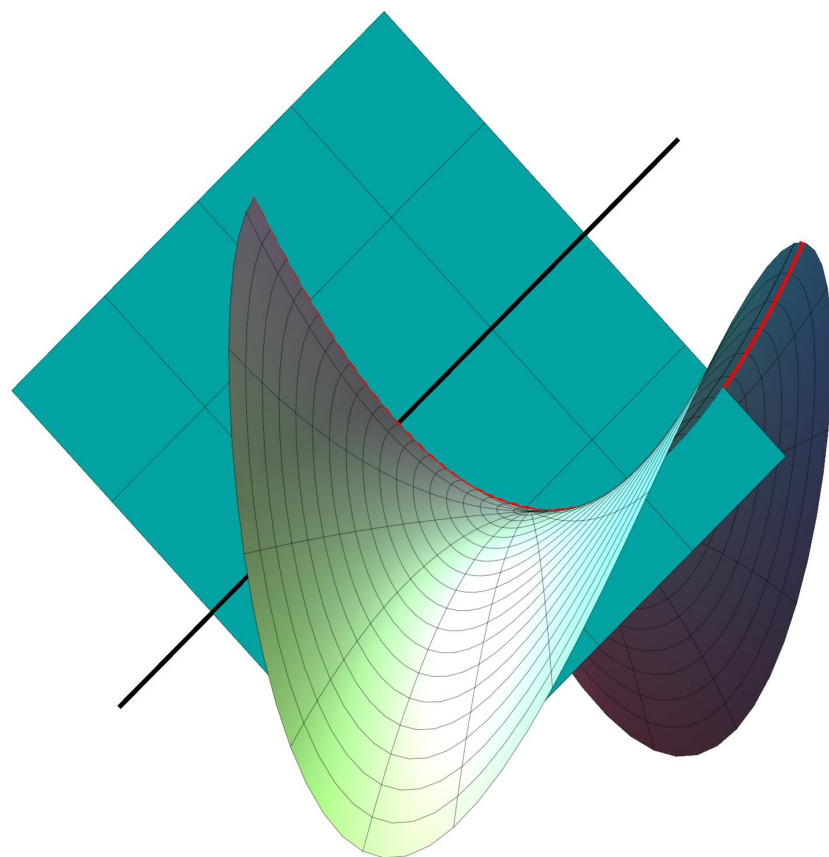
$$g(u) = a \cdot (u - (1/3) \cdot u^3)$$
$$h(u) = b + c \cdot u^2 - (b + c) \cdot u^3$$

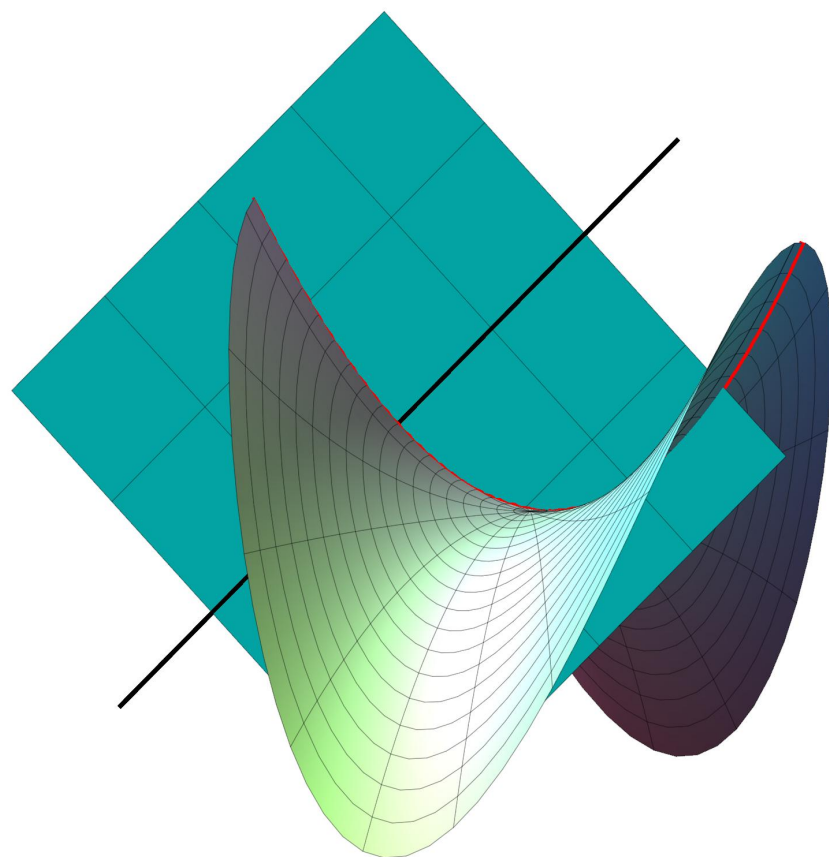
Lidt om middelkrumningen generelt

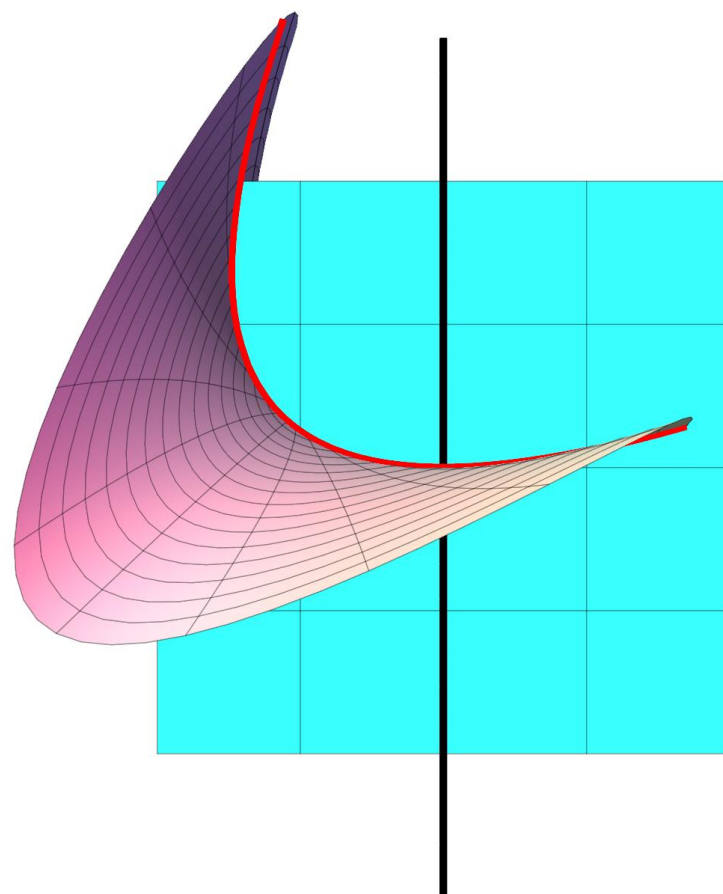
- ⊙ Fra en Matematisk fortælling om middelkrumning:

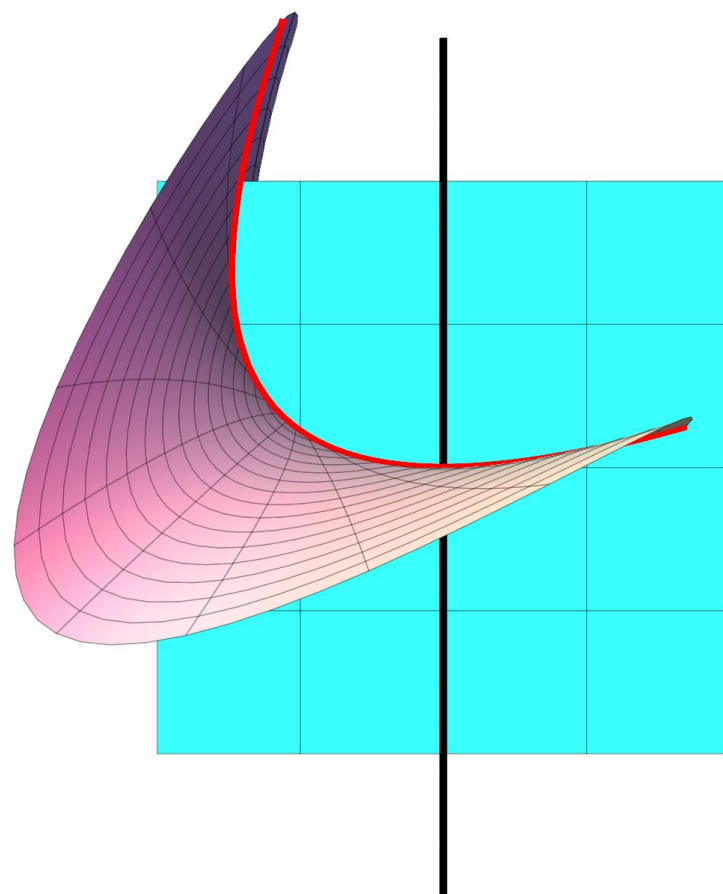


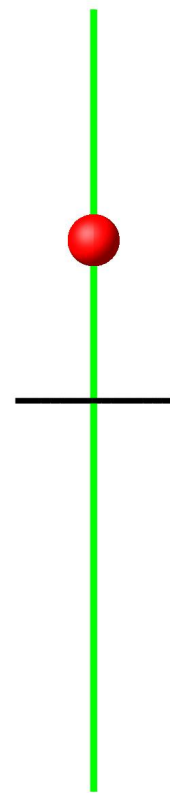
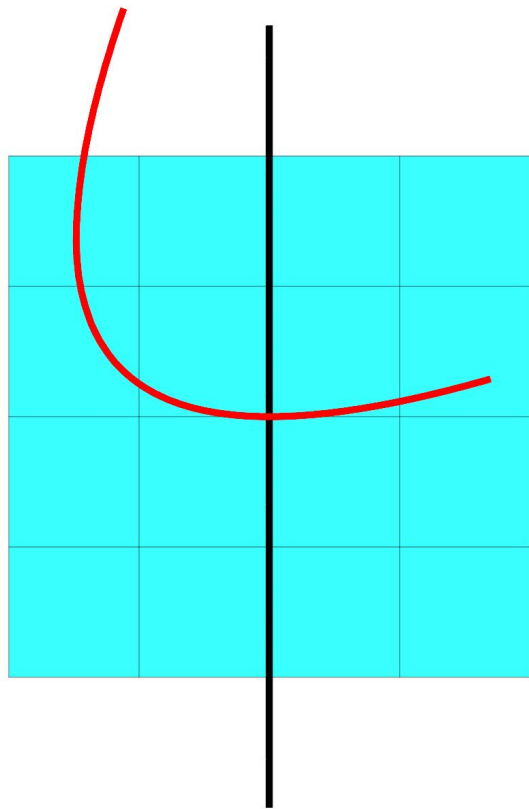


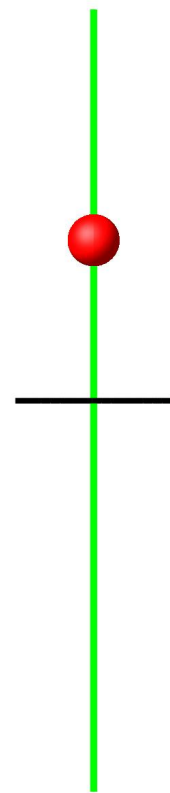
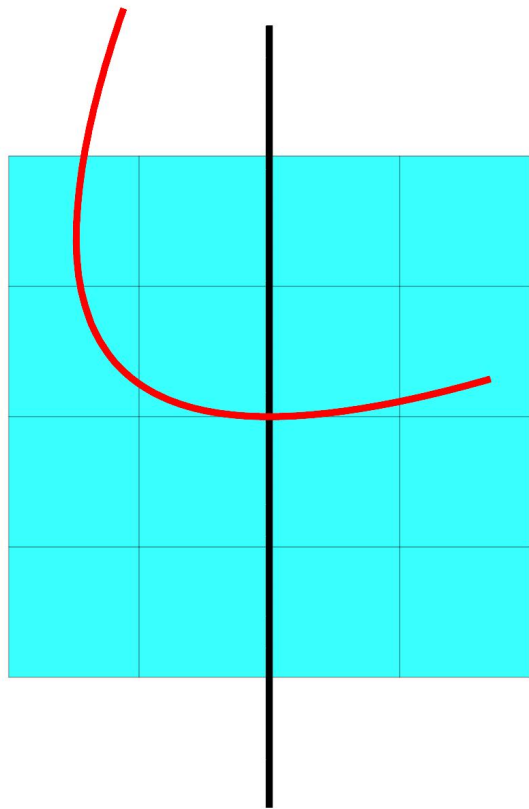












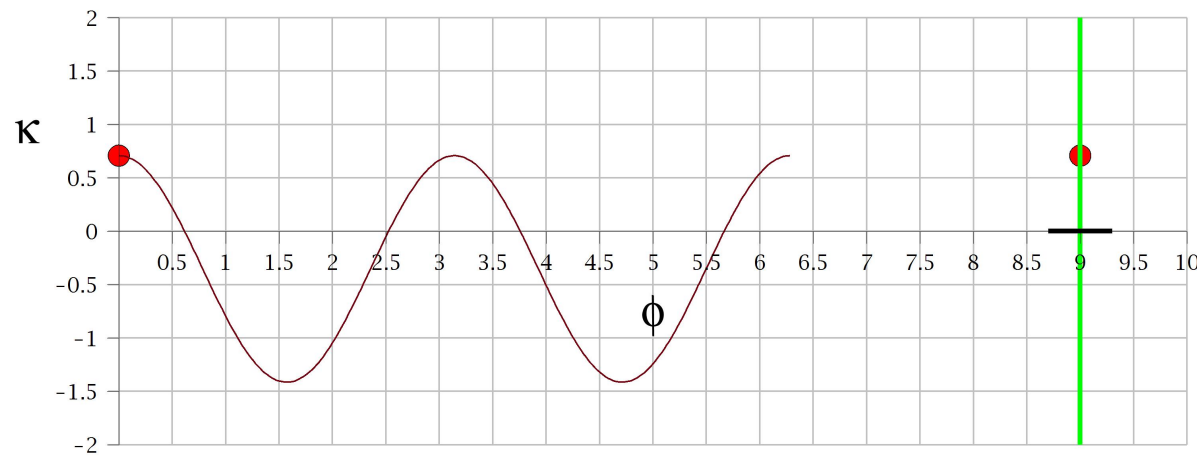
Lidt om middelkrumningen generelt

Sætning 1 (Euler's sætning om snitkrumningerne).
*Snitkrumningen med en plan der er drejet ϕ om normalen er bestemt ved to **principale krumninger** i punktet, κ_1 og κ_2 :*

$$\kappa_n(\phi) = \kappa_1 \cos^2(\phi) + \kappa_2 \sin^2(\phi) \quad . \quad (1)$$

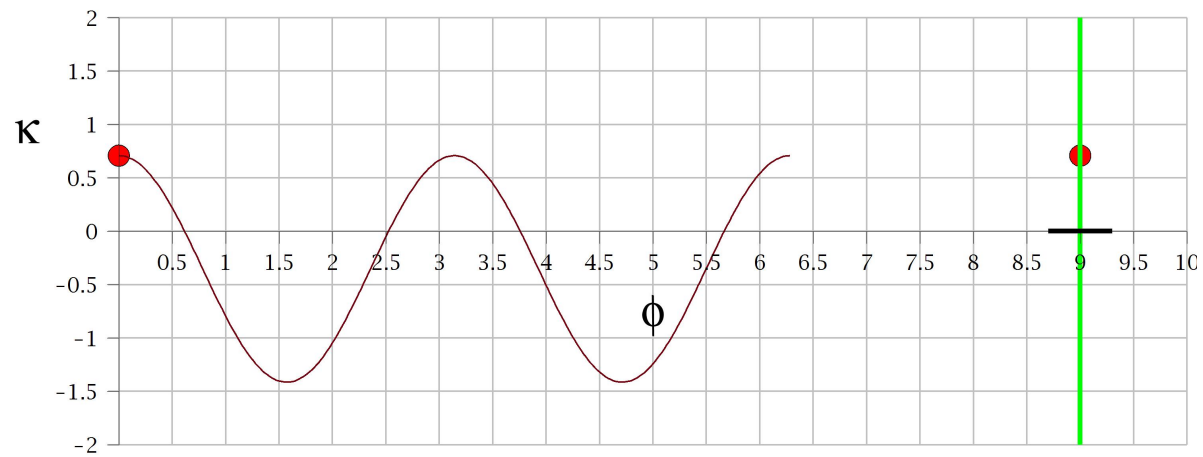
Middelkrumningsfunktionen

- Snitkrumningen som funktion af drejningsvinklen



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Middelkrumningsfunktioner

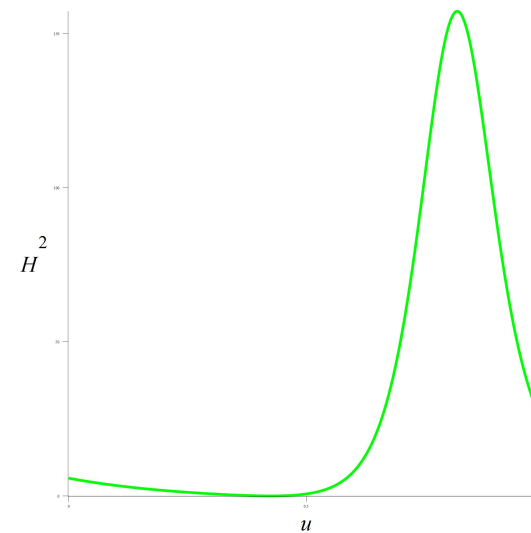
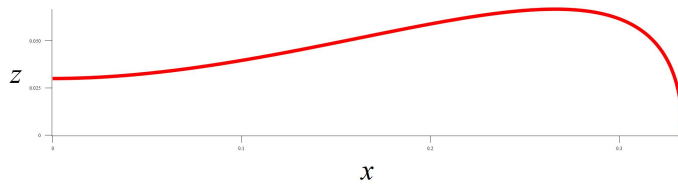
- ⊙ Definition af middelkrumningsfunktionen $H(u, v)$

Definition 2. Middelkrumningen H i et punkt **defineres** ved middeltallet:

$$\begin{aligned} H &= \frac{1}{2\pi} \int_0^{2\pi} \kappa_n(\phi) d\phi \\ &= \frac{1}{2} (\kappa_1 + \kappa_2) \end{aligned}$$

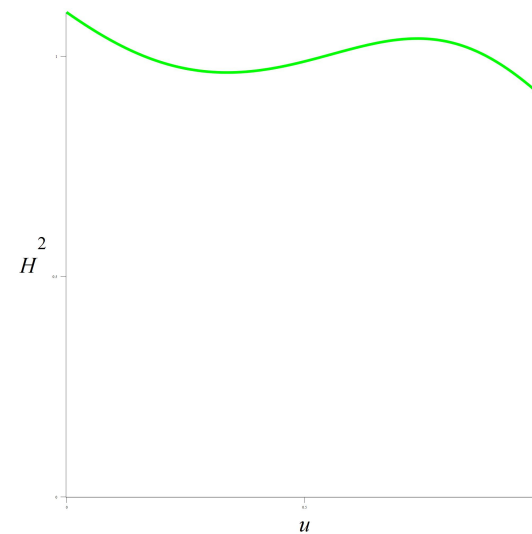
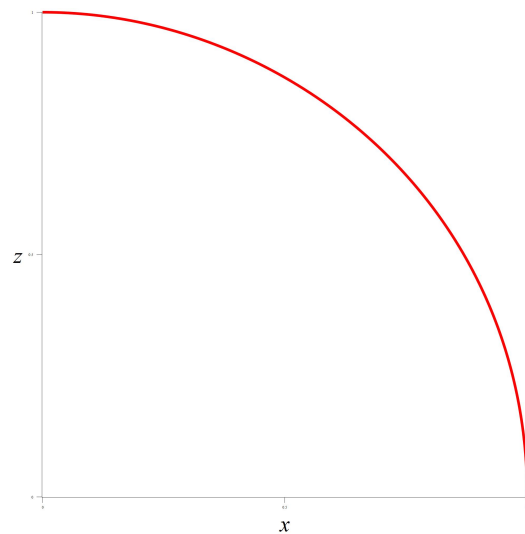
Middelkrumningsfunktioner

- Middelkrumningsfunktioner
for typiske RBC-omdrejningsflader



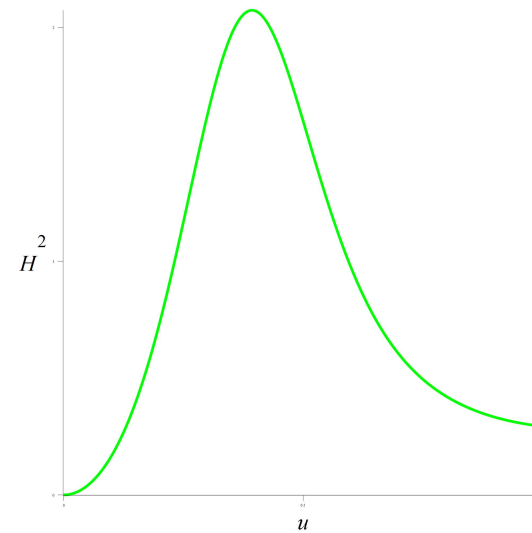
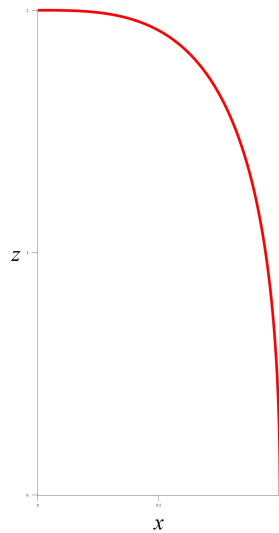
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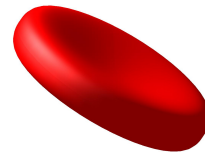
Middelkrumningsfunktioner

- Middelkrumningsfunktioner
for typiske RBC-omdrejningsflader



Optimering af E_{CH}

- ⊙ Med 3 designparametre (a , b , c) og 2 betingelser (fast areal $A = 20$ og fast rumfang $V = 3$) er der (typisk) **én fri parameter** tilbage - som kan bruges til at minimere den tredje og vigtigste størrelse: CH-energien.



Videregående overvejelser

- ⊙ Andre A og V værdier
- ⊙ Spontan krumningsbidrag
- ⊙ Topologisk fasediagram
- ⊙ Willmore formodningen

Tak for opmærksomheden!

God fornøjelse med projektet!

