On stability of viscoelastic elements of thin-shelled constructions under aeronydrodynamic action

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Abstract
Dynamic stability of viscoelastic elements of thin-shelled constructions (pipeline; a wing; a wing profile; channel walls) under interaction with subsonic flow of fluid or gas is studied.

Introduction

Dynamic stability of viscoelastic elements of thin-shelled constructions under interaction with flow of fluid or gas is studied. It is considered subsonic regime. For the deduction of the viscoelastic plates oscillating equations the model of aging viscoelastic body is used. According to this model connection between tensions and deformations described by the Volterra-Feucht integro-differential equation. Aerodynamic load is determined by asymptotic aerohydromechanics equations.

Statements and investigation methods offered for dynamical damping viscoelastic bodies, being in contact with subsonic flow of the fluid or the gas, lead to the study of linked initial boundary problems to systems of partial integro-differential equations. Having been based on the construction of functionals, corresponding to these equation, solutions' stability conditions are obtained for some aerohydroelastical problems, in particular for dynamics of elements of a plane channel, through which fluid flows; of elements of a wing profile; of elements of a facilatated weight wing; pipeline.

Dynamic stability of pipeline

Transverse fluctuations of a viscoelastic aging peg, connected with aging base, is described by the equation for a deflection $w(x,t)$

$$M\ddot{w}(x,t)+\left\{D\left[w''(x,t)-\int\limits_0^t \frac{\partial Q(x,\tau,t)}{\partial \tau} w''(x,\tau)\,d\tau\right]+\xi\dot{w}''(x,t)\right\}''-(\eta\ddot{w}'(x,t))'+(Pw'(x,t))'+$$

$$+\beta\left[w(x,t)-\int\limits_0^t \frac{\partial V(x,\tau,t)}{\partial \tau} w(x,\tau)\,d\tau\right]+\gamma\dot{w}(x,t)+\alpha\dot{w}'(x,t)+f(x,t,w)+$$

$$+g(x,t,w,\dot{w},w',w'',\dot{w}')-w''(x,t)\left[\mu\int\limits_0^\ell w'^2(x,t)\,dx+\nu\frac{\partial}{\partial t}\int\limits_0^\ell w'^2(x,t)\,dx\right]=0,$$

where $0\le x\le \ell$; $M=\rho F$ - specific mass of pipeline ($\rho(x)$ - density, $F(x)$ - area of cross-section); $D=EJ$ - bending rigidity ($E(x)$ - module of bounce, $J(x)$ - moment of inertias of section to comparatively neutral axis); $\xi(x)$ - coefficient of internal damping (models Feucht); $\eta=\rho(x)J(x)$ - coefficient, accounting inertia of rotation; $P(x,t)$ - portioned longitudal compressing (decompressing) effort; $\gamma(x,t)$ - coefficient of linear external damping (basis); $\beta(x,t)$ - coefficient of linear rigidity of basis; $Q(x,\tau,t), V(x,\tau,t)$ - measures to relaxations of lumpy aging peg and basis accordingly; μ, ν - some functions of time or constant; $f(x,t,w), g(x,t,w,\dot{w},w',w'',\dot{w}')$ - other nonlinear efforts, acting on peg (for instance, nonlinear forming effort of reaction and damping of basis, in general event is possible to expect their dependency from displacement w, velocities of displacement $\dot{w}$, corner of tumbling w', velocities of rotating $\dot{w}'$, curvatures w''). Nonlinear integral members take a nonlinear longitudal effort, appearing because of restrictions, assessed on moving the ends of peg $x=0, x=\ell$. In the event of absolutely rolling ends $\nu=\mu=0$. Point denotes private derivative on a time t; prime - on the coordinate x.

Member $\alpha\dot{w}'(x,t)$ appears in that event, when inwardly hollow peg (pipes) flows a fluid. In this case

$$M = m_0(x) + m_*, \quad P = P_0(x,t) + m_* U^2, \quad \alpha = 2Um_*$$

where $m_0(x)$ - a specific mass of peg, m_* - a specific mass of fluid, U - a velocity of moving a fluid, $P_0(x,t)$ - portioned (given) longitudal effort. Members $2Um_*\dot{w}'$ and m_*U^2w'' accordingly characterize Coriolis and centrifugal forces, acting on the fluid.

Consider a functional

$$J(t) = \frac{1}{2}\int_0^\ell \Bigg\{ M\dot{w}^2 + D\Big[1 + Q(x,0,t) - Q(x,t,t)\Big]w''^2 + \eta\dot{w}'^2 - Pw'^2 +$$

$$+\beta\Big[1 + V(x,0,t) - V(x,t,t)\Big]w^2 + D\int_0^t \frac{\partial Q}{\partial\tau}\Big[w''(x,t) - w''(x,\tau)\Big]^2 d\tau +$$

$$\beta\int_0^t \frac{\partial V}{\partial\tau}\Big[w(x,t) - w(x,\tau)\Big]^2 d\tau + 2\int_0^w f(x,t,z)\,dz \Bigg\} dx + \frac{1}{4}\mu\left(\int_0^\ell w'^2\,dx\right)^2.$$

Herewith are superimposed restrictions on the type of fastening the ends of a pipeline:

	$x=0$	$x=\ell$		$x=0$	$x=\ell$
1.	r	r	2.	r	h
3.	r	f.e.	4.	h	r
5.	h	h			

Here accepted following indications: "r" - the rigid fastening ($w=0,\ w'=0$); "h" - the higed fastening ($w=0, \tilde{M}=0$); "fe" - the rigid fastening in a free element ($w'=0, \tilde{Q}=0$). Here $\tilde{M}$ - bending moment, $\tilde{Q}$ - transverse effort.

Bring simplest variant of conditions of stability, expecting simultaneously that M, D, ξ, η, γ, β, U hang from x, parameters μ, ν are constant, but effort P_0 depends on t

$$M>0,\ D>0,\ \eta\geq 0,\ \xi\geq 0,\ \beta\geq 0,\ \mu\geq 0,\ \nu\geq 0,\ m_*U'<\gamma$$

$$\frac{\partial Q(x,\tau,t)}{\partial\tau}\geq 0,\ \frac{\partial V(x,\tau,t)}{\partial\tau}\geq 0,\ \frac{\partial Q(x,0,t)}{\partial t} - \frac{dQ(x,t,t)}{dt}\leq 0,\ \frac{\partial V(x,0,t)}{\partial t} - \frac{dV(x,t,t)}{dt}\leq 0,$$

$$\frac{\partial^2 Q(x,\tau,t)}{\partial t\partial\tau}\leq 0,\ \frac{\partial^2 V(x,\tau,t)}{\partial t\partial\tau}\leq 0,\ 1+Q(x,0,t)-Q(x,t,t)\geq 0,\ 1+V(x,0,t)-V(x,t,t)\geq 0,$$

$$P_0 < \frac{2}{\ell^2}\inf_{x,t}\Big[D(1+Q(x,0,t)-Q(x,t,t))\Big] - m_* U^2, \quad \frac{dP_0}{dt} \geq 0$$

$$\int_0^w f(x,t,z)dz \geq 0, \int_0^w \frac{\partial f(x,t,z)}{\partial t}dz \leq 0, \dot{w}g(x,t,w,\dot{w},w',\dot{w}',w'') \geq 0$$

Enumerated condition must be executed for all x, t, τ.

Indicate examples of the functions $f(x,t,w)$,$g(x,t,w,\dot{w},w',\dot{w}',w'')$, satisfying conditions

$$f(x,t,w) = \sum_{k=1}^{n} f_k(x,t)w^{2k+1}, \quad f_k(x,t) \geq 0, \quad \frac{\partial f_k(x,t)}{\partial t} \leq 0$$

$$g = \sum_{i=0}^{m_1}\sum_{j=0}^{m_2} g_{ij}(x,t)w^{2j}(\dot{w})^{2i+1}, \quad g_{00} \equiv 0, \quad g_{ij}(x,t) \geq 0$$

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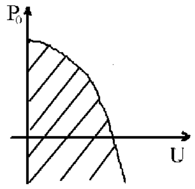
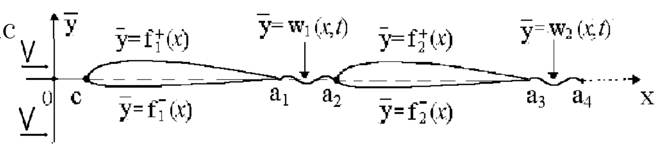


Dynamic Stability of Viscoelastic Elements of Facilatated Weight Wing

Let us c(... ıt dynamic stability of viscoelastic elements of the fa ... ow around a thin profile in an unbounded subsonic



In the statement corresponding to small perturbations of the homogeneous fluid stream directed along the x - axis and to small deviations of the plates this problem is formulated by the following way

$$\triangle\varphi \equiv \varphi_{xx} + \varphi_{yy} = 0, \ (x,y) \in J = R^2 \backslash [c,d], \qquad |\bigtriangledown\varphi|^2_\infty = (\varphi_x^2 + \varphi_y^2 + \varphi_t^2)_\infty = 0, \tag{1}$$

$$\varphi_y^{\pm}(x,0,t) = \lim_{y\to\pm 0} \varphi_y(x,y,t) = V f_1^{\pm\prime}(x), \qquad x \in (c, a_1), \tag{2}$$

$$\varphi_y^{\pm}(x,0,t) = w_{kt}(x,t) + V w_{kx}(x,t), \ x \in (a_{2k-1}, a_{2k}), k = 1 \div n, \tag{3}$$

$$\varphi_y^{\pm}(x,0,t) = V {f_{k+1}^{\pm}}'(x), \quad x \in (a_{2k}, a_{2k+1}), \ k = 1 \div (n-1) \tag{4}$$

$$\varphi_y^{\pm}(x,0,t) = V {f_{n+1}^{\pm}}'(x), \qquad x \in (a_{2n}, d), \tag{5}$$

$$L_k(w_k)=\rho\left(\varphi_t^+-\varphi_t^-\right)+\rho V\left(\varphi_x^+-\varphi_x^-\right),\ y=0, x\in(a_{2k-1},a_{2k}), \tag{6}$$

The operators $L_k(w_k)$ will be assigned the following nonlinear integro-differential expressions

$$L_k(w_k)\equiv M_k(x)\ddot{w}_k(x,t)+\left[D_k(x)\left(w_k''(x,t)-\int_0^t R_{k1}(x,\tau,t)w_k''(x,\tau)d\tau\right)+\beta_{2k}(x)\,\dot{w}_k{}''(x,t)\right]''+$$

$$+(N_k(x,t)w_k'(x,t))'+\beta_{1k}(x,t)\dot{w}_k(x,t)+\beta_{0k}(x,t)\left(w_k(x,t)-\int_0^t R_{k2}(x,\tau,t)w_k(x,\tau)\,d\tau\right)+ \tag{7}$$

$$+g_k(x,t,w_k,\dot{w}_k)+h_k(x,t,w_k)-w_k''(x,t)\left[\mu_k\int_{a_{2k-1}}^{a_{2k}} {w_k'}^2dx+\nu_k\frac{\partial}{\partial t}\int_{a_{2k-1}}^{a_{2k}} {w_k'}^2dx\right].$$

Here $\varphi(x,y,t)$ is the potential of the fluid velocity; $w_k(x,t)$ the plate deflections function; x,y are Cartesian coordinates, t is the time variable; $f_k^{\pm}$ are given functions determining the form of undeformable parts of a wing; V is unperturbed homogeneous flow velocity; ρ is the density of the fluid; D and M are the deflection rigidity and the specific mass of the plate; N - longitudal compressing (decompressing) effort; β_0 is the stiffness coefficient of the base; β_1 is the rotational inertia coefficient, β_2 is damping coefficient of the plate material; $R_1(\tau,t), R_2(\tau,t)$ are the relaxation kernels accounting for the aging of the materials of the plate and the base; the functions h,g represent nonlinear components reactions of the basis or other nonlinear influences (managements); the nonlinear integrated members take into account influence of longitudinal efforts; a point over letters denotes time derivative, a prime is used for the derivative with respect to x or τ, subindices x,y,t designate partial derivatives with respect to the corresponding variables.

Using methods of the theory of complex variable functions, according to the formulas (6), the solusion of a problem can be reduced to a system of equations for determination of $w_k(x,t)$

$$L_i(w_i)=\frac{2\rho\sqrt{(x-c)(d-x)}}{\pi}\int_{a_1}^{d}\frac{\partial\omega^1(\tau,t)}{\partial t}\frac{d\tau}{\sqrt{(\tau-c)(d-\tau)(x-\tau)}}+\frac{2\rho V}{\pi\sqrt{(x-c)(d-x)}}\times$$

$$\times\sum_{k=1}^{n}\int_{a_{2k-1}}^{a_{2k}}\frac{\partial\omega^1(\tau,t)}{\partial\tau}\frac{\sqrt{(\tau-c)(d-\tau)}}{x-\tau}d\tau+\frac{V\rho}{2\pi}\int_c^d(\omega_+^2(\tau)+\omega_-^2(\tau))\left(1+\frac{\sqrt{(d-\tau)(\tau-c)}}{\sqrt{(d-x)(x-c)}}\right)\frac{d\tau}{x-\tau}, \tag{8}$$

$$x\in(a_{2i-1},a_{2i}),$$

$$\omega^{\mathbf{1}}(\mathbf{x},\mathbf{t})=\begin{cases}0, & x\in[c,a_1],\\ \int_{a_1}^{x}(w_{1t}+Vw_{1x})\,dx, & x\in(a_1,a_2),\\ \sum_{k=1}^{r}\int_{a_{2k-1}}^{a_{2k}}(w_{kt}+Vw_{kx})dx, & x\in(a_{2r},a_{2r+1}), r=1\div n-1,\\ \sum_{k=1}^{r-1}\int_{a_{2k-1}}^{a_{2k}}(w_{kt}+Vw_{kx})dx+\int_{a_{2r-1}}^{x}(w_{rt}+Vw_{rx})dx, & x\in(a_{2r-1},a_{2r}), r=2\div n,\\ \sum_{k=1}^{n}\int_{a_{2k-1}}^{a_{2k}}(w_{kt}+Vw_{kx})dx, & x\in(a_{2n},d);\end{cases} \tag{9}$$

$$\omega^{\mathbf{2}}_{\pm}(\mathbf{x}) = \begin{cases} V{f_1^{\pm}}'(x)\ x \in [c, a_1], \\ V{f_{k+1}^{\pm}}'(x),\ x \in [a_{2k}, a_{2k+1}],\ k = 1 \div n-1, \\ 0, \quad x \in (a_{2k-1}, a_{2k}), r = 1 \div n, \\ V{f_{n+1}^{\pm}}'(x),\ x \in [a_{2n}, d], \end{cases}$$

This system of equations is obtained with any modes of the fastening of viscoelastic plates.

Let us obtain the sufficient stability conditions of the solutions of a system of integro-differential equations (8) with respect to perturbations of the initial conditions. If the system of equation (8) is linear or the form of the cross-section of the wing is symmetrical (then are absent the two last summand), it is enough to investigate a stability of trivial solution of the appropriate system of the homogeneous equations

$$L_i(w_i) = \frac{2\rho\sqrt{(x-c)(d-x)}}{\pi} \int\limits_{a_1}^{d} \frac{\partial \omega^1(\tau,t)}{\partial t} \frac{d\tau}{\sqrt{(\tau-c)(d-\tau)(x-\tau)}} + \\ + \frac{2\rho V}{\pi\sqrt{(x-c)(d-x)}} \sum_{k=1}^{n} \int\limits_{a_{2k-1}}^{a_{2k}} \frac{\partial \omega^1(\tau,t)}{\partial \tau} \frac{\sqrt{(\tau-c)(d-\tau)}}{x-\tau} d\tau, \quad x \in (a_{2i-1}, a_{2i}),\ i = 1 \div n. \tag{10}$$

where $L_i(w_i)$ and $\omega^1(\tau,t)$ are defined by expressions (7) and (9) accordingly.

Let us introduce the function

$$K(\tau,x) = 2\,ln \frac{\sqrt{(x-c)(d-\tau)} + \sqrt{(\tau-c)(d-x)}}{\mid \sqrt{(x-c)(d-\tau)} - \sqrt{(\tau-c)(d-x)} \mid}. \tag{11}$$

It is easy to see, that $K(\tau,x) \geq 0, K(\tau,x) = K(x,\tau)$. Substituting $\omega^1(\tau,t)$ in the system of equation (10), we shall reduce it to the form

$$L_i(w_i) = -\frac{\rho}{\pi} \sum_{k=1}^{n} \int\limits_{a_{2k-1}}^{a_{2k}} (\ddot{w}_k + V\ \dot{w}_k{}')K(\tau,x)\,d\tau - \frac{V\rho}{\pi} \sum_{k=1}^{n} \int\limits_{a_{2k-1}}^{a_{2k}} (\dot{w}_k + V\ w_k{}')\frac{\partial K(\tau,x)}{\partial x}\,d\tau, \tag{12}$$

$$x \in (a_{2i-1}, a_{2i}), \quad k = 1 \div n.$$

Let us introduce the functional

$$\Phi(t) = \sum_{k=1}^{n} \int\limits_{a_{2k-1}}^{a_{2k}} \Bigg\{ M_k\ \dot{w}_k{}^2 + D_k\Bigg[(1+Q_{k1}(x,0,t))w_k''^2 + \int\limits_0^t \frac{\partial Q_{k1}}{\partial \tau}(w_k''(x,t) - w_k''(x,\tau))^2 d\tau\Bigg] + \\ +\beta_{0k}\Bigg[(1+Q_{k2}(x,0,t))w_k^2 + \int\limits_0^t \frac{\partial Q_{k2}}{\partial \tau}(w_k(x,t) - w_k(x,\tau))^2 d\tau\Bigg] + 2\int\limits_0^{w_k} h_k(x,t,z)dz\Bigg\} dx + \\ + \frac{1}{4}\sum_{k=1}^{n} \mu_k \left(\int\limits_{a_{2k-1}}^{a_{2k}} w_k'^{\,2} dx \right)^2 + I(t) + J(t), \tag{13}$$

$$I(t) = \frac{\rho}{2\pi} \sum_{i=1}^{n}\sum_{j=1}^{n} \int\limits_{a_{2i-1}}^{a_{2i}} dx \int\limits_{a_{2j-1}}^{a_{2j}} (\dot{w}_i(x,t) + \dot{w}_j(\tau,t))^2 K(\tau,x)\,d\tau,$$

$$J(t) = \frac{\rho V^2}{2\pi} \sum_{i=1}^{n} \sum_{j=1}^{n} \int\limits_{a_{2i-1}}^{a_{2i}} dx \int\limits_{a_{2j-1}}^{a_{2j}} (w_i'(x,t) - w_j'(\tau,t))^2 K(\tau,x)\, d\tau. \quad (14)$$

The following theorem is proved on a base of study this functional on stability of viscoelactic lightened weight wing elements

THEOREM. Lets assume, that the extremities of plates are fixed rigidly or hingedly

$$w_k(x,t) = {w_k}'(x,t) = 0,\ w_k(x,t) = {w_k}''(x,t) = 0,\ x = a_{2k-1} \,\text{or}\, x = a_{2k}. \quad (15)$$

the kernels of a relaxation $R_{ki}(x,\tau,t) (i = 1,2)$ with $0 \le \tau \le t$ satisfy to conditions

$$R_{ki}(x,\tau,t) = \frac{\partial Q_{ki}}{\partial \tau}(x,\tau,t), \quad Q_{ki}(x,t,t) = 0, \quad \frac{\partial Q_{ki}}{\partial t}(x,0,t) \le 0,$$

$$\frac{\partial Q_{ki}}{\partial \tau}(x,\tau,t) \ge 0,\ \frac{\partial^2 Q_{ki}}{\partial \tau \partial t}(x,\tau,t) \le 0,\ 1 + Q_{ki}(x,0,\infty) > 0, \quad a_{2k-1} \le x \le a_{2k}. \quad (16)$$

and are executed inequalities

$$\int\limits_0^{w_i} h_i(x,t,z)dz \ge 0,\ \int\limits_0^{w_i} \frac{\partial h_i(x,t,z)}{\partial t} dz \le 0,\ \dot{w}_i g_i(x,t,w_i,\dot{w}_i) \ge 0,\ M_i \ge \frac{\rho K_i}{\pi},\ \mu_i \ge 0,\ \nu_i \ge 0, \quad (17)$$

$$N_i{}^* < \lambda_{1i} D_{i*} - \frac{\rho K_i V^2}{\pi},\ D_i > 0,\ \beta_{ji} \ge 0\,(j = 0,1,2),\ \dot{\beta}_{0i} \le 0, \dot{N}_i(x,t) \ge 0,\ i = 1 \div n, \quad (18)$$

where $K_i = \sup\limits_{x \in (a_{2i-1}, a_{2i})} \sum\limits_{j=1}^{n} \int\limits_{a_{2j-1}}^{a_{2j}} K(\tau,x)\, d\tau,\ i = 1 \div n; D_{i*} = \inf\limits_x D_i(x)(1 + Q_{i1}(x,0,\infty));$

$$N_i{}^* = \sup_{x,t} N_i(x,t), \quad x \in [a_{2i-1}, a_{2i}], t \in [0,\infty);$$

λ_{1i} are least own values of marginal problem $\psi^{IV}(x) = -\lambda \psi''(x)$ with boundary conditions, corresponding specified types of fastening. Then decision $w_i(x,t)$ equation systems (12) are stability with respect to outraging the initial values of velocity and curvatures $\dot{w}_{0i} = \dot{w}_i(x,0)$, $w''_{0i} = w''_i(x,0)$ $(i = 1 \div n)$.

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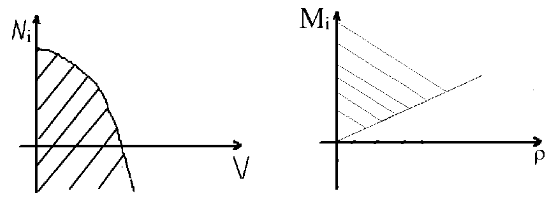
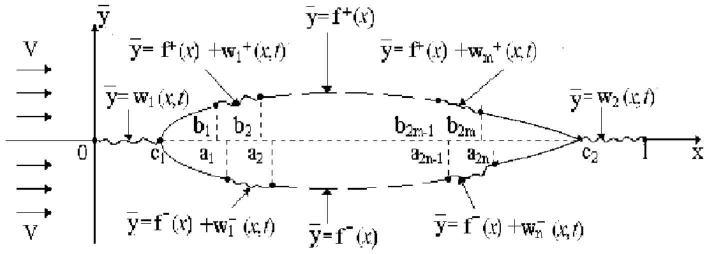


Dynamic Stability of Viscoelastic Elements of the Wing Profile

We consider the planar problem about dynamic stability of viscoelastic elements of the wing without a detachment flow around a thin profile in an unbounded subsonic fluid stream.



This problem is formulated by the following way

$$\triangle\varphi \equiv \varphi_{xx} + \varphi_{yy} = 0, \quad (x,y) \in \Im = R^2 \setminus [0,d]$$

$$\varphi_y^+(x,0,t) = w_{kt}^+(x,t) + Vw_{kx}^+(x,t) + Vf_+'(x),\ x \in (b_{2k-1}, b_{2k}), \quad k = 1 \div m,$$

$$\varphi_y^+(x,0,t) = Vf_+'(x),\ x \in [0,] \setminus \left(\bigcup_{k=1}^{m} [b_{2k-1}, b_{2k}] \right),$$

$$\varphi_y^-(x,0,t) = w_{kt}^-(x,t) + Vw_{kx}^-(x,t) + Vf_-'(x),\ x \in (a_{2k-1}, a_{2k}), \quad k = 1 \div n,$$

$$\varphi_y^-(x,0,t) = Vf_-'(x),\ x \in [0,] \setminus \left(\bigcup_{k=1}^{n} [a_{2k-1}, a_{2k}] \right),$$

$$\varphi_y^\pm(x,0,t) = w_{1t}(x,t) + Vw_{1x}(x,t),\ x \in (a,b),$$

$$\varphi_y^\pm(x,0,t) = w_{2t}(x,t) + Vw_{2x}(x,t),\ x \in (,d),$$

$$|\nabla\varphi|_\infty^2 = (\varphi_x^2 + \varphi_y^2 + \varphi_t^2)_\infty = 0$$

$$\mathbf{L_1}(w_1) = \rho\left(\varphi_t^+ - \varphi_t^-\right) + \rho V\left(\varphi_x^+ - \varphi_x^-\right), x \in (a,b),$$

$$\mathbf{L_2}(w_2) = \rho\left(\varphi_t^+ - \varphi_t^-\right) + \rho V\left(\varphi_x^+ - \varphi_x^-\right), x \in (c,d),$$

$$L_k^+(w_k^+) = \rho(\varphi_t^+ + V\varphi_x^+),\ x \in (b_{2k-1}, b_{2k}),$$

$$L_k^-(w_k^-) = -\rho(\varphi_t^- + V\varphi_x^-), x \in (a_{2k-1}, a_{2k}),$$

Here $\varphi(x,y,t)$ is the potential of fluid velocity; $w_k^+(x,t), w_k^-(x,t)$ - the deflection functions of the plates on the upper and lower parts of a profile; $w_1(x,t), w_2(x,t)$ - the plate deflection functions of the trailing-edge and advancing-edge; $f_\pm(x)$ are given functions determining the form of the profile; $c - b$ is the length of profile chord; the operators $L_k(w_k), L_k^\pm(w_k^\pm)$ will be assigned the nonlinear integro-differential expressions (7).

Similarly to the previous cases, using methods of the theory of complex variable functions, the solution of the problem can be reduced to a system of equations

$$L_k(w_k) = -\frac{2\rho}{\pi}\int_a^b \bar{w}_{1t}K(\tau,x)d\tau - \frac{2V\rho}{\pi}\int_a^b \bar{w}_1 K_x(\tau,x)d\tau - \frac{2\rho}{\pi}\int_c^d \bar{w}_{2t}K(\tau,x)d\tau - \frac{2V\rho}{\pi}\int_c^d \bar{w}_2 K_x(\tau,x)d\tau -$$

$$-\frac{\rho}{\pi}\sum_{k=1}^{m}\int\limits_{b_{2k-1}}^{b_{2k}}\bar{w}_{kt}^{+}K(\tau,x)d\tau-\frac{V\rho}{\pi}\sum_{k=1}^{m}\int\limits_{b_{2k-1}}^{b_{2k}}\bar{w}_{k}^{+}K_x(\tau,x)d\tau-\frac{\rho}{\pi}\sum_{k=1}^{n}\int\limits_{a_{2k-1}}^{a_{2k}}\bar{w}_{kt}^{-}K(\tau,x)d\tau-$$

$$-\frac{V\rho}{\pi}\sum_{k=1}^{n}\int\limits_{a_{2k-1}}^{a_{2k}}\bar{w}_{k}^{-}K_x(\tau,x)d\tau,\qquad x\in(a,b)\,\text{for}\,k=1,\ x\in(c,d)\,\text{for}\,k=2;$$

$$L_k^{\pm}(w_k^{\pm})=-\frac{\rho}{\pi}\int\limits_a^b\bar{w}_{1t}K(\tau,x)d\tau-\frac{V\rho}{\pi}\int\limits_a^b\bar{w}_{1}K_x(\tau,x)d\tau-\frac{\rho}{\pi}\int\limits_c^d\bar{w}_{2t}K(\tau,x)d\tau-\frac{V\rho}{\pi}\int\limits_c^d\bar{w}_{2}K_x(\tau,x)d\tau-$$

$$-\frac{\rho}{\pi}\sum_{k=1}^{m}\int\limits_{b_{2k-1}}^{b_{2k}}\bar{w}_{kt}^{+}K^{\pm}(\tau,x)d\tau-\frac{V\rho}{\pi}\sum_{k=1}^{m}\int\limits_{b_{2k-1}}^{b_{2k}}\bar{w}_{k}^{+}K_x^{\pm}(\tau,x)d\tau-\frac{\rho}{\pi}\sum_{k=1}^{n}\int\limits_{a_{2k-1}}^{a_{2k}}\bar{w}_{kt}^{-}K^{\mp}(\tau,x)d\tau-$$

$$-\frac{V\rho}{\pi}\sum_{k=1}^{n}\int\limits_{a_{2k-1}}^{a_{2k}}\bar{w}_{k}^{-}K_x^{\mp}(\tau,x)d\tau,\qquad x\in(b_{2k-1},b_{2k})\,\text{for}\,w_k^{+}(x,t);\ \ x\in(a_{2k-1},a_{2k})\,\text{for}\,w_k^{-}(x,t),$$

$$\text{where}\ \ \bar{w}_k^{\pm}(\tau,t)=w_{kt}^{\pm}(\tau,t)+Vw_{k\tau}^{\pm}(\tau,t),\ \bar{w}_1(\tau,t)=w_{1t}(\tau,t)+Vw_{1\tau}(\tau,t),$$

$$\bar{w}_2(\tau,t)=w_{2t}(\tau,t)+Vw_{2\tau}(\tau,t),\quad K(\tau,x)=\ln\frac{\sqrt{(d-\tau)(x-a)}+\sqrt{(d-x)(\tau-a)}}{|\sqrt{(d-\tau)(x-a)}-\sqrt{(d-x)(\tau-a)}|},\quad \tau\neq x,$$

$$K^{\pm}(\tau,x)=\ln\frac{d-a}{|\sqrt{(d-\tau)(x-a)}\mp\sqrt{(d-x)(\tau-a)}|},\quad \tau\neq x.$$

The dynamic stability of solutions of this system of integro-differential equations is investigated under the assumption that at every end of the plate one of the following boundary conditions is fulfilled: $1)w=w_x=0; 2)w=w_{xx}=0$. Using for this system of equations a functional of Liapunov's type, we will obtain the following stability conditions for the plates oscillations $w_1(x,t), w_2(x,t), w_k^{+}(x,t)(k=1\div m), w_k^{-}(x,t)(k=1\div n)$ with respect to small changes of initial values $\dot{w}_k(x,0), w_k''(x,0)\,(k=1,2),\ \dot{w}_k^{+}(x,0), w_k^{+''}(x,0)(k=1\div m),\ \dot{w}_k^{-}(x,0), w_k^{-''}(x,0)\,(k=1\div n)$:

$$R_{ki}(x,\tau,t)=\frac{\partial Q_{ki}}{\partial\tau}(x,\tau,t), Q_{ki}(x,t,t)=0, \frac{\partial Q_{ki}}{\partial t}(x,0,t)\leq 0, \frac{\partial Q_{ki}}{\partial\tau}(x,\tau,t)\geq 0, \frac{\partial^2 Q_{ki}}{\partial\tau\partial t}(x,\tau,t)\leq 0,$$

$$1+Q_{ki}(x,0,\infty)>0,\ \tau\in[0,t], i=1,2;\ \beta_{ik}(x,t)\geq 0\,(i=0,1,2),\ \dot{\beta}_{0k}(x,t)\leq 0, \dot{N}_k(x,t)\geq 0,$$

$$M_k\geq\frac{\rho K_{0k}}{\pi}, N_k^{*}<\lambda_{1k}D_{*k}-\frac{\rho K_{0k}V^2}{\pi}, \int\limits_0^{w_k}h_k(x,t,z)dz\geq 0, \int\limits_0^{w_k}\frac{\partial h_k(x,t,z)}{\partial t}dz\leq 0,$$

$$\dot{w}_k g_k(x,t,w_k,\dot{w}_k)\geq 0, \mu_k\geq 0,\ \nu_k\geq 0,\quad x\in(a,b)\,\text{for}\ \,k=1,\quad x\in(c,d)\,\text{for}\ \,k=2;$$

$$R_{ki}^{\pm}(x,\tau,t)=\frac{\partial Q_{ki}^{\pm}}{\partial\tau}(x,\tau,t), Q_{ki}^{\pm}(x,t,t)=0, \frac{\partial Q_{ki}^{\pm}}{\partial t}(x,0,t)\leq 0, \frac{\partial Q_{ki}^{\pm}}{\partial\tau}(x,\tau,t)\geq 0, \frac{\partial^2 Q_{ki}^{\pm}}{\partial\tau\partial t}(x,\tau,t)\leq 0,$$

$$1+Q_{ki}^{\pm}(x,0,\infty)>0,\ \tau\in[0,t],\ i=1,2;\ \beta_{ik}^{\pm}(x,t)\geq 0\,(i=0,1,2), \dot{\beta}_{0k}^{\pm}(x,t)\leq 0, \dot{N}_k^{\pm}(x,t)\leq 0,$$

$$M_k^{\pm}\geq\frac{\rho K_{0k}^{\pm}}{\pi}, N_k^{\pm *}<\lambda_{1k}^{\pm}D_{k\,*}^{\pm}-\frac{\rho K_{0k}^{\pm}V^2}{\pi}, \int\limits_0^{w_k^{\pm}}h_k^{\pm}(x,t,z)dz\geq 0, \int\limits_0^{w_k^{\pm}}\frac{\partial h_k^{\pm}(x,t,z)}{\partial t}dz\leq 0,$$

$$\dot{w}_k^{\pm} g^{\pm}(x,t,w_k^{\pm},\dot{w}_k^{\pm}) \geq 0, \mu_k^{\pm} \geq 0,\ \nu_k^{\pm} \geq 0,\quad x \in (b_{2k-1}, b_{2k}) \text{ for } '+',\ x \in (a_{2k-1}, a_{2k}) \text{ for } '-',$$

where $K_{01} = \sup\limits_{x \in (a,b)} K_1(x)$, $K_{02} = \sup\limits_{x \in (c,d)} K_2(x)$, $K_{0k}^{+} = \sup\limits_{x \in (b_{2k-1}, b_{2k})} K^{+}(x)$, $K_{0k}^{-} = \sup\limits_{x \in (a_{2k-1}, a_{2k})} K^{-}(x)$,

$$K_k(x) = 2\int_a^b K(\tau,x)d\tau + 2\int_c^d K(\tau,x)d\tau + \sum_{k=1}^{m} \int_{b_{2k-1}}^{b_{2k}} K(\tau,x)d\tau + \sum_{k=1}^{n} \int_{a_{2k-1}}^{a_{2k}} K(\tau,x)d\tau,$$

$$K^{\pm}(x) = \int_a^b K(\tau,x)d\tau + \int_c^d K(\tau,x)d\tau + \sum_{k=1}^{m} \int_{b_{2k-1}}^{b_{2k}} K^{\pm}(\tau,x)d\tau + \sum_{k=1}^{n} \int_{a_{2k-1}}^{a_{2k}} K^{\mp}(\tau,x)d\tau,$$

$$D_{k*} = \inf_x [D_k(x)(1 + Q_{1k}(x,0,\infty))],\ N_k^{*} = \sup_{x,t} N_k(x,t),\ x \in (a,b) \text{ for } k=1,\ x \in (c,d) \text{ for } k=2,$$

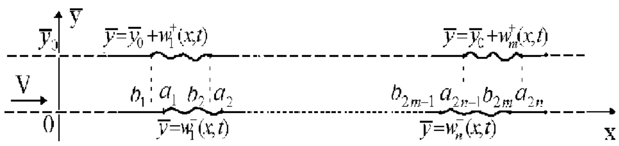
$$D_{k*}^{\pm} = \inf_x \left[D_k^{\pm}(x)(1 + Q_{1k}^{\pm}(x,0,\infty))\right], N_k^{\pm *} = \sup_{x,t} N_k^{\pm}(x,t),$$

$$x \in (b_{2k-1}, b_{2k}) \text{ for } '+',\ x \in (a_{2k-1}, a_{2k}) \text{ for } '-',\ t \in [0,\infty),$$

$\lambda_{1k}, \lambda_{1k}^{\pm}$ are the minimal eigenvalues of the boundary value problem $y^{IV}(x) = -\lambda y''(x)$ with boundary conditions to which the functions $w_k(x,t), w_k^{\pm}$ correspond. The conditions impose restrictions on the magnitude of relaxation, the homogeneous flow velocity V, longitudal compressing (decompressing) efforts and other parameters of a mechanical system.

Dynamic Stability of Viscoelastic Elements of the Channel Walls

We consider the planar problem about dynamic stability of viscoelastic elements of the walls of an infinit ble fluid flows.



This problem is formulated in the following way

$$\triangle\varphi = 0,\ (x,y) \in R^2 : |x| < \infty, y \in [0, y_0];$$

$$\varphi_y(x,y_0,t) = w_{kt}^{+}(x,t) + V w_{kx}^{+}(x,t), \quad x \in (b_{2k-1}, b_{2k}), k = 1 \div m,$$

$$\varphi_y(x,y_0,t) = 0,\ x \in R \setminus \left(\bigcup_{k=1}^{m} [b_{2k-1}, b_{2k}]\right),$$

$$\varphi_y(x,0,t) = w_{kt}^{-}(x,t) + V w_{kx}^{-}(x,t), \quad x \in (a_{2k-1}, a_{2k}), k = 1 \div n,$$

$$\varphi_y(x,0,t) = 0,\ x \in R \setminus \left(\bigcup_{k=1}^{n} [a_{2k-1}, a_{2k}]\right),$$

$$\left(\varphi_x^2+\varphi_y^2\right)_{x=\pm\infty}=0,\ (\varphi_t)_{x=-\infty}=0,\ y\in(0,y_0);$$

$$L_k^+(w_k^+)=-\rho(\varphi_t+V\varphi_x)_{y=y_0},\ x\in(b_{2k-1},b_{2k}),$$

$$L_k^-(w_k^-)=\rho(\varphi_t+V\varphi_x)_{y=0},\ x\in(a_{2k-1},a_{2k}),$$

Here $w_k^{\pm}(x,t)$ are the deflections functions of the plates on the bottom and top walls of the channel respectively; $\varphi(x,y,t)$ is the potential of fluid velocity; the operators $L_k^{\pm}(w_k^{\pm})$ will be assigned the nonlinear integro-differential expressions (7).

Using methods of the theory of complex variable functions, the solution of the problem can be reduced to a system of equations for determination of $w_k^+(x,t), w_k^-(x,t)$. Dynamic stability of the solutions of this system of equations is investigated under the assumption that on every end of the plate one of the following boundary conditions is fulfilled: $1)w=w_x=0; 2)w=w_{xx}=0$. Using a functional of Liapunov's type, we will obtain the following stability conditions for the plate oscillations $w_k^+(x,t)\,(k=1\div m), w_k^-(x,t)\,(k=1\div n)$ with respect to small changes of initial values $\dot{w}_k^+(x,0), w_k^{+\prime\prime}(x,0)\,(k{=}1{\div}m),\ \dot{w}_k^-(x,0), w_k^{-\prime\prime}(x,0)\,(k=1\div n)$:

$$R_{ki}^{\pm}(x,\tau,t)=\frac{\partial Q_{ki}^{\pm}}{\partial\tau}(x,\tau,t),Q_{ki}^{\pm}(x,t,t)=0,\frac{\partial Q_{ki}^{\pm}}{\partial t}(x,0,t)\le 0,\frac{\partial Q_{ki}^{\pm}}{\partial\tau}(x,\tau,t)\ge 0,\frac{\partial^2 Q_{ki}^{\pm}}{\partial\tau\partial t}(x,\tau,t)\le 0,$$

$$1+Q_{ki}^{\pm}(x,0,\infty)>0,\ \tau\in[0,t],\ i=1,2;\ \beta_{ik}^{\pm}(x,t)\ge 0\,(i=0,1,2),\dot{\beta}_{0k}^{\pm}(x,t)\le 0,\ \dot{N}_k^{\pm}(x,t)\ge 0,$$

$$M_k^{\pm}\ge\frac{\rho K_{0k}^{\pm}}{\pi},N_k^{\pm *}<\lambda_{1k}^{\pm}D_{k\,*}^{\pm}-\frac{\rho K_{0k}^{\pm}V^2}{\pi},\int\limits_0^{w_k^{\pm}}h_k^{\pm}(x,t,z)dz\ge 0,\int\limits_0^{w_k^{\pm}}\frac{\partial h_k^{\pm}(x,t,z)}{\partial t}dz\le 0,$$

$$\dot{w}_k^{\pm}g_k^{\pm}(x,t,w_k^{\pm},\dot{w}_k^{\pm})\ge 0,\mu_k^{\pm}\ge 0,\ \nu_k^{\pm}\ge 0,\quad x\in(b_{2k-1},b_{2k})\text{ for }'+',\ x\in(a_{2k-1},a_{2k})\text{ for }'-',$$

where $K_{0k}^+=\sup\limits_{x\in(b_{2k-1},b_{2k})}K_1^+(x), K_{0k}^-=\sup\limits_{x\in(a_{2k-1},a_{2k})}K_1^-(x), K_1^{\pm}(x)=\sum\limits_{k=1}^{m}\int\limits_{b_{2k-1}}^{b_{2k}}K^{\pm}(\tau,x)d\tau+$

$$+\sum_{k=1}^{n}\int\limits_{a_{2k-1}}^{a_{2k}}K^{\mp}(\tau,x)d\tau, K^{\pm}=ln\left|\frac{e^{\frac{-\pi a_1}{y_0}}+e^{\frac{-\pi b_1}{y_0}}}{e^{\frac{-\pi\tau}{y_0}}\mp e^{\frac{-\pi x}{y_0}}}\right|,\ N_k^{\pm *}=\sup_{x,t}N_k^{\pm}(x,t),$$

$$D_{k\,*}^{\pm}=\inf_x\left[D_k^{\pm}(x)(1+Q_{1k}^{\pm}(x,0,\infty))\right],t\in[0,\infty),\ x\in(b_{2k-1},b_{2k})\text{ for }'+',\ x\in(a_{2k-1},a_{2k})\text{ for }'-',$$

$\lambda_{1k}^{\pm}$ are the minimal eigenvalues of the boundary value problem $y^{IV}(x)=-\lambda y''(x)$ with boundary conditions to which the functions $w_k^{\pm}$ correspond. The conditions impose restrictions on a measures of relaxation, speed of a homogeneous flow V, longitudal compressing (decompressing) efforts and other parameters of the mechanical system.

Conclusion

Dynamic stability conditions of viscoelactic elements of a plane channel, a wing profile, of a facilatated weight wing, pipeline are obtained. The conditions impose restrictions on a measures of relaxation, speed of a flow, longitudal compressing (decompressing) efforts and other parameters of the mechanical system.

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