Geometric Morphology

Geometric Design with IT

v./ Steen Markvorsen, DTU Compute, Mathematics

http://geometry.compute.dtu.dk/

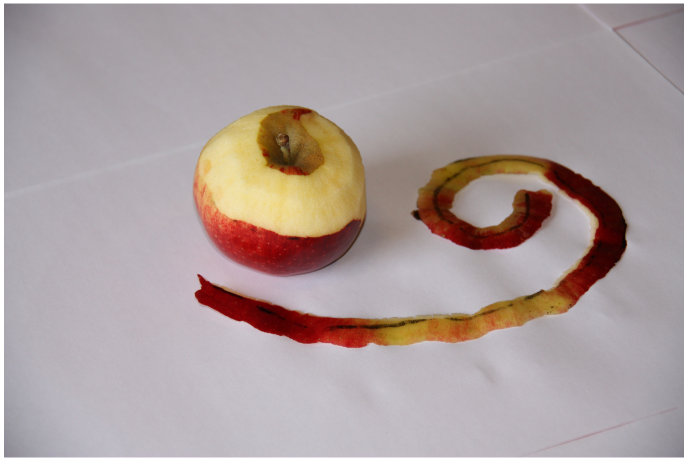
IT Camp for Girls, DTU Compute, October 18. 2019

Intro

- When you peel a potato

Intro

- - or an apple

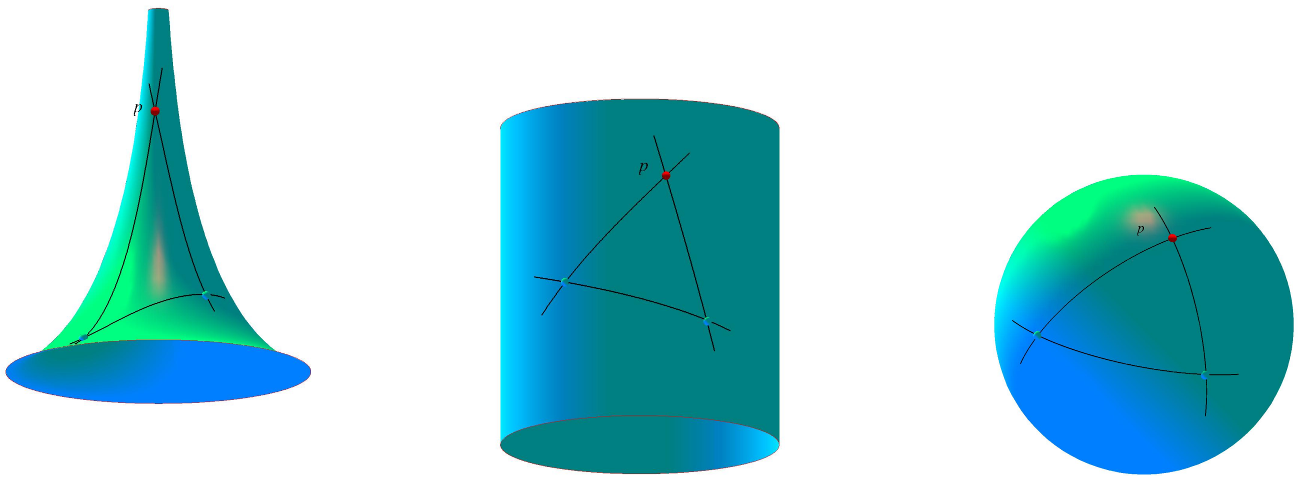


Intro

- - or an apple

What is the curvature of the cut-surface?

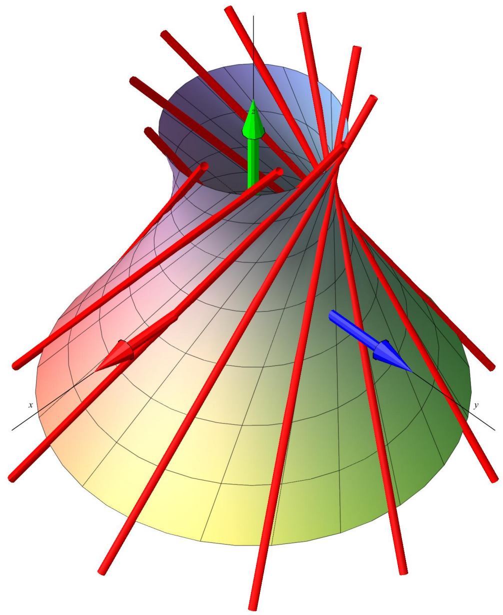
- Negative, zero, or positive curvature?

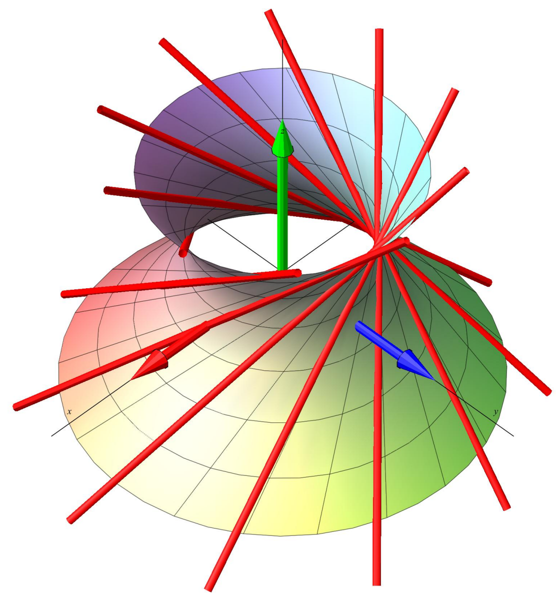


What is the curvature of the cut-surface?

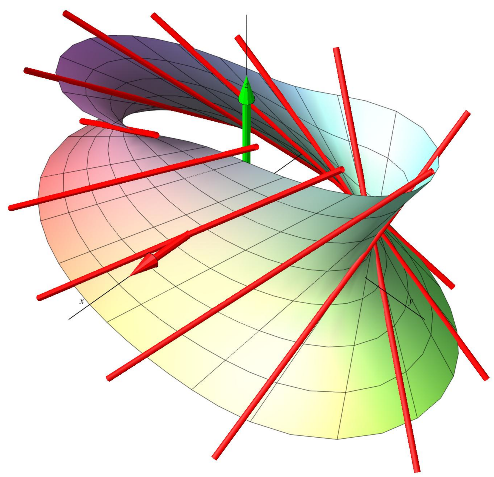
- When using a (straight) knife or string for cutting, the ensuing curvature is typically negative!

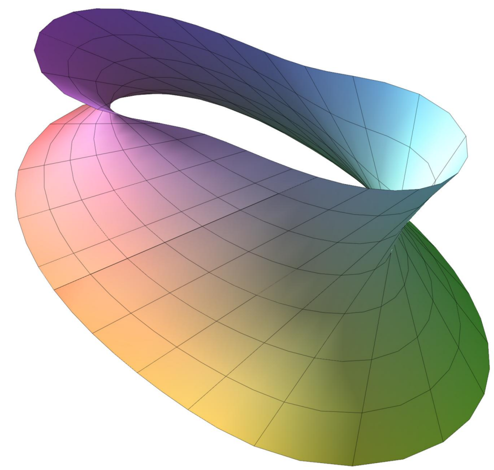
What is the curvature of the cut-surface?



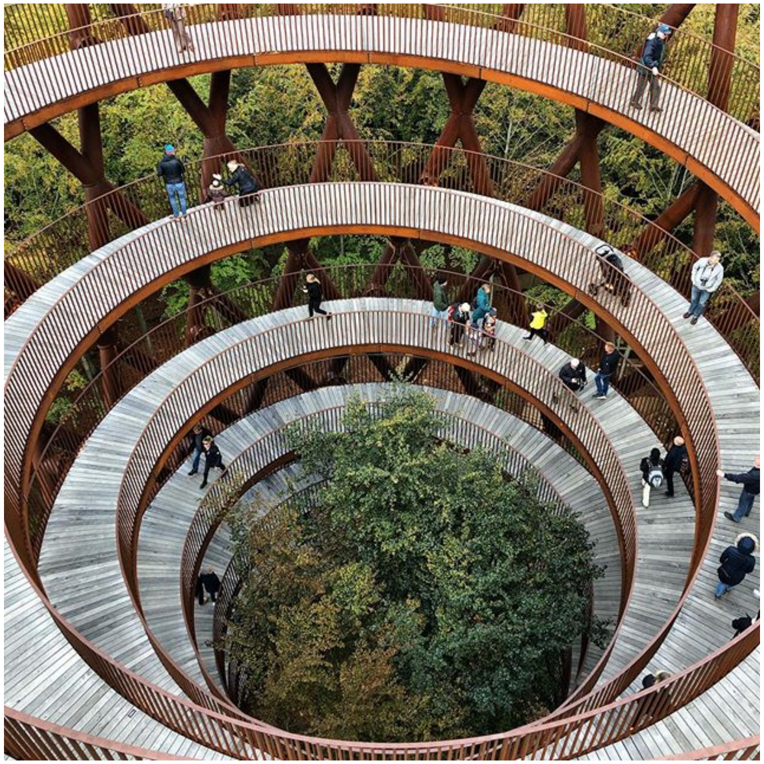


What is the curvature of the cut-surface?





Applications

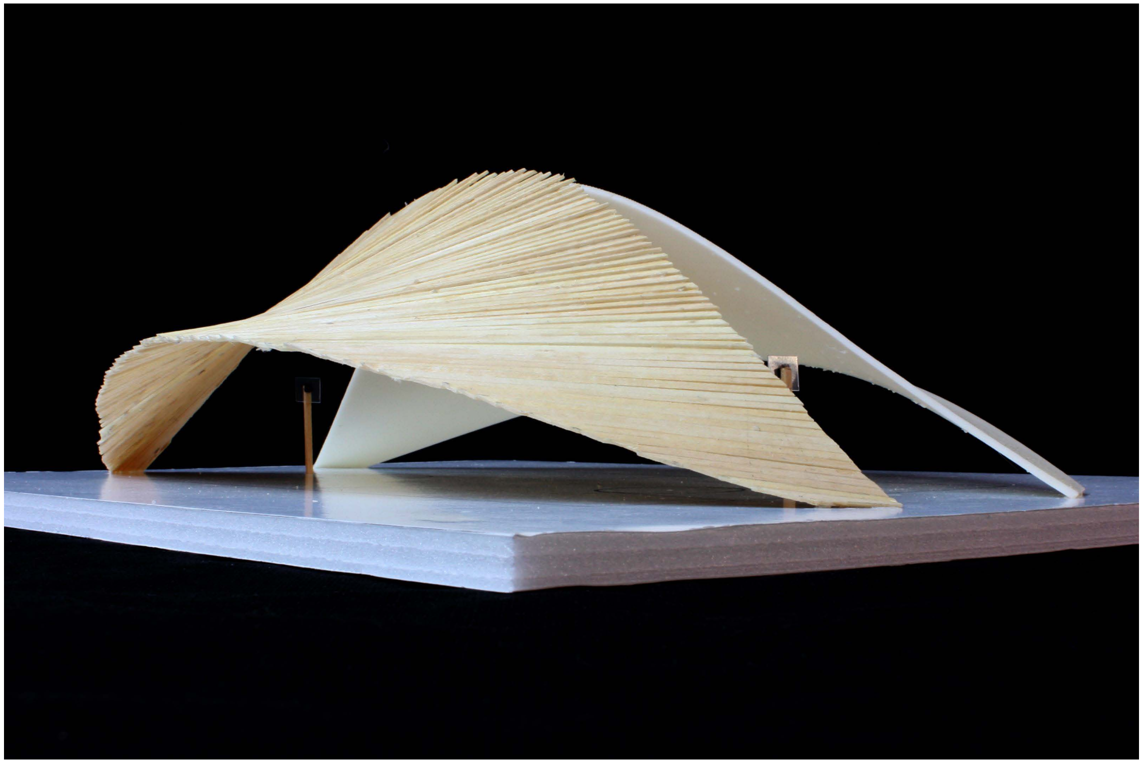


Applications

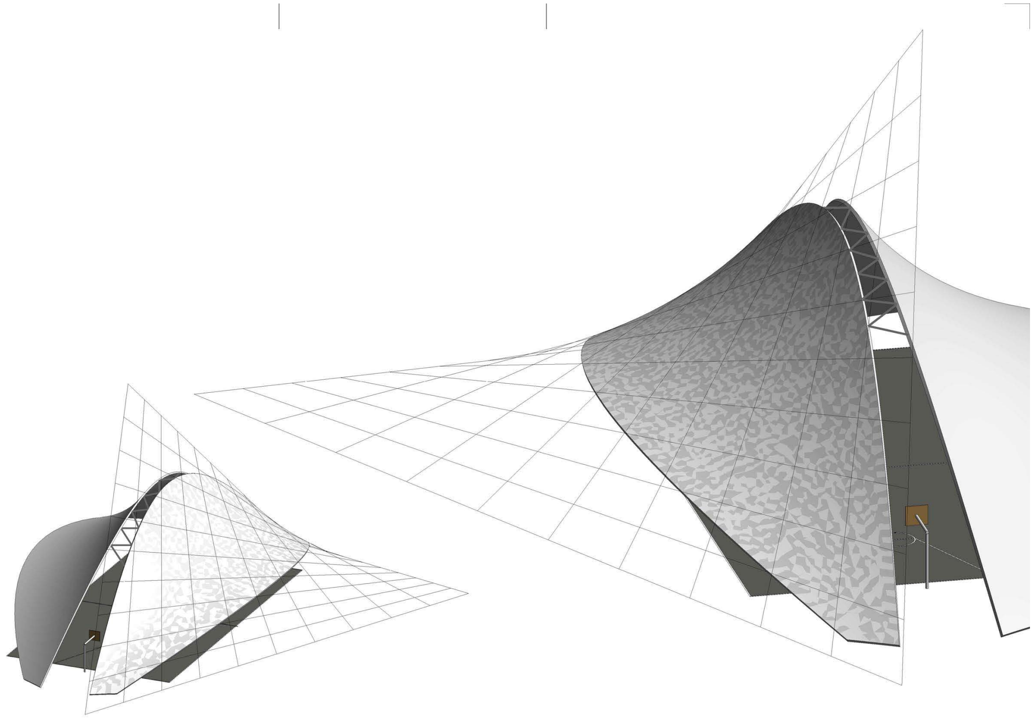
- The Roof

  A 3. semester project at
  DTU Architectural Engineering

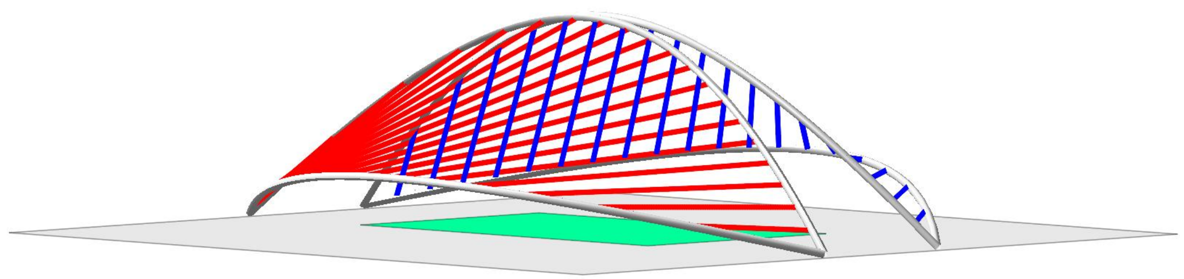
Applications



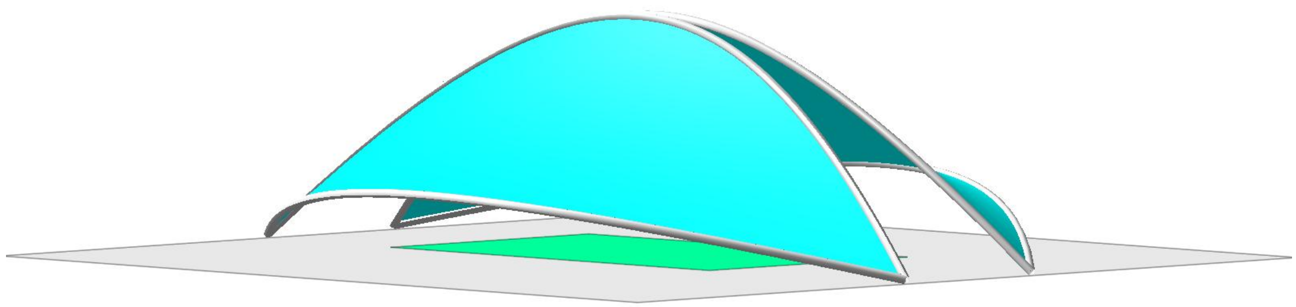
Applications



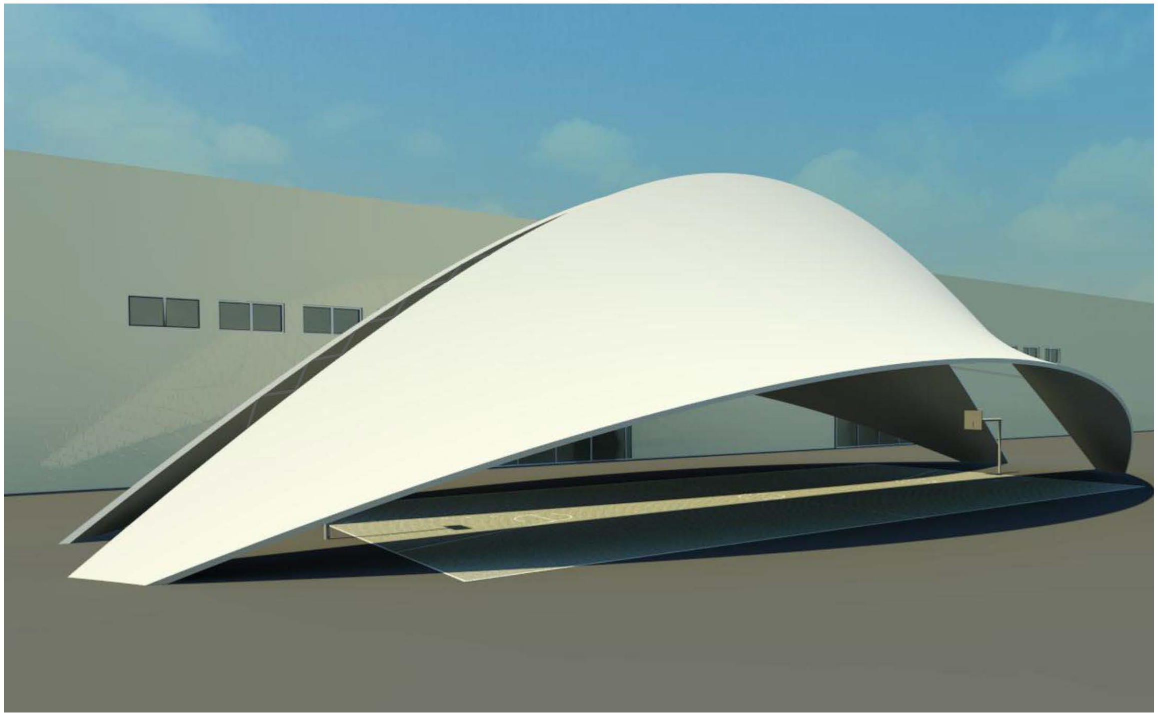
Applications



Applications



Applications



Applications – The Robot

Applications – The Robot

Applications

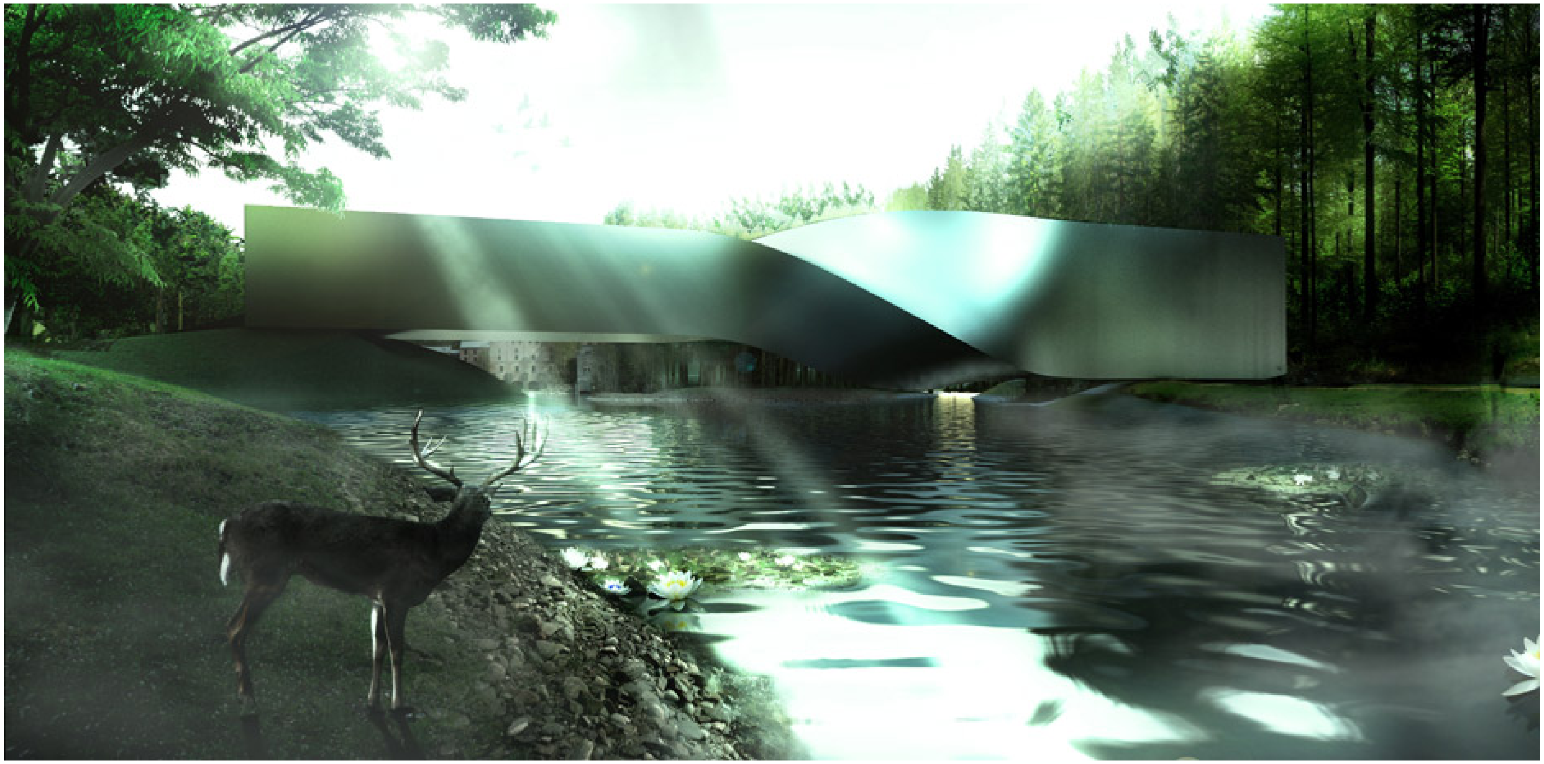
Hot Wire Cutting

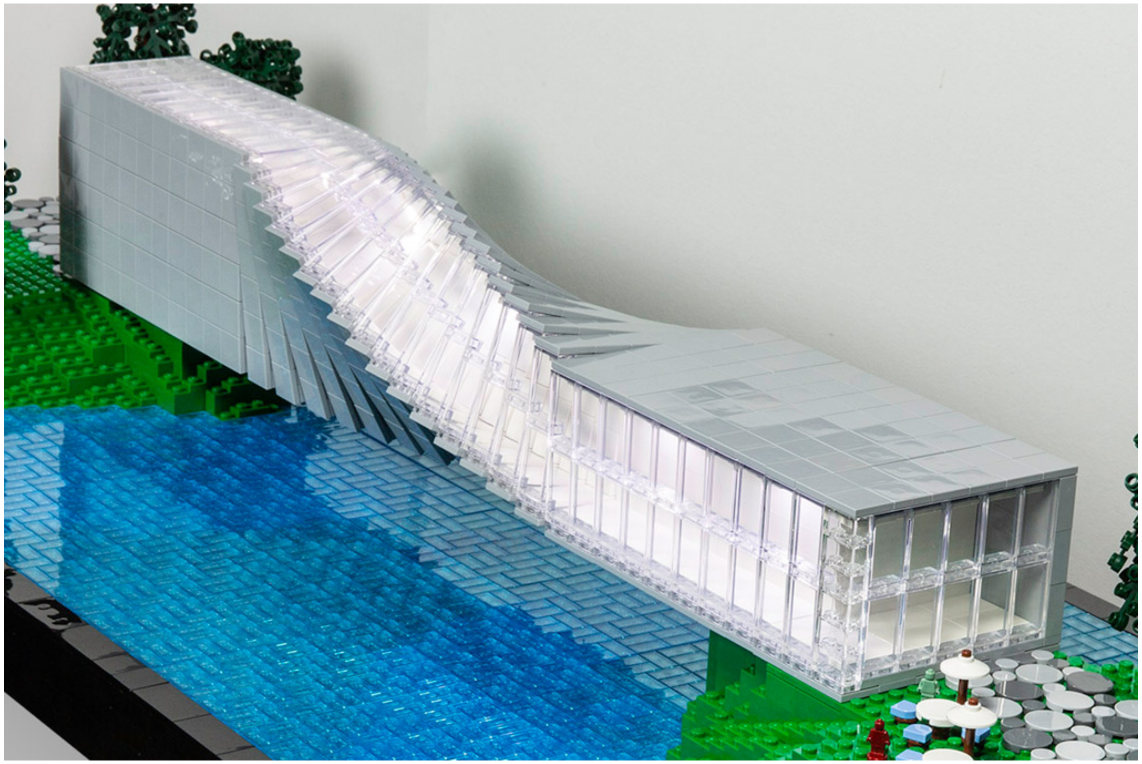
The almost flat reindeer

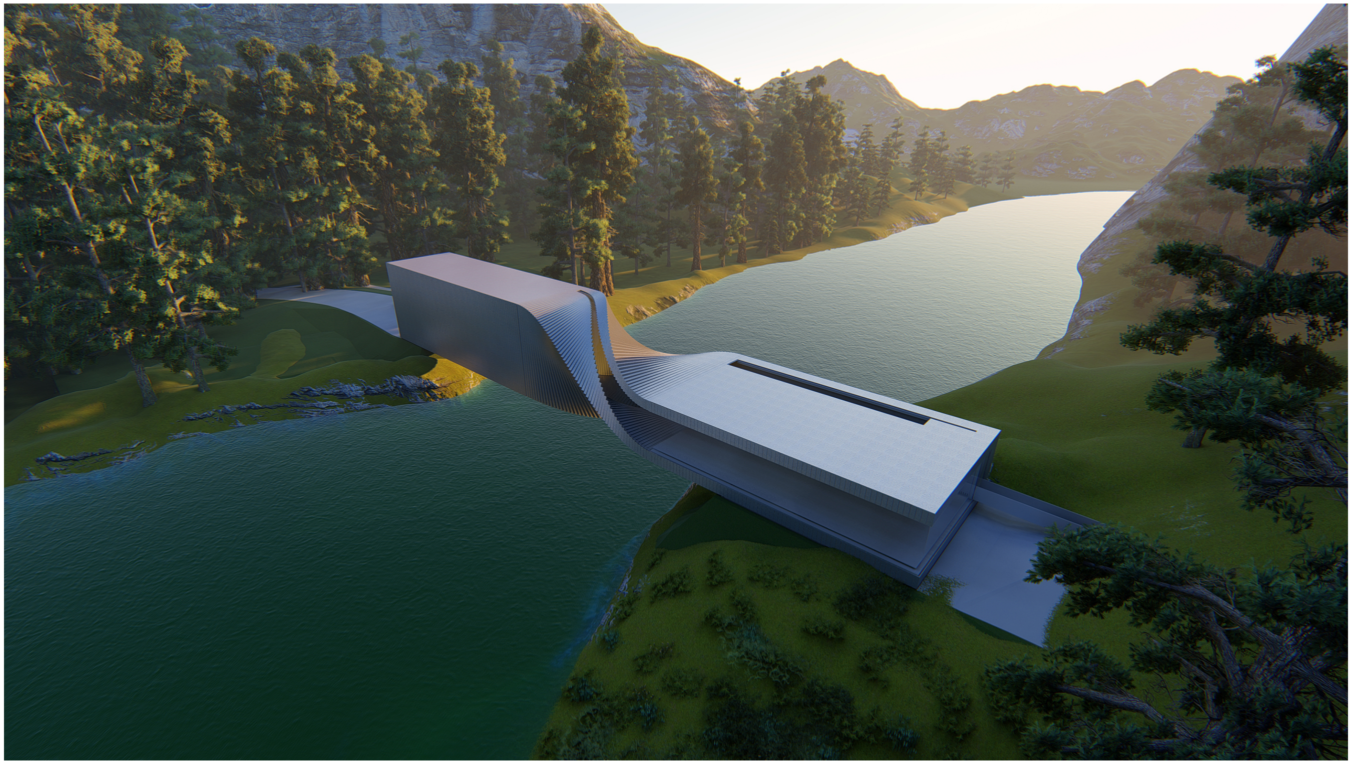
Patent: Extension to curved hot wire cutting, 2017.

PhD thesis: Ann-Sofie Fisker, 2018: Surface design and rationalization for robotic hot-blade cutting.

The Kistefos Museum in Norway







The problem of bending a flat ribbon in space

- Parametrization of a surface in space:

$$R(u,v) = (R_1(u,v), R_2(u,v), R_3(u,v)) \ , \ u \in [a,b] \ , \ v \in [c,d] \quad .$$

Geometric Analysis

- Parametrization of a curve
  on a surface in space:

$$\gamma(t) = R(u(t), v(t)) = R(\Gamma(t))$$
$$= (R_1(u(t), v(t)), R_2(u(t), v(t)), R_3(u(t), v(t)))\ ,\ t \in [0, T] \quad .$$

Geometric Analysis

- Parametrization of a ruled surface in space – as obtained from peeling by a straight knife or string:

$$r(t, v) = \gamma(t) + v \cdot \beta(t) \ , \ t \in [0, T] \ , \ v \in [c, d] \quad ,$$

where β is a vector function of t.

Geometric Analysis

- The task:

Project 1. Approximate a given surface $R(u, v)$ by a ruled surface $r(t, v)$ along a given curve $\gamma(t)$ on the given surface.

Geometric Analysis

- Cartan approximations can be constructed via rolling:

Theorem 2 (K. Nomizu (1978), R-B-M (2017)). *The given surface is rolled on a plane so that the given curve on the surface has a unique contact point on the plane. The track of this contact point is then the center curve of the plane (developed) Cartan approksimation.*

The instantaneous rotation vector for the rolling is precisely the ruling vector for the developable Cartan approximation:

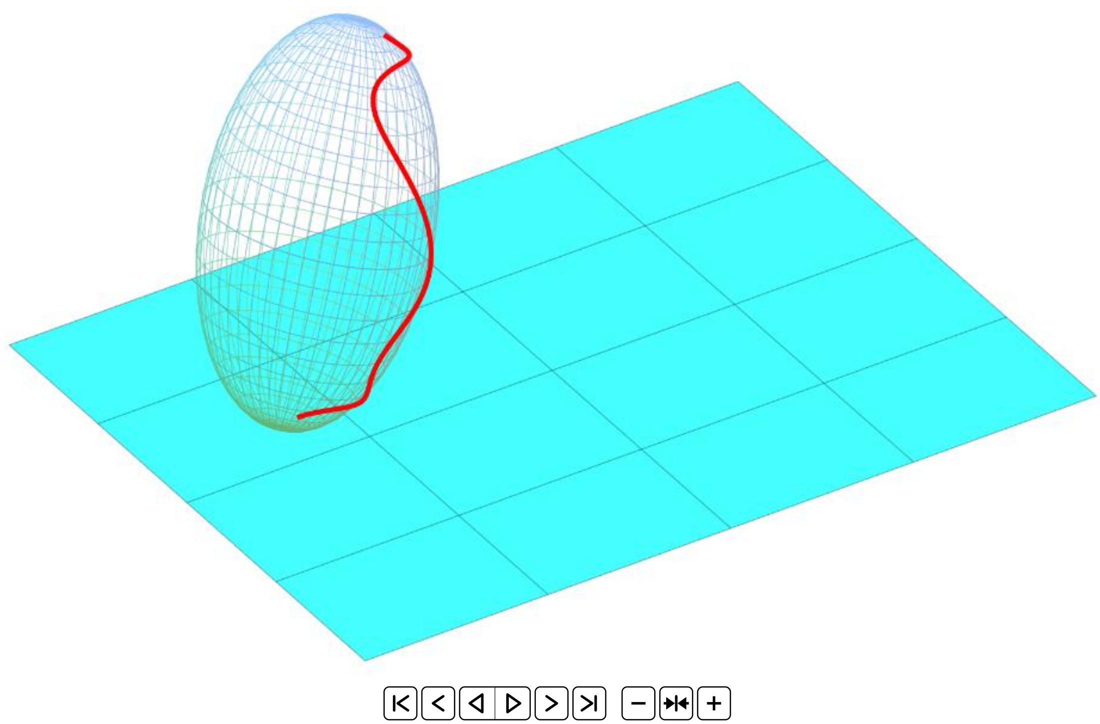
$$\begin{aligned}\omega(t) &= \|\gamma'(t)\| \cdot \beta(t) \\ &= \|\gamma'(t)\| \cdot (\kappa_n(t) \cdot h(t) - \tau_g(t) \cdot e(t)) \quad .\end{aligned}$$

Geometric Analysis

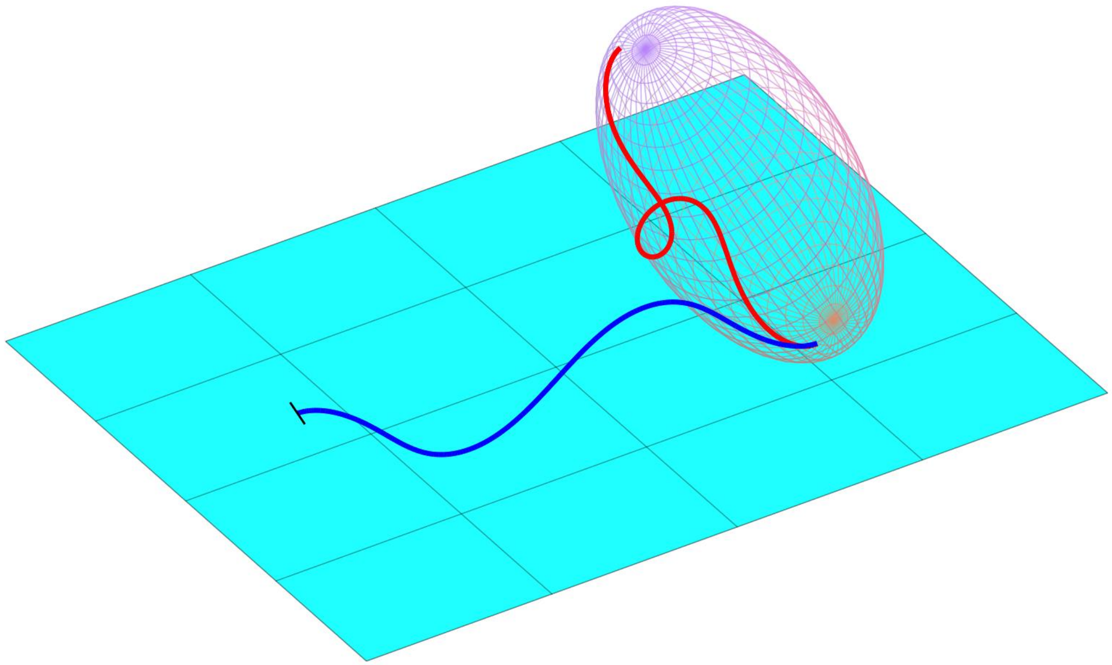
- Cartan approksimations via rolling

- Illustrated:

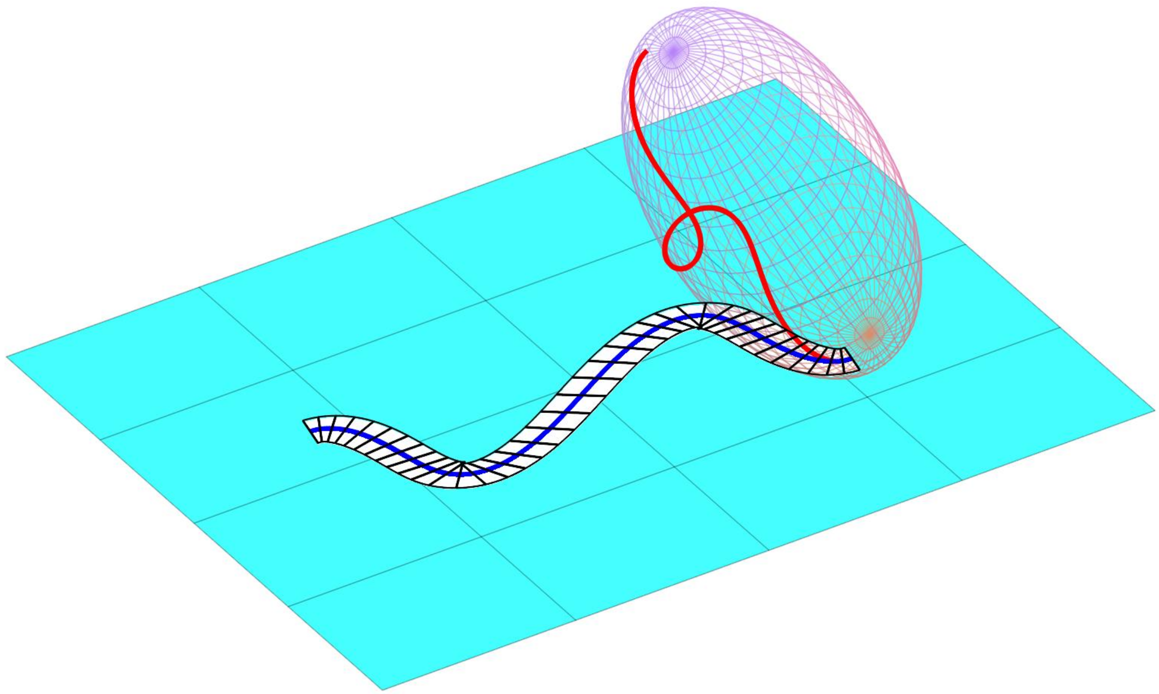
Rolling I



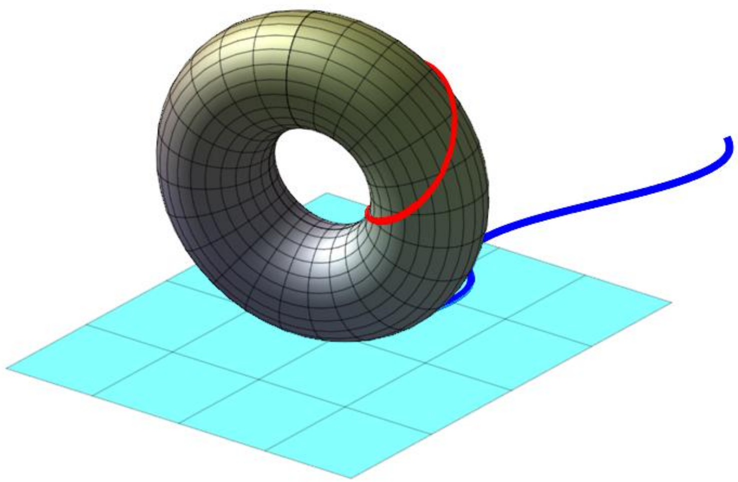
Rolling II



Rolling III

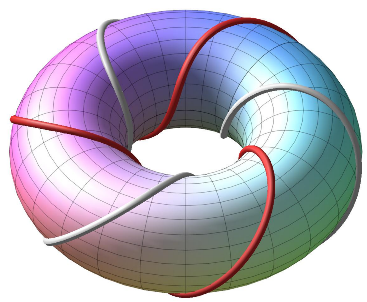
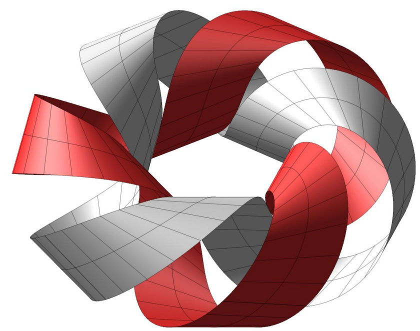


Exotic example: Rolling of a torus



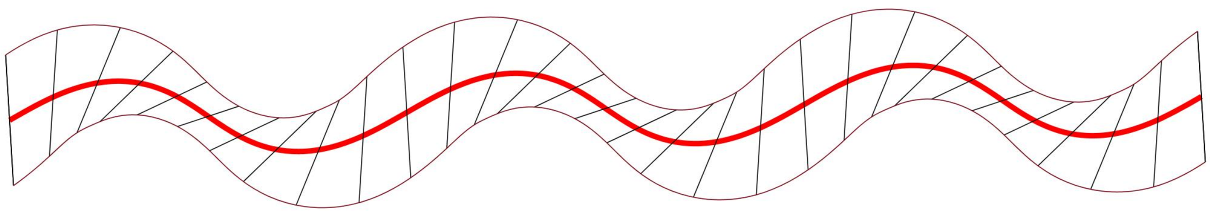
Exotic example: Rolling of a torus

- Cartan approximation of a torus along two center curves:

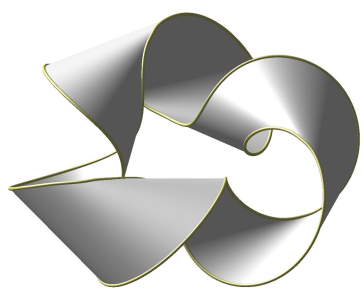
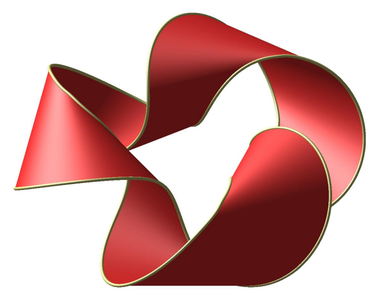
Example: The torus

- Cartan approximation – the developed ribbon:



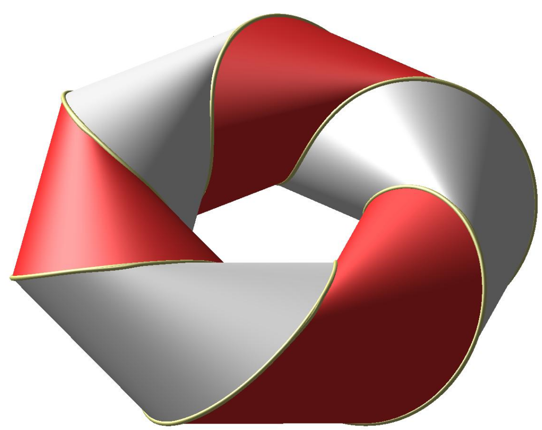
Example: The torus

- Cartan approximation of a torus along two center curves:

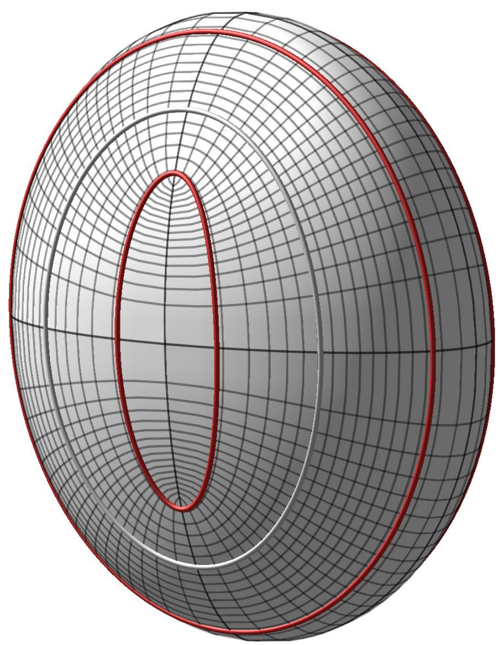
Example: The torus

- Cartan approximation of a torus along two center curves:



Example: Ellipsoid

- Cartan approximation of an ellipsoid along the so-called curvature lines:

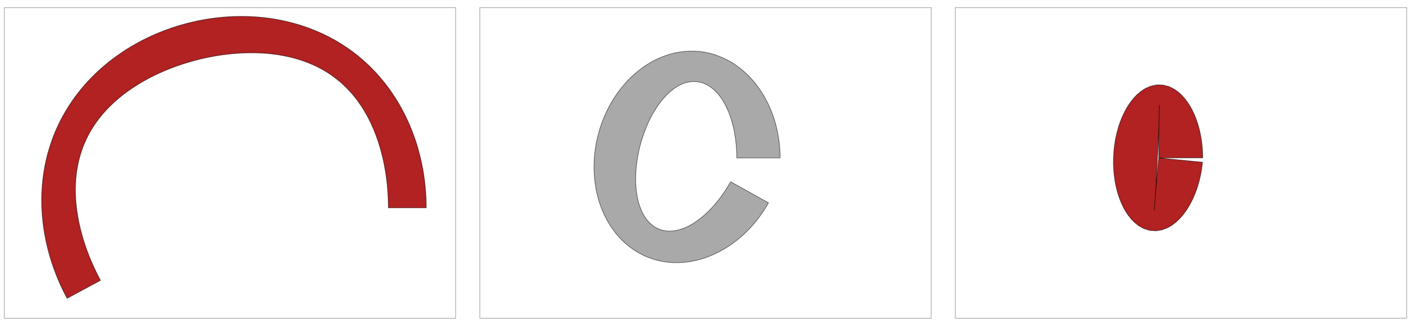


Example: Ellipsoid

- Cartan approximation of an ellipsoid along the so-called curvature lines:

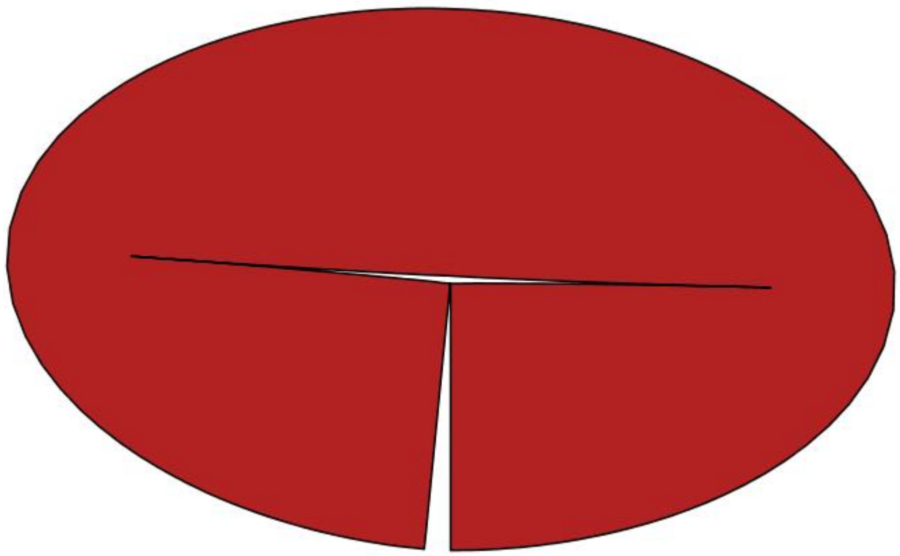
Example: Ellipsoid

- Cartan approximation: The planar developed ribbons:



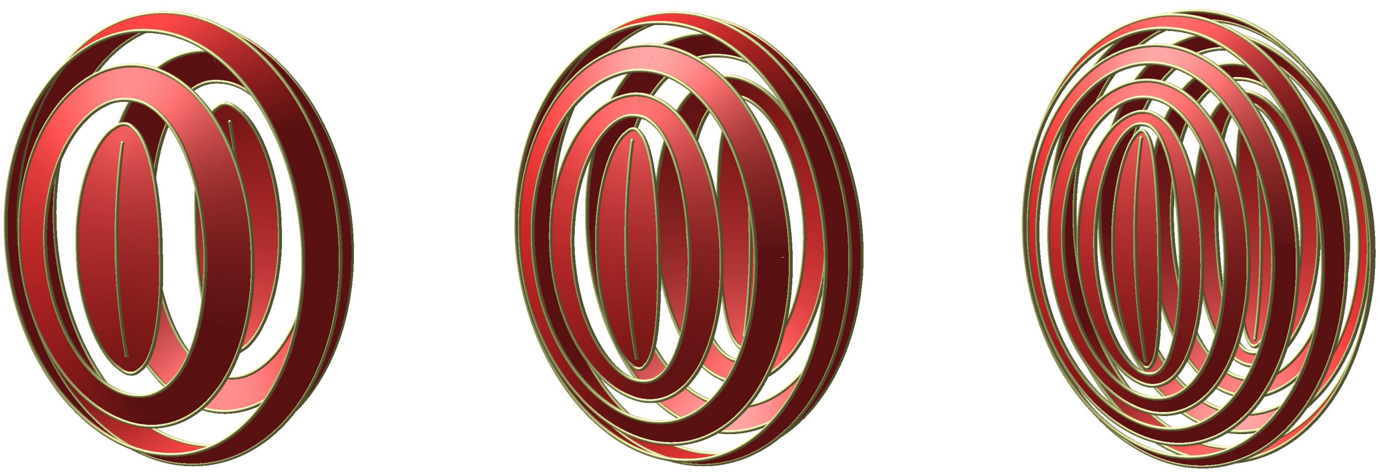
Example: Ellipsoid

- Cartan approximation: The caps:



Example: Ellipsoid

- Cartan approximation of an ellipsoid along the so-called curvature lines:

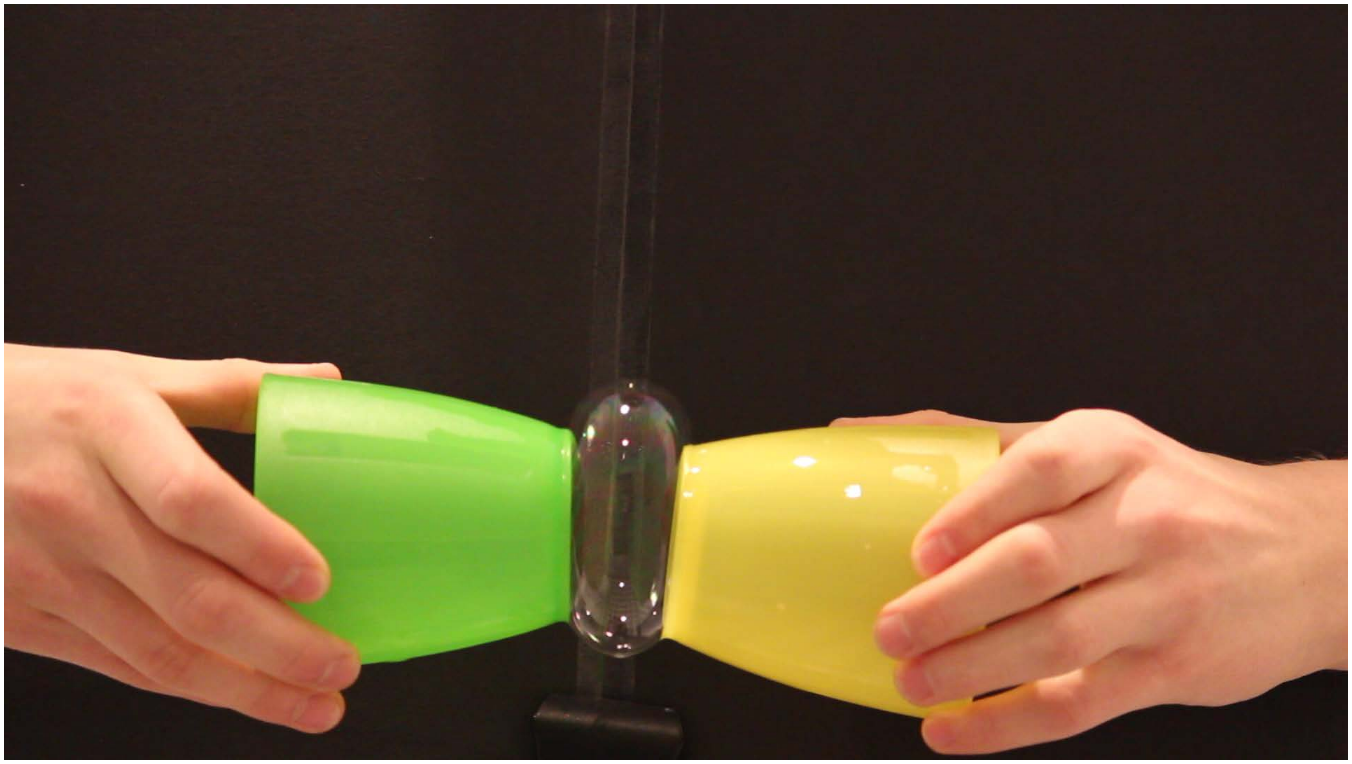


Example: Ellipsoid

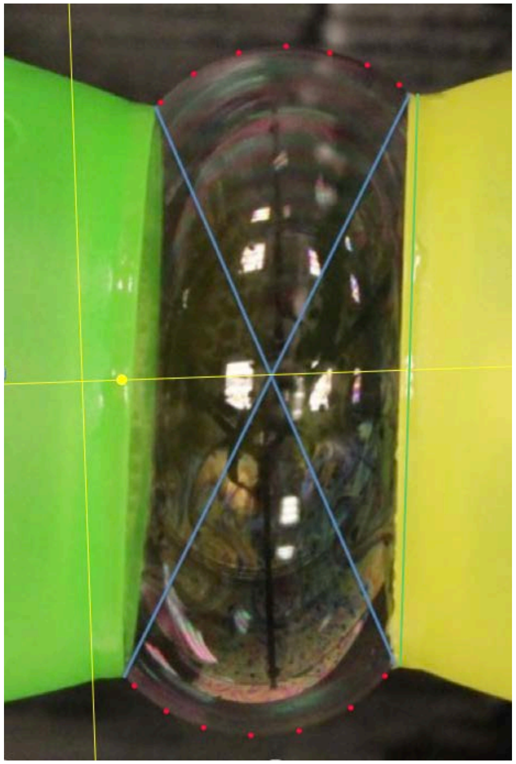
- Cartan approximation of an ellipsoid along the so-called curvature lines:

SRP: How to catch a soap bubble

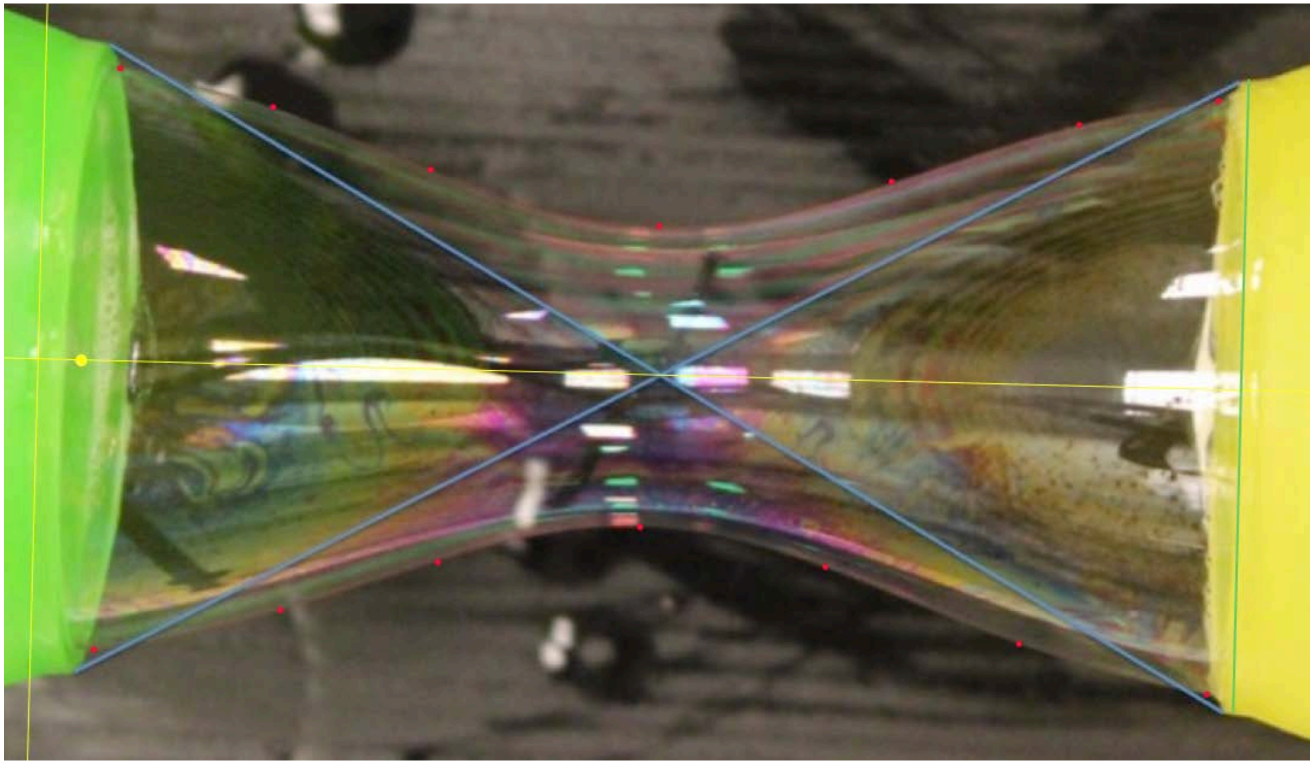
SRP: How to catch a soap bubble



SRP: How to catch a soap bubble



SRP: How to catch a soap bubble



Delaunay–Cosserat sweeping

- Rotation-sweeping with a special curve:

Delaunay–Cosserat sweeping

- Rotation-sweeping with a special curve:

Delaunay–Cosserat sweeping

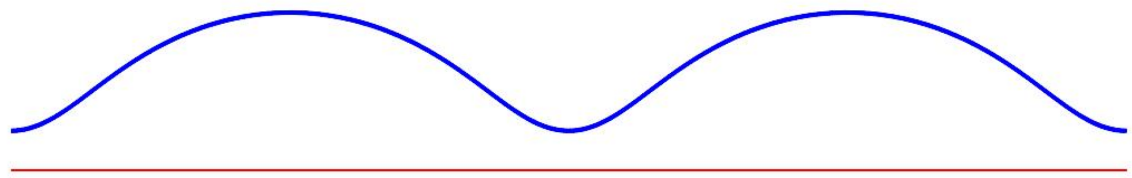
- Rotation-sweeping with a special curve:

Delaunay–Cosserat sweeping

- Rotation-sweeping with a special curve:

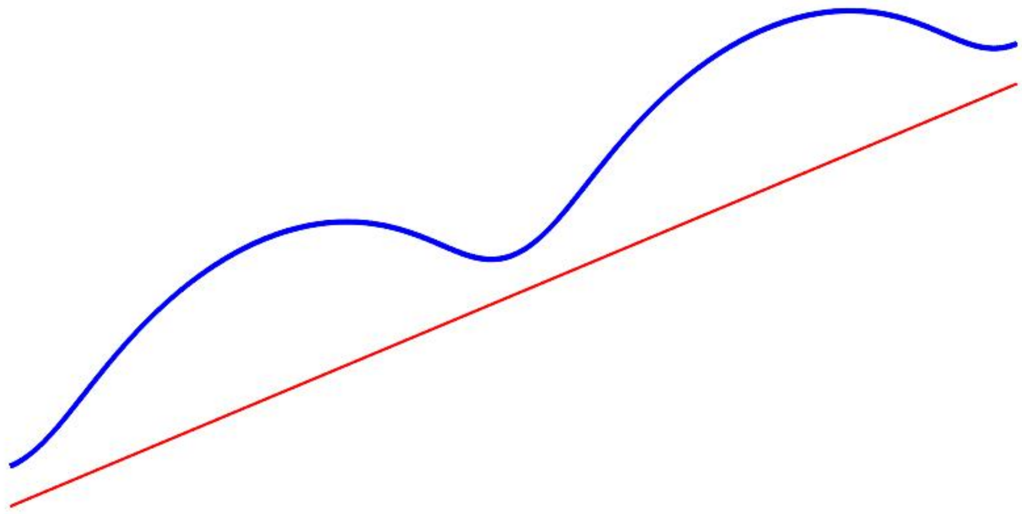
Delaunay–Cosserat sweeping

- Rotation-sweeping with a special curve:

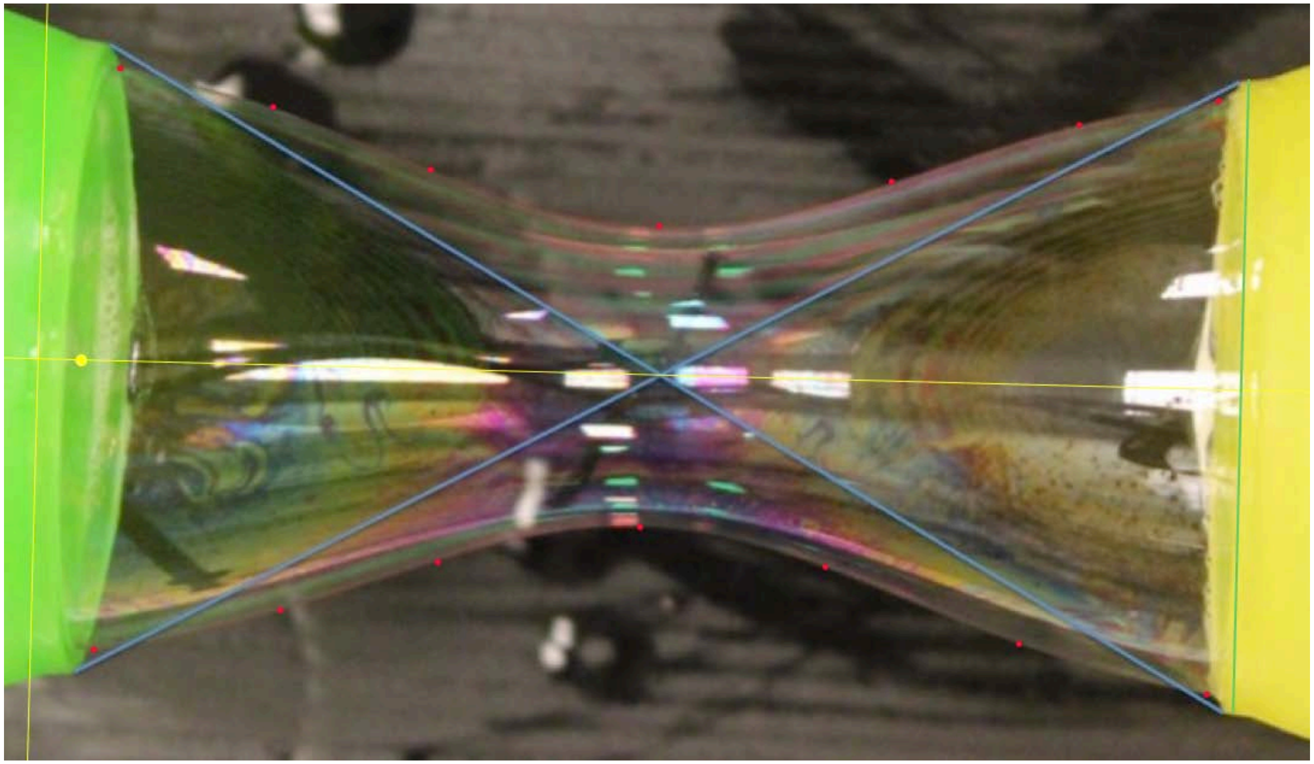


Delaunay–Cosserat sweeping

- Rotation-sweeping with a special curve:



SRP: How to catch a soap bubble

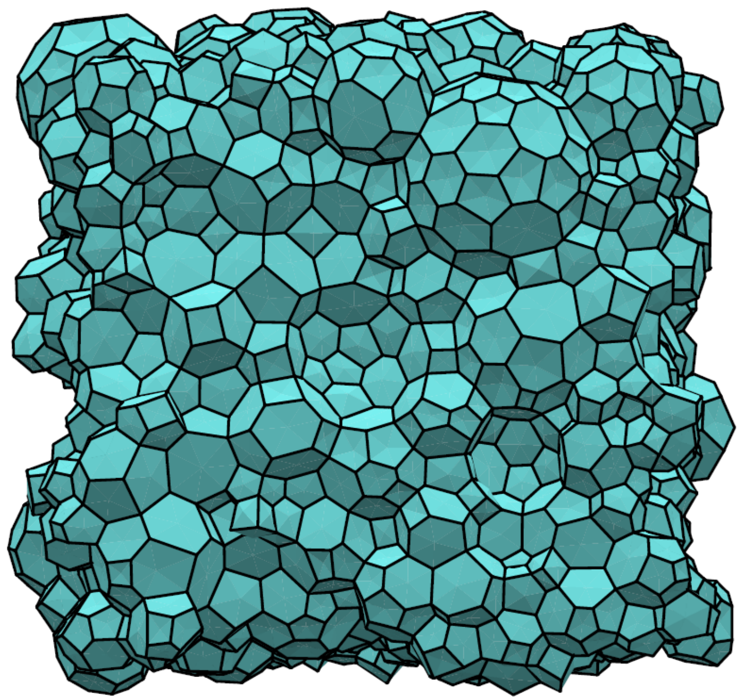


Multi-bubbles

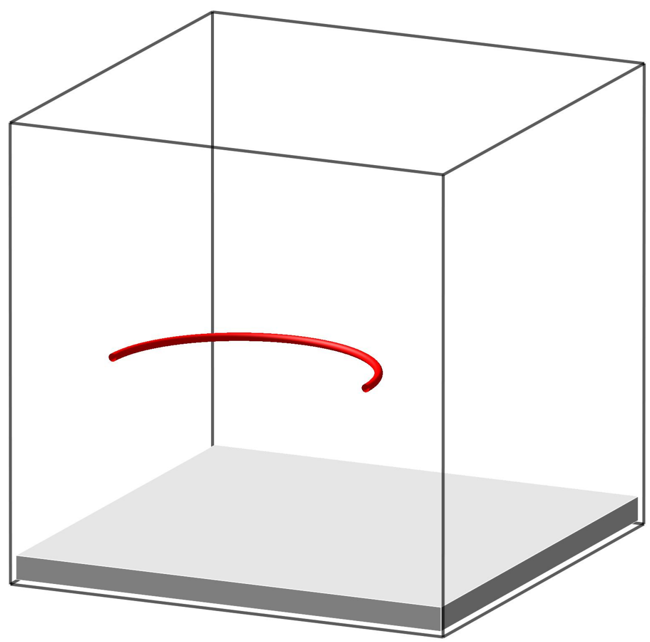
- Foam

Multi-bubbles

- Foam



SRP: How to 3D-print a formula



Half of a unit circle in space

SRP: How to 3D-print a formula

Half of a unit circle in space

$$Eksplicit \; : \quad y = \sqrt{1 - x^2} \quad , \quad x \in \,]-1, 1[$$

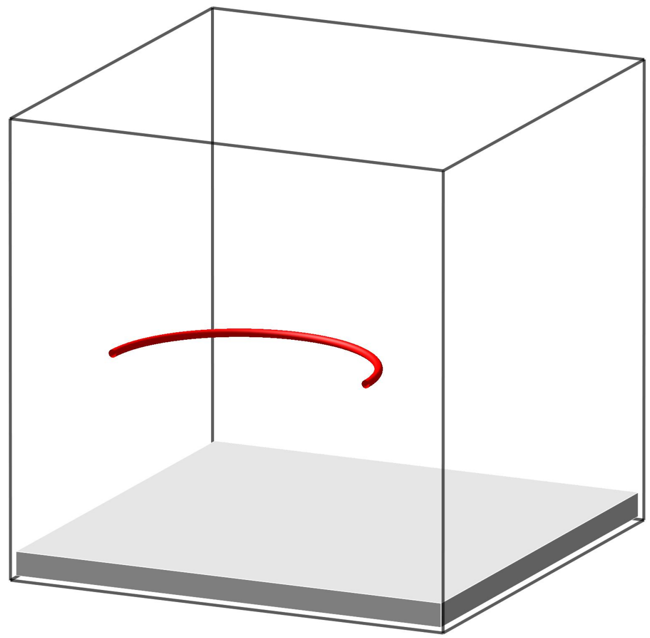
$$Implicit \; : \quad x^2 + y^2 = 1 \quad , \quad y > 0$$

$$Parametric \; : \quad \mathbf{r}(t) = (\cos(t), \sin(t), 0) \quad , \quad t \in \,]0, \pi[$$

Half of a unit circle in space

$$Implicit \;:\quad \mathcal{C} \;:\quad x^2 + y^2 = 1 \quad , \quad y > 0 \quad , \quad z = 0$$

Half of a unit circle in space



Den halve enhedscirkel præpareres

3D printning kræver, at objektet fedes op til en veldefineret solid – et massivt 3D legeme.

Opfedningen foretages typisk i 3 trin:

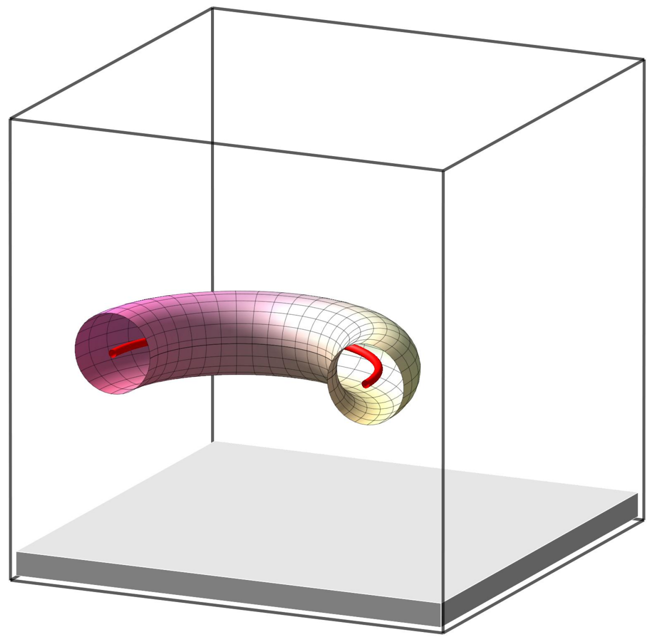
Den halve enhedscirkel, opfedet I

$$Implicit \; : \quad \mathcal{C} \; : \quad x^2 + y^2 = 1 \quad , \quad y > 0 \quad , \quad z = 0$$

$$Implicit \; : \quad \mathcal{T} \; : \quad \mathsf{dist}^2_{\mathcal{C}}(x, y, z) = d^2 \quad , \quad y > 0$$

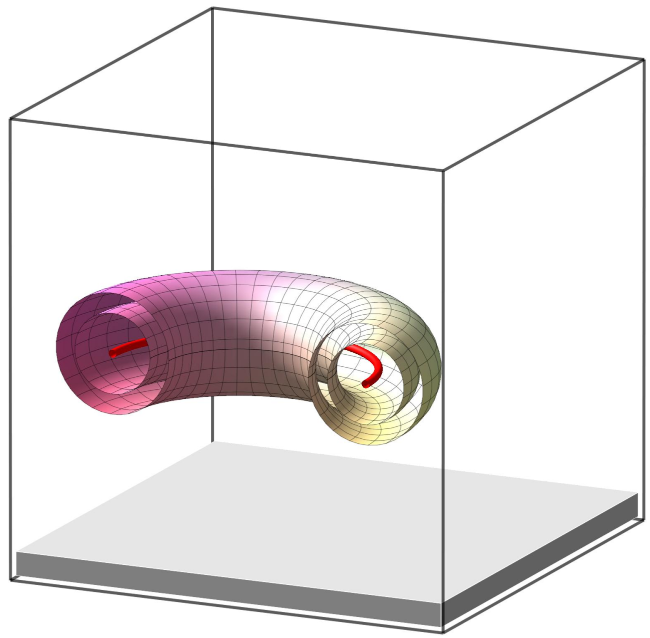
$$Implicit \; : \quad \mathcal{T} \; : \quad \left(\sqrt{x^2 + y^2} - 1\right)^2 + z^2 = d^2 \quad , \quad y > 0$$

Den halve enhedscirkel, opfedet I



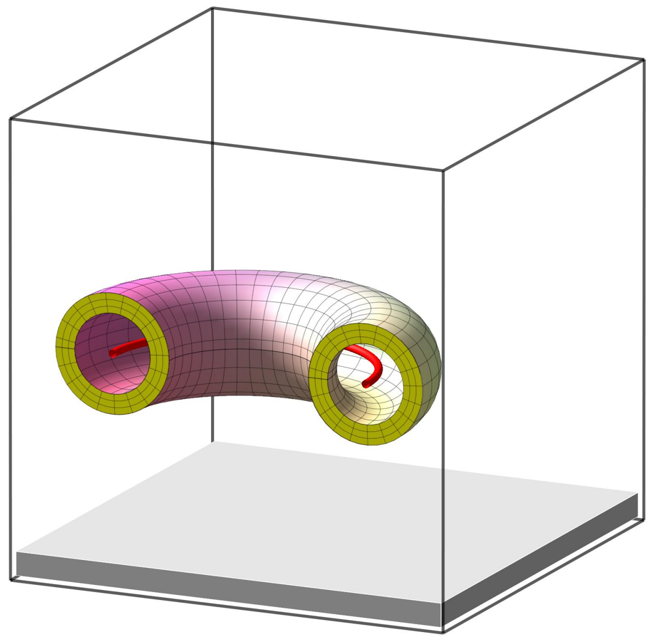
Halv enhedscirkel i rummet opfedet til en halv torus flade

Den halve enhedscirkel, opfedet II



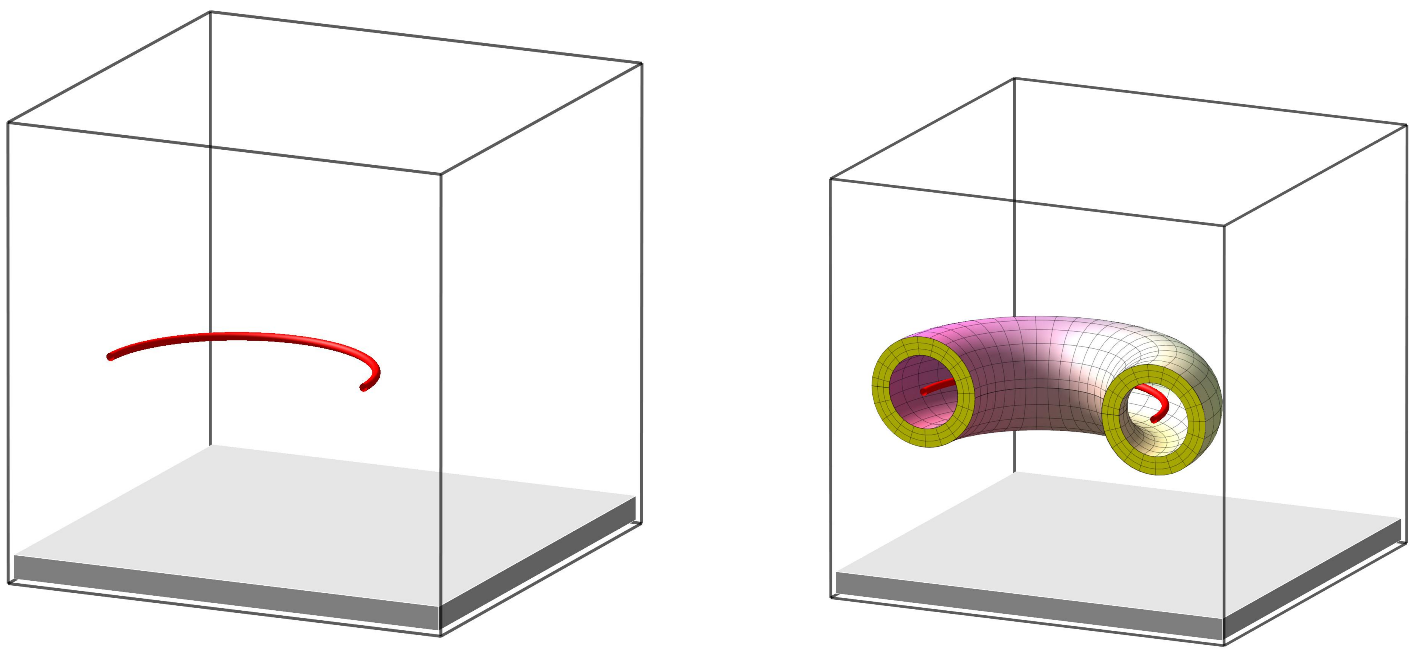
Halv enhedscirkel i rummet dobbelt opfedet

Den halve enhedscirkel, opfedet III



Halv enhedscirkel i rummet dobbelt opfedet og plomberet til en halv massiv torus-solid

Den halve enhedscirkel parat til print?



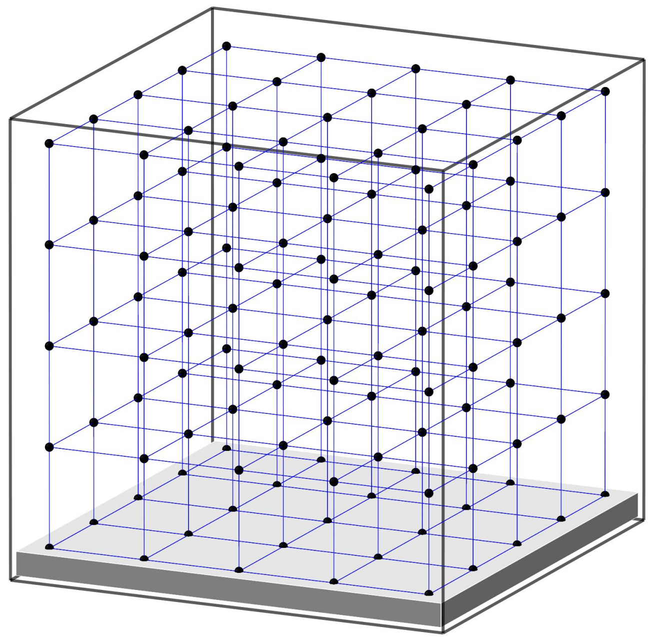
Halv enhedscirkel i rummet og et tilsvarende legeme

Den halve enhedscirkel parat til print?

Ikke helt!

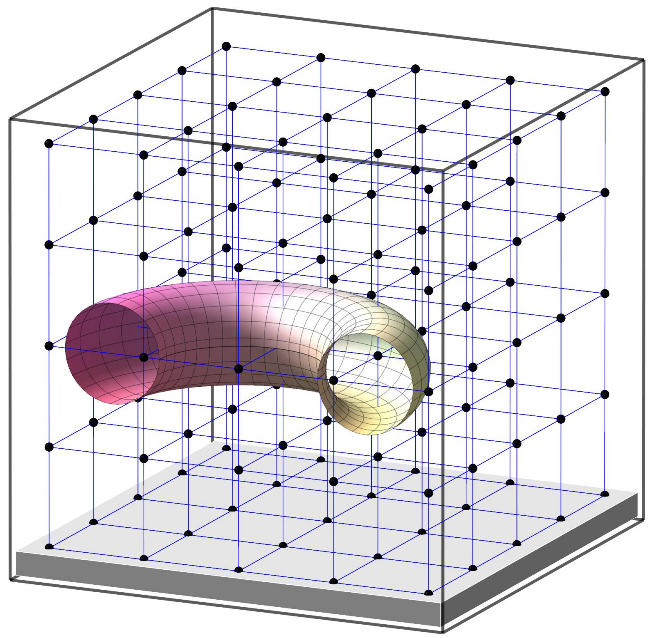
Vi skal triangulere den resulterende solid direkte ud fra den implicitte repræsentation:

Den halve enhedscirkel; Triangulering



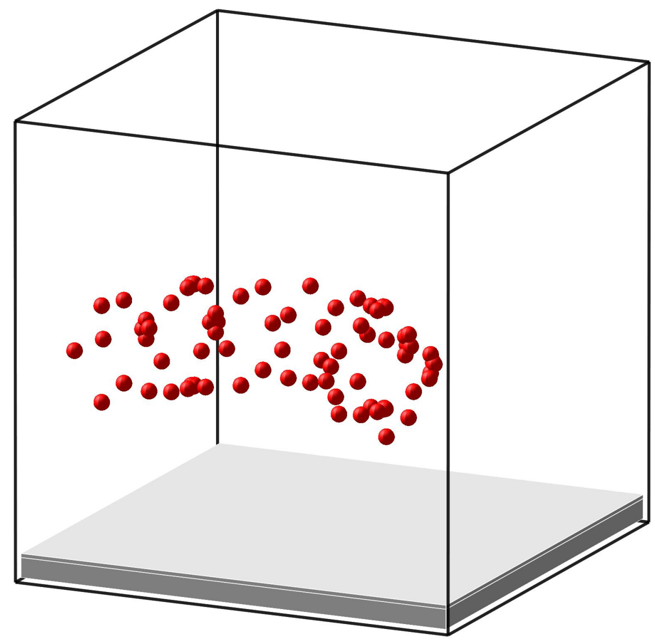
3D gitteret

Den halve enhedscirkel; Triangulering



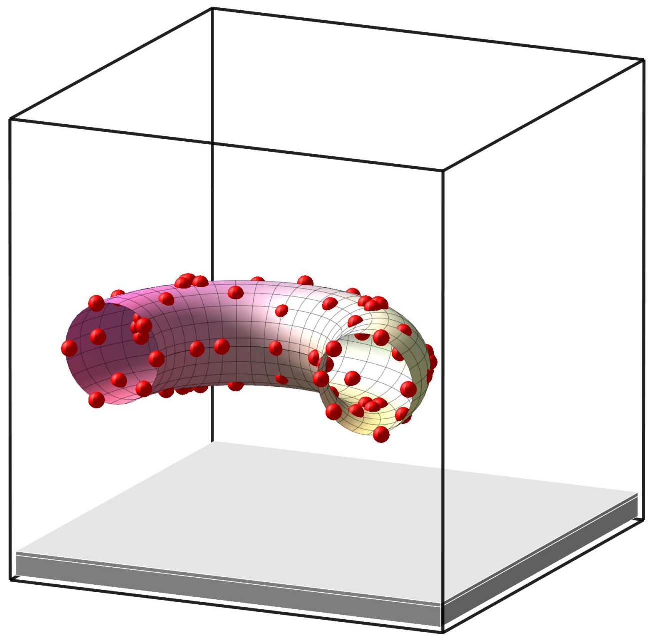
3D gitteret møder den implicit givne første torus-opfedning

Den halve enhedscirkel; Triangulering



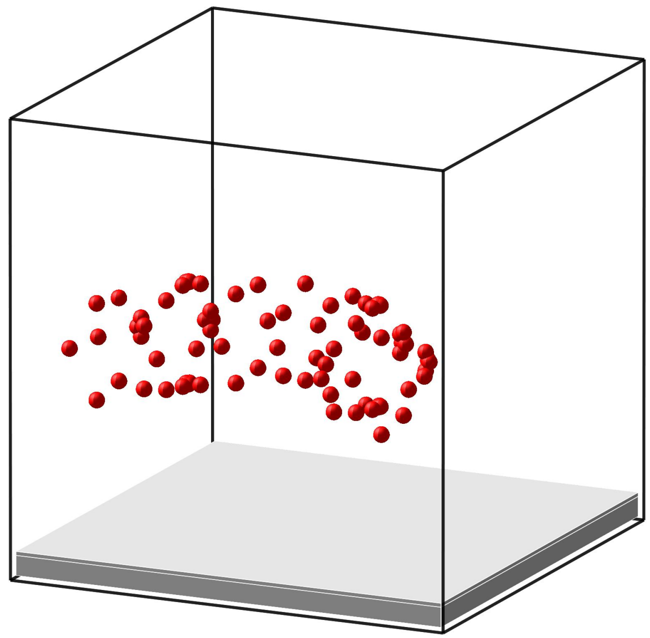
Skæringspunkterne

Den halve enhedscirkel; Triangulering



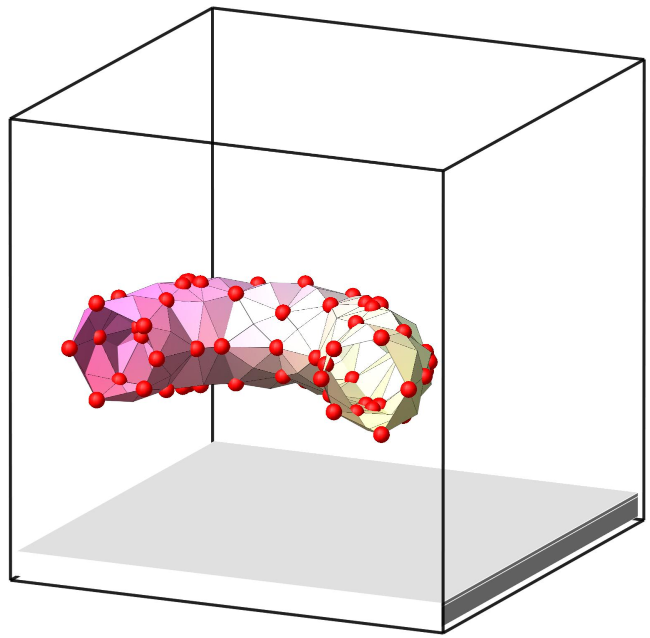
Check: Skæringspunkterne ligger på fladen

Den halve enhedscirkel; Triangulering



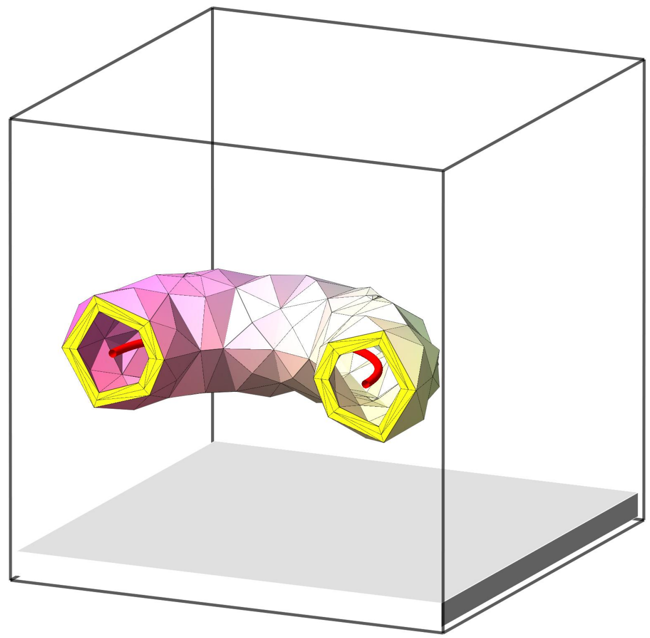
Skæringspunkterne

Den halve enhedscirkel; Triangulering



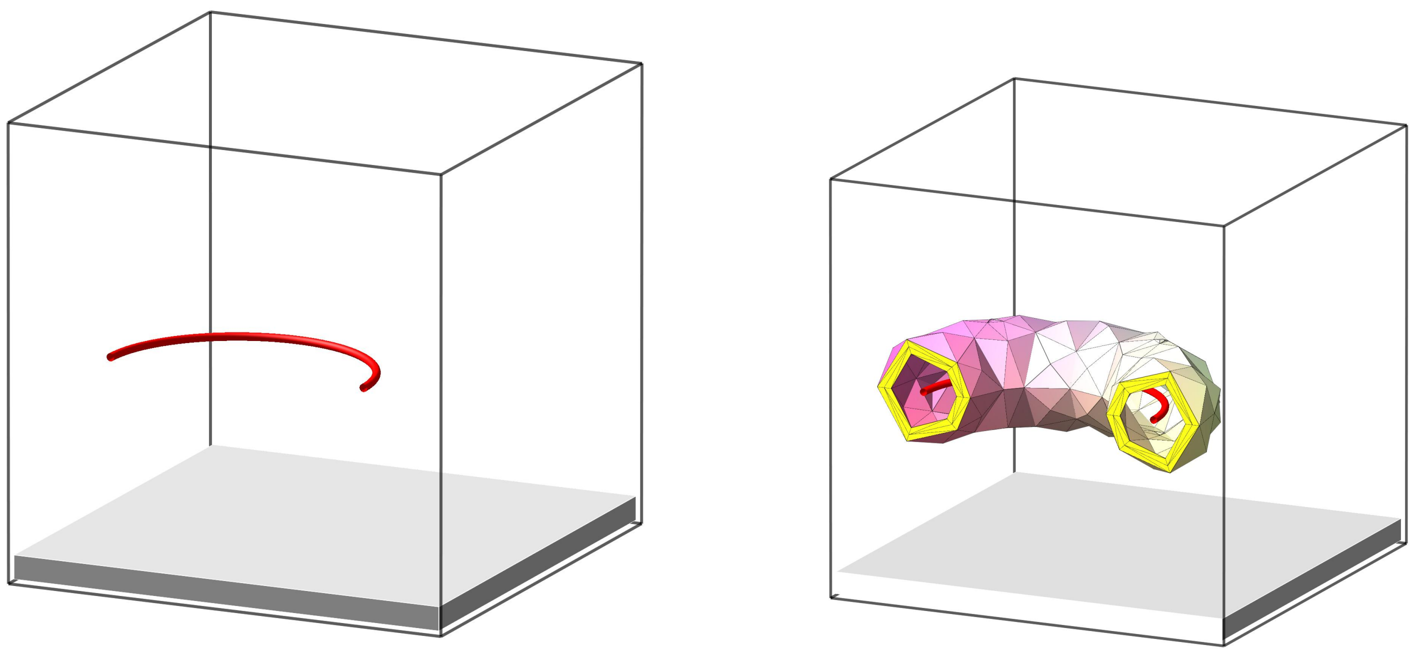
Forbindes til triangulering

Den halve enhedscirkel; Trianguleret



Trianguleret dobbelt opfedning med plombering

Den halve enhedscirkel; Trianguleret

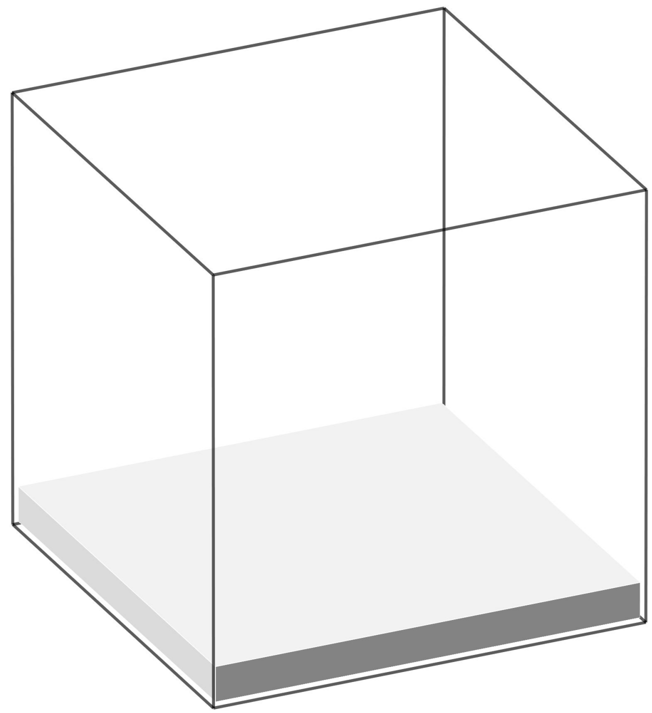


Dobbelt opfedning og plombering – nu med trekanter

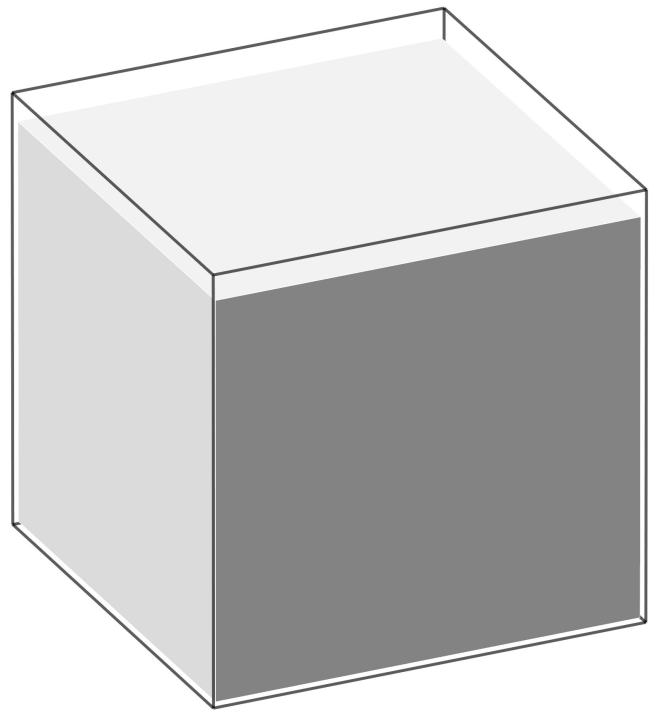
Den halve enhedscirkel 3D printes

Den triangulerede dobbelte opfedning med plombering kan nu sendes til 3D-printeren:

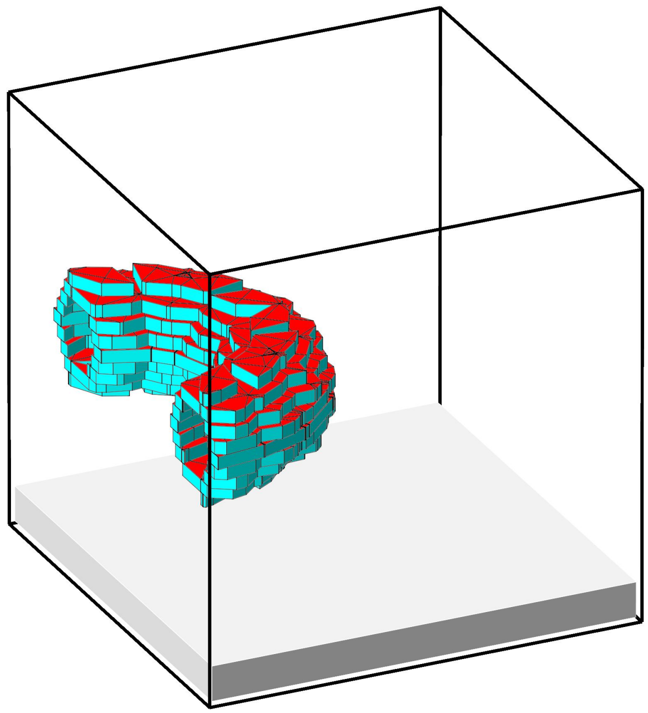
Den halve enhedscirkel 3D printes

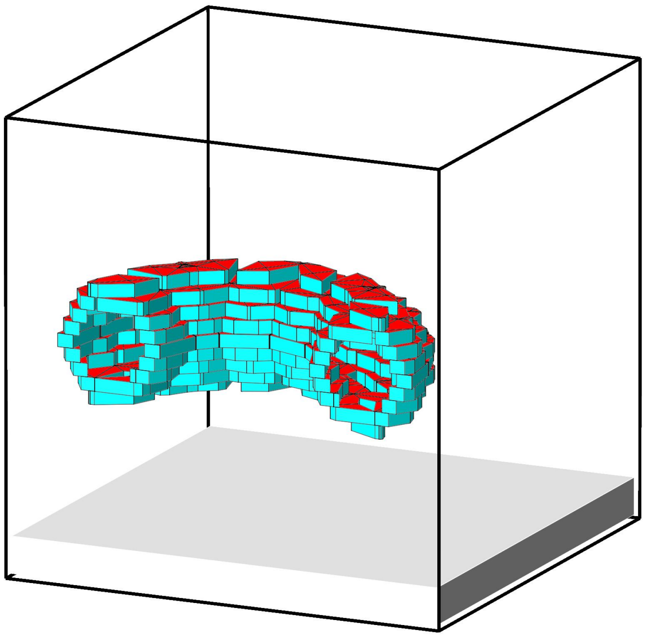


Den halve enhedscirkel 3D printes



Den halve enhedscirkel 3D printet

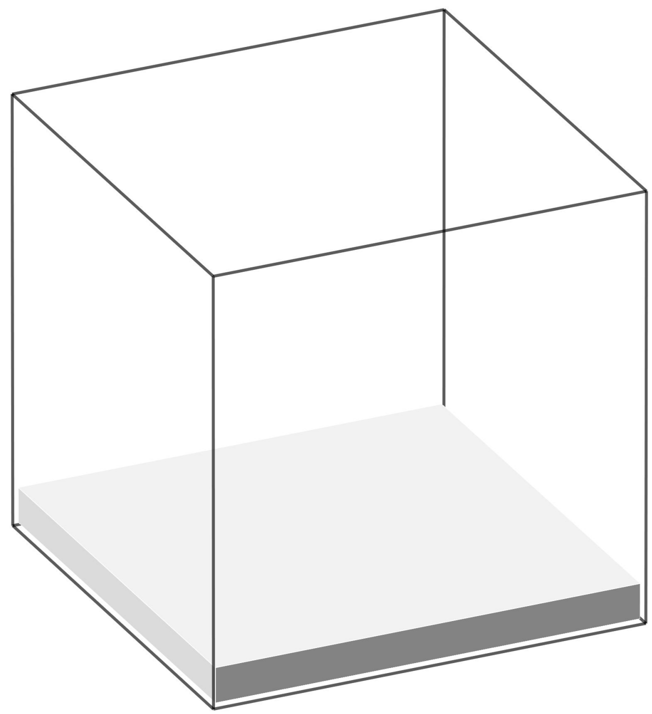




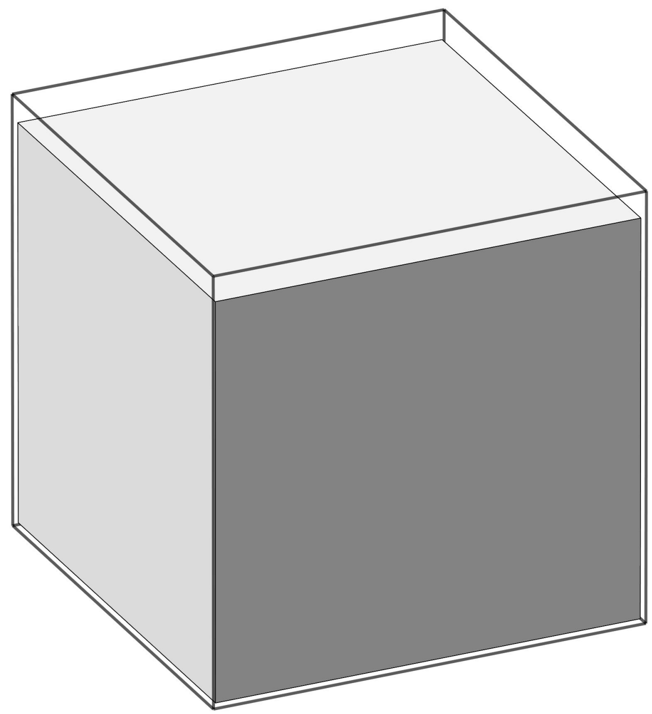
Den halve enhedscirkel

Der er herefter (mindst) to oplagte muligheder for udbedring af den tydelige 'ruhed' af 3D printet:

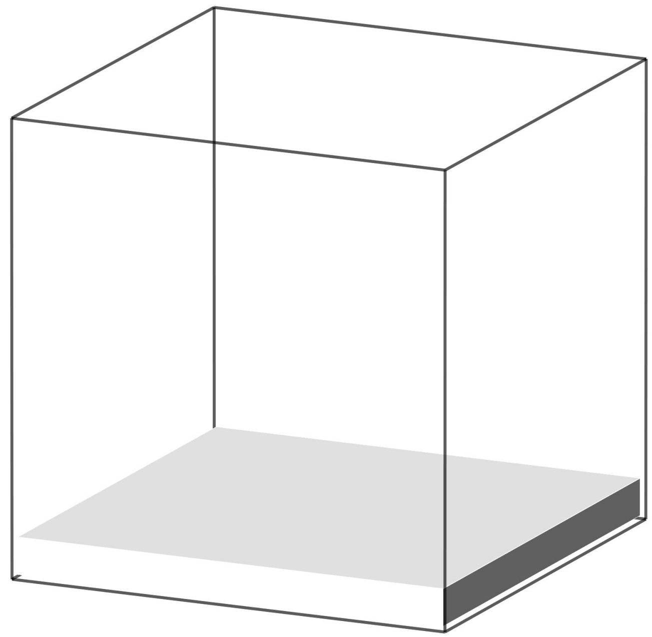
Den halve enhedscirkel; Finere gitter



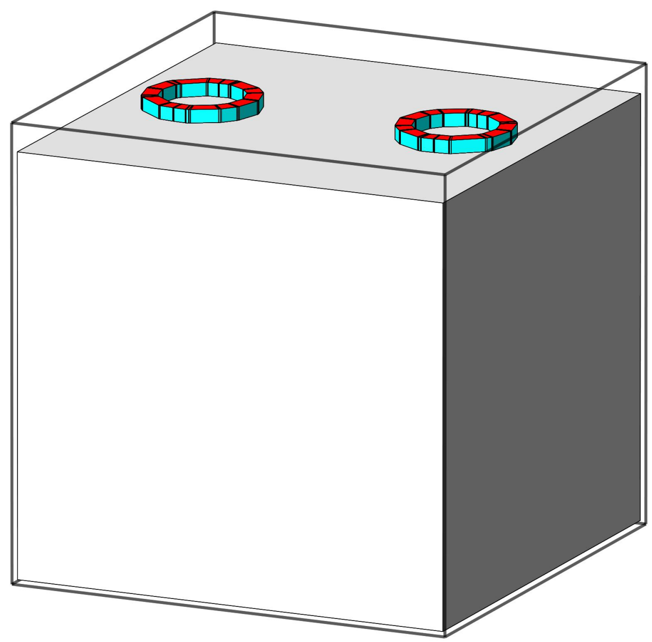
Den halve enhedscirkel; Finere gitter

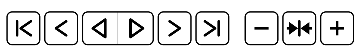


Den halve enhedscirkel; Ny orientering

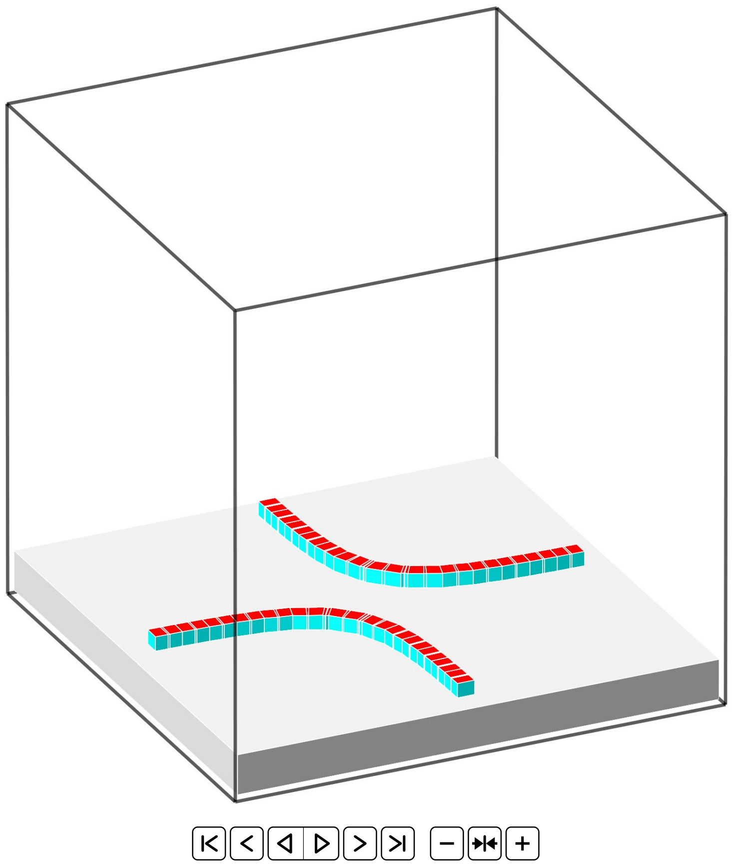


Den halve enhedscirkel; Ny orientering



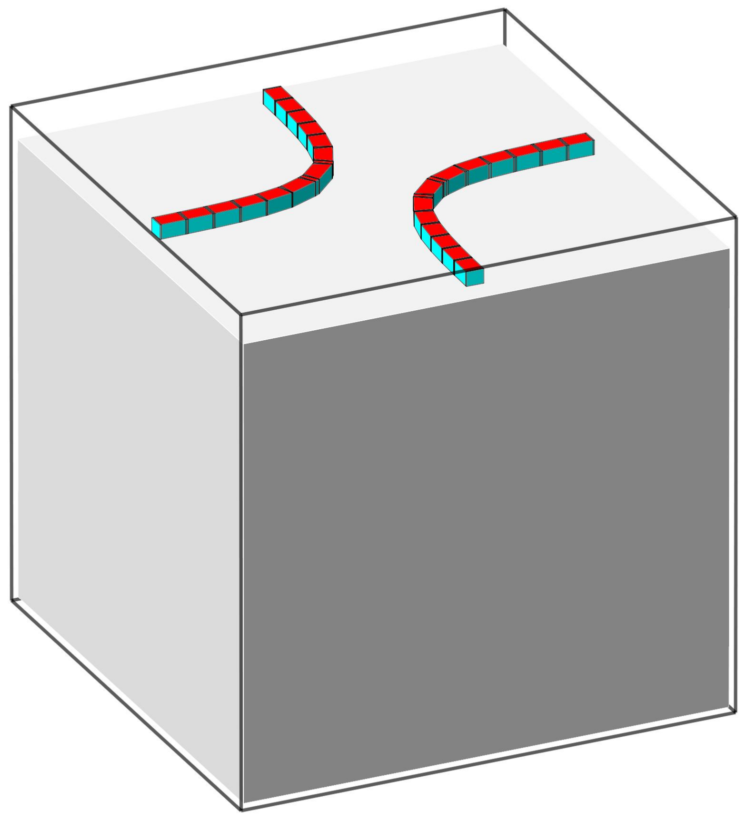


Flere, mere interessante, flader



Scherk's enkelt-periodiske minimalflade

Mere interessante flader



Scherk's enkelt-periodiske minimalflade

Princip

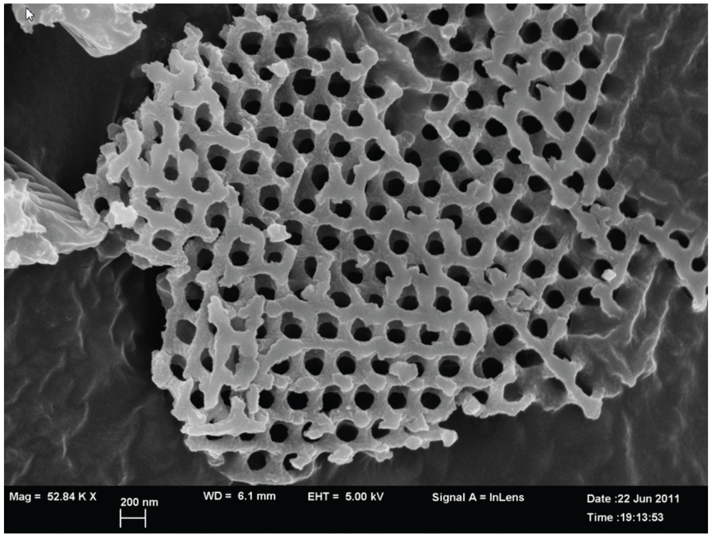
Ethvert passende regulært objekt, som er givet ved en af repræsentationerne:

- Eksplicit
- Implicit
- Parametrisk

– kan i sin helhed 3D-printes på den angivne måde.

Gyroiden

Grøn busk sommerfugl



Thank you for your attention!

Comments, questions?