INTERACTION OF TWO STRONG NONLINEARITIES

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Abstract: *A model of mechatronic system with two degrees of freedom containing hardening spring nonlinearity in mechanical part and softening nonlinearity in electrical feedback loop is presented. The hardening spring nonlinearity is caused by the impacts on the stops with finite stiffness, the softening nonlinearity describes the dependence of output current on voltage input of the power amplifier, in which the limitation of output current occur.*
The response of the system, the mathematical model of which is transformed into the dimensionless form, is studied for different parameters of nonlinear characteristics.

Introduction

Modern machines contain very often constrains, the characteristics of which are strongly nonlinear. Constrains in mechanical part are e. g. stops, clearances, loose parts, which properties can be described by a hardening piecewise linear dependence of force on deformation [1,2]. The advanced up-to-date machines and technological devices used to be mechatronic systems, where mechanical part is equipped by electronic control subsystem with feedback loop, having usually also nonlinear properties. These nonlinearities are often of analytical type, but there exist also strong softening nonlinearities caused e. g. by limitation of output current of power amplifier or by saturation of electromagnets.
As an example of mechatronic system containing both hardening and softening nonlinearities let us investigate active magnetic bearing (AMB), supporting rotor of high-speed machine [3,4].
The main advantages of magnetic bearings against the supports based on rolling elements or oil journal bearings are: reduction of bearing friction and wear to zero, applicability for very high revolutions, for very low temperature, vacuum etc. Large advantage is also the possibility to control the stiffness and damping of bearing system according to operating conditions and so to avoid critical cases.
Magnetic bearings are in their nature unstable, but by suitable adjusting of feedback loop's parameters, the system can be stabilised. With increase of displacements and/or of control current, the nonlinearities can cause the lost of stability of periodic oscillations. The irregular oscillations in the form of jumps, beats and chaotic motions are investigated in this paper for various values of system parameters.
These properties will be presented in the form of response curves, time histories, phase trajectories and Poincaré mapping.

Studied system

The most simple electro-mechanical system of AMB is in Fig. 1, where the stiff rotor is modelled by mass *m*. Signal from rotor motion x is measured by pick up *1* , transformed in proportional-integral-derivative PID control device *2* and with combination of bias current I_0 is led to the power amplifiers *3* with feed-back loops *4* . Resulting currents supply the electromagnets, the controlled attractive forces of which return the rotor into the equilibrium position.
The nonlinear properties of AMB at small deviation from equilibrium position are caused by nonlinear characteristic of magnetic fields and belong to the analytical weak nonlinearities. The more important are strong nonlinearities, occurred at amplitudes higher than x_0 , when the journal impacts on the safety retainer bearing *6.* Another strong nonlinearity is introduced by the current limitation of the power amplifier. Black arrows in Fig. 1 point the positions of these strong nonlinearities.

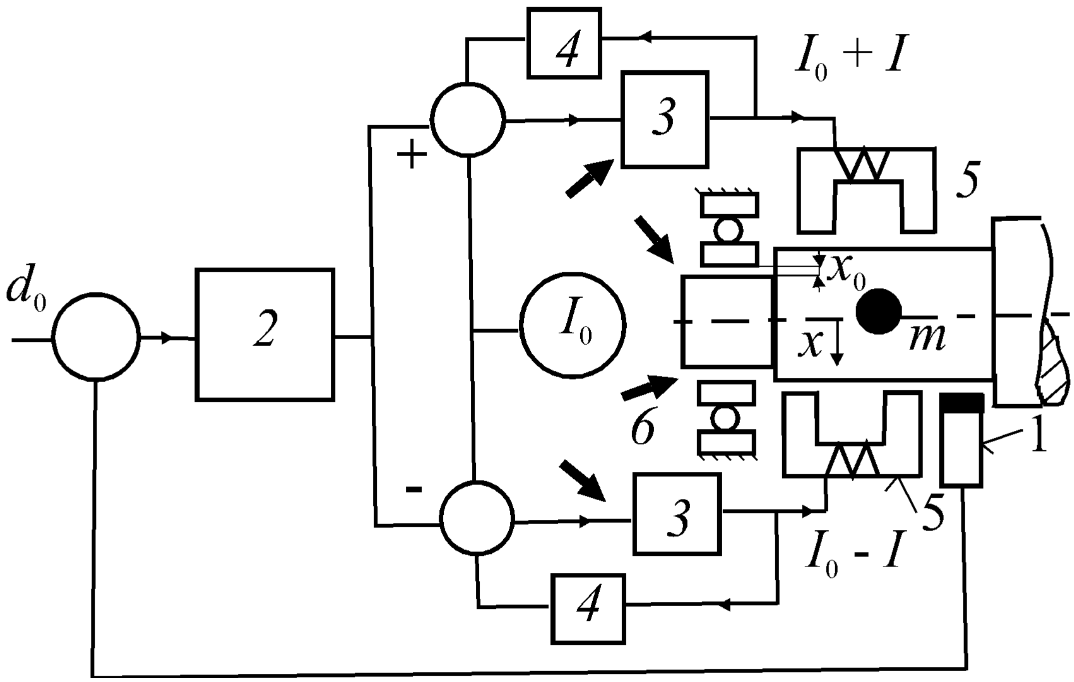


Fig. 1

Mathematical model of AMB simplified to two degrees of freedom system is described by differential equations

$$
\begin{aligned}
& m\ddot{x} + F(x, I) = me\omega^2 \cos \omega t \\
& i_z = \alpha_1 \int x dt + \alpha_2 + \alpha_3 \dot{x} \\
& \dot{I} = (I_z - I) T_r
\end{aligned}
\tag{1}
$$

First equation corresponds to the motion of rotor with the eccentricity e, second one to the properties PID controller with adjustable parameters α_1, α_2, α_3 and the last one concerns the properties of power amplifier with time constant T_r.. We suppose the rotor motions in vertical and horizontal planes are independent.

The function $F(x, I)$ describes the force acting on the journal. This one contains both the linear parts of electromagnetic force (in which weak nonlinearity is neglected) and at large displacement also the contact forces of safety retainer bearing:

$$F(x, I) = -p_1 x + p_2 I + k_1 (x - x_0 \operatorname{sgn} x) H(|x| - x_0)\,, \tag{2}$$

where

$H(\xi)$ is Heaviside´s function,

k_1 stiffness of the retainer bearing and its support

p_1, p_2 coefficients of magnetic force dependence on journal displacement x and feeding current I.

The current limitation of power amplifier at I_m can be expressed by means of i_z and I_z

$$I_z = i_z - (i_z - I_m \operatorname{sgn} i_z) H(|i_z| - I_m) \tag{3}$$

Introducing dimensionless variables and parameters

$$
\begin{aligned}
& y_1 = x / x_0, \;\; y_2 = I p_2 / (x_0 p_1), \;\; e_c = e / x_0, \;\; y_{2m} = I_m p_2 / (x_0 p_1), \;\; \chi = T_r \sqrt{p_1 / m} \\
& \eta = \omega \sqrt{m / p_1}, \;\; \tau = t \sqrt{p_1 / m}, \;\; (') = d / d\tau, \;\; y_3 = i_z p_2 / (x_0 p_1), \;\; k_n = k_1 / p_1 \\
& \beta_1 = \alpha_1 p_2 / p_1 \sqrt{m / p_1}, \;\; \beta_2 = \alpha_2 p_2 / p_1, \;\; \beta_3 = \alpha_3 p_2 / p_1 \sqrt{p_1 / m}, \;\; F_n = F / p_1 x_0,
\end{aligned}
\tag{4}
$$

we get equation of motion in dimensionless form

$$y_1'' - y_1 + y_2 + k_n(y_1 - \operatorname{sgn} y_1)H(|y_1| - 1) = e_c\eta^2 \cos\eta\tau$$
$$y_2' = \lfloor y_3 - (y_3 - y_{2m} \operatorname{sgn} y_3)H(|y_3| - y_{2m}) - y_2 \rfloor / \chi \qquad (5)$$
$$y_3' = \beta_1 y_1 + \beta_2 y_1' + \beta_3 y_1''.$$

Strongly nonlinear functions are shown in left-hand side of Fig. 2 both for dimensional and non-dimensional variables and parameters.

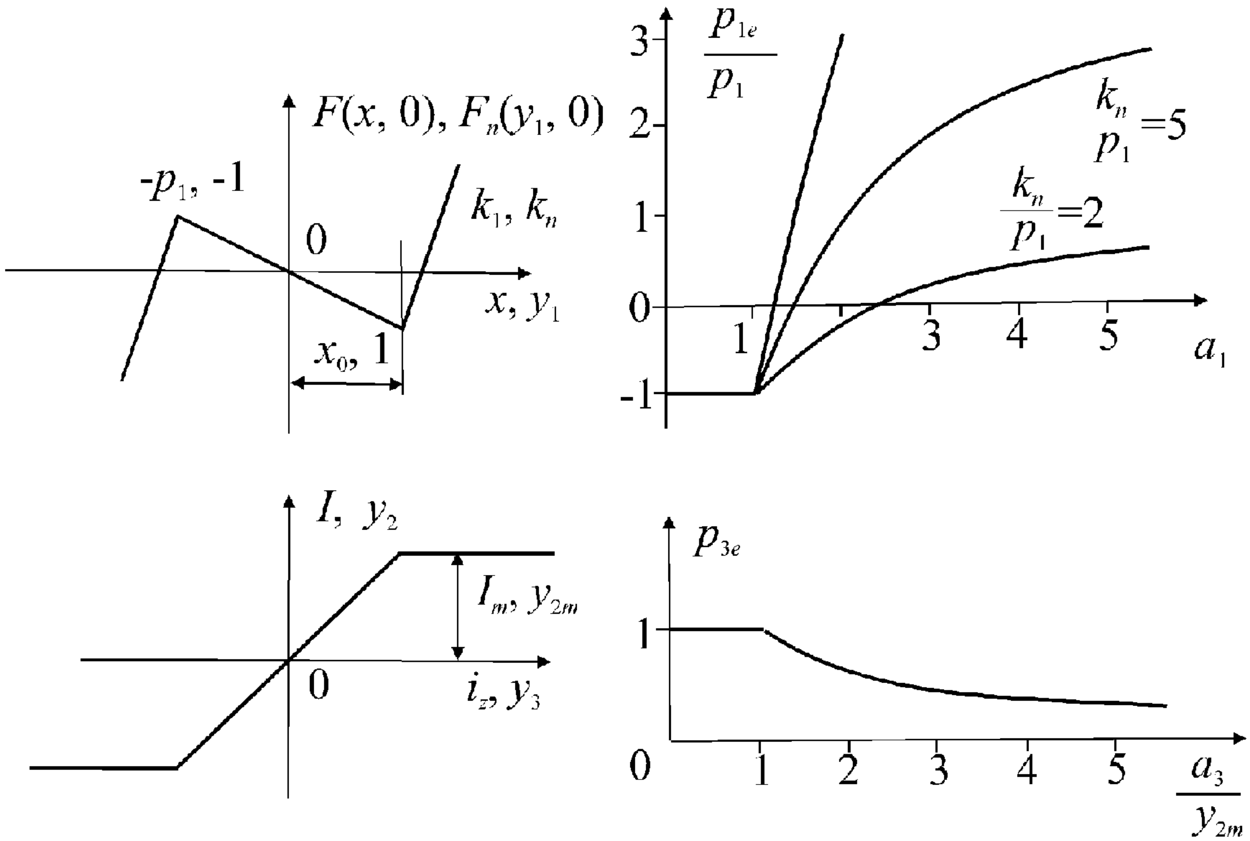


Fig. 2

For the preliminary analysis, where we suppose the nearly harmonic signals

$$y_i = a_i \cos(\eta\tau + \varphi_i) = C_i \cos\eta\tau + S_i \sin\eta\tau \qquad (6)$$
$$a_i = \sqrt{C_i^2 + S_i^2} \qquad i = 1, 2, 3,$$

the equations (5) can be replaced by equivalent linearised equations

$$y_1'' + p_{1e} y_1 + y_2 = e_c\eta^2 \cos\eta\tau$$
$$y_2' = (p_{2e} y_3 - y_2)/\chi \qquad (7)$$
$$y_3' = \beta_1 y_1 + \beta_2 y_1' + \beta_3 y_1''\ .$$

Connecting last two equations we get

$$y_1'' + p_{1e} y_1 + y_2 = e_c\eta^2 \cos\eta\tau$$
$$y_2'' - \lfloor p_{3e}(\beta_1 y_1 + \beta_2 y_1' + (-p_{1e} y_1 - y_2 + e_c\eta^2 \cos\eta\tau)) - y_2 \rfloor / \chi\ . \qquad (7a)$$

These equations contain equivalent linear coefficients [1,5,6]

$$p_{1e}(a_1) = -p_1 + 2k_n/\pi \lfloor \arccos(1/a_1) - (1/a_1)\sqrt{1 - (1/a_1)^2} \rfloor H(a_1 - 1) \qquad (8)$$
$$p_{3e}(a_3) = 1 - 2/\pi \lfloor \arccos(y_{2m}/a_3) - (y_{2m}/a_3)\sqrt{1 - (y_{2m}/a_3)^2} \rfloor H(a_3 - y_{2m}) \qquad (8a)$$

where $a_3 = \sqrt{(\beta_1/\eta - \eta\beta_3)^2 + \beta_2^2}\, a_1$ according to the last equation (7).

The plots of dimensionless values $p_{1e}(a_1)$ and $p_{3e}(a_3)$ versus a_1 and a_3 / y_{2m} are in the right hand side of Fig. 2.

Response curves in the first approximation

Because the investigated system contains two strong nonlinearities, it is very difficult to solve it analytically in general form. But in the case of active magnetic bearing, where the softening nonlinearity in control electrical feedback loop depends also on the displacement x and the amplitudes a_1 and a_3 are connected by the third equation 7, the solution is more easy. It is possible to solve both nonlinearities simultaneously.

As an example let us show here the result of analytical study of stiffness bearing influence. From Fig. 3, calculated for values $\beta_1 = 3., \beta_2 = 2.7, \beta_3 = 2.5, \chi = .25$ and for several stiffness ratios $k_n = 0.0;\ 0.2;\ 1;10$ and $y_{2m} = \infty$ it is seen that the stiffness of retainer bearing makes lower the resonance peak.

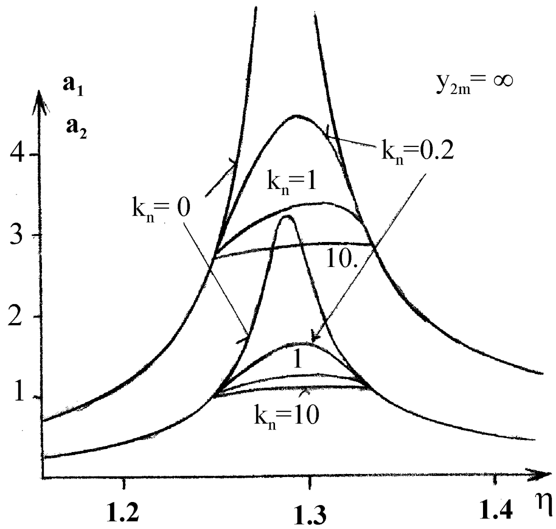
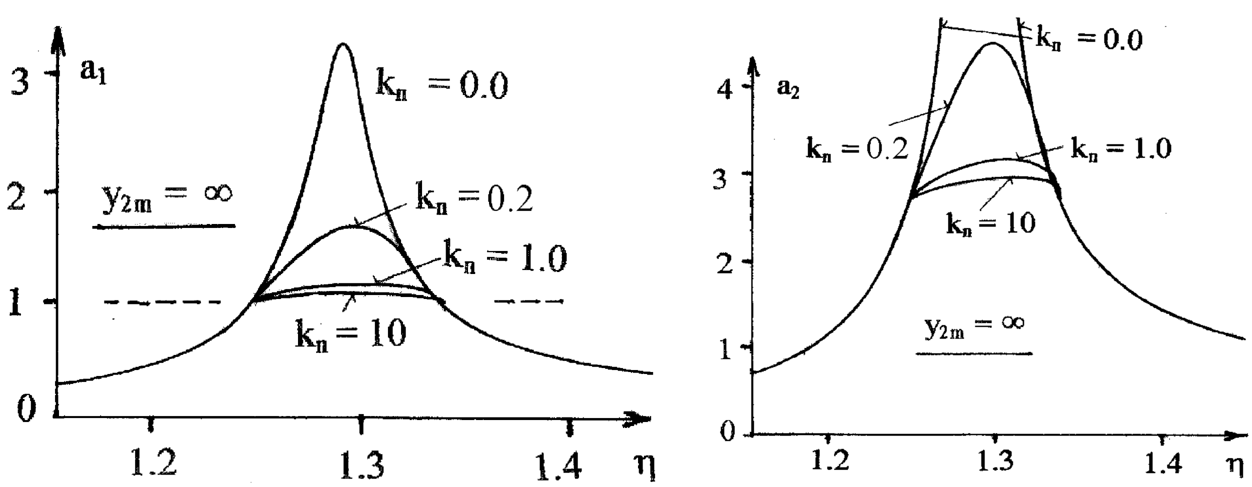


Fig. 3

The same result is obtained from the numerical simulation [7], where in addition the stability and behaviour of system in instability domain can be shown.

In Fig. 4a, b there are the results of simulation of the same system, where again the current feeding electromagnets is without limits ($y_{2m} = \infty$). We see that the equivalent linearization method gives approximately as exact results as numerical simulation does.



a) b)

Fig. 4

In the real systems always exists current limitation. Introducing this limitation of currents $(y_2 \le y_{2m})$ into the same system, but without any stops in the rotor motion $(k_n = 0)$, then the system became unstable after reaching the current limit. The amplitudes of rotor oscillations rise beyond all values and the computer overflows. Corresponding response curves calculated for $k_n = 0$ and $y_{2m} = 3$ are seen in Fig. 5. We can deduce that existence of only softening nonlinear element in the feedback control loop without the hardening element (stop) in the mechanical part is very dangerous and magnetic bearing of such a structure cannot be used in practise.

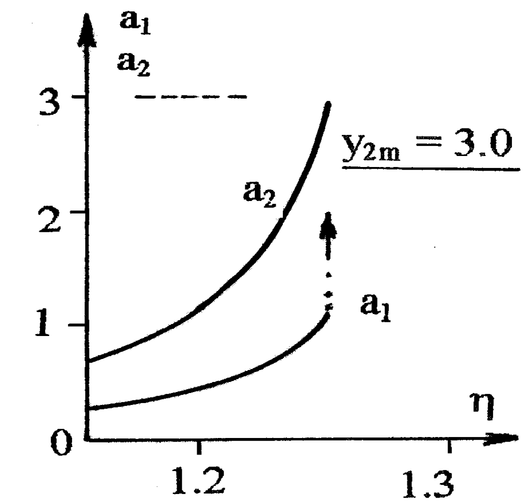


Fig. 5

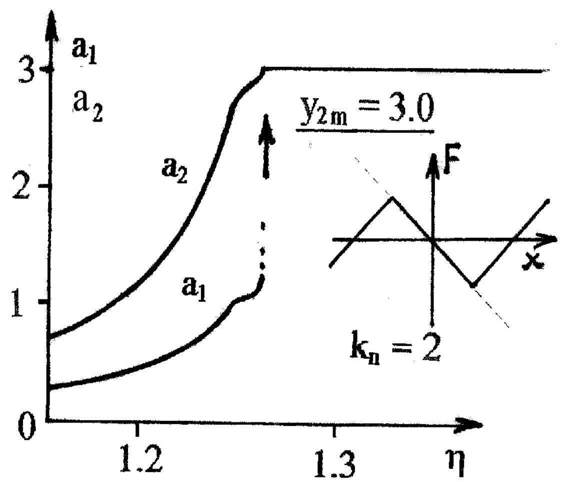
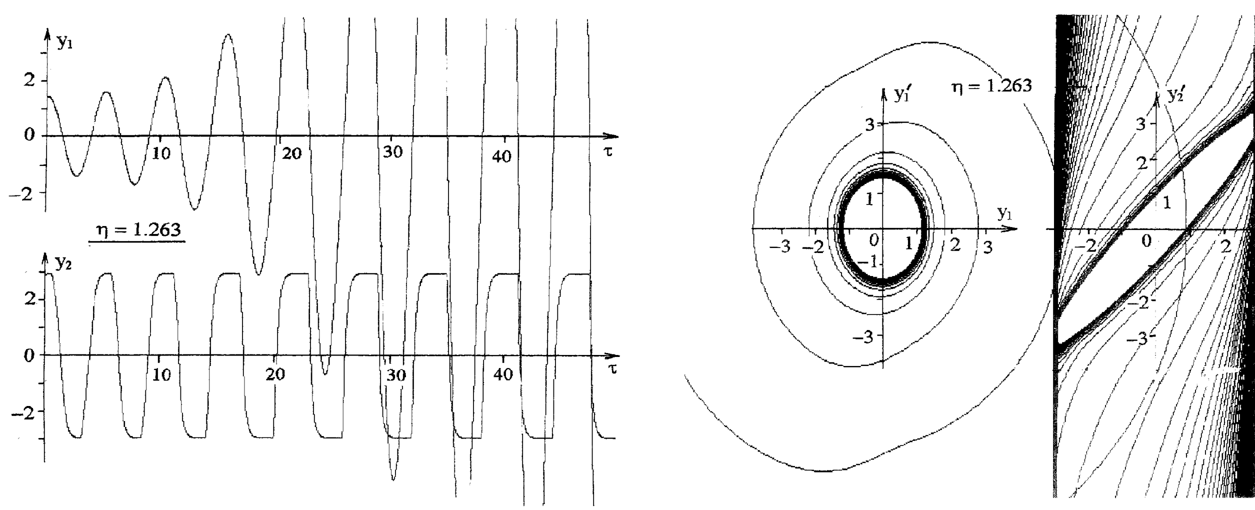


Fig. 6

Adding to this system very weak stops (e.g. $k_n = 2$), which begin to work before the current reaches its limit, this instability can be removed and no overflow happens, but the mechanical oscillations of the rotor again increase to very high values, as is seen from Fig. 6. This combination of nonlinear characteristics is therefore very dangerous, too. After motion y_1 exceeds value $y_1 = 1$, the amplitude a_1 stabilises by sudden jump on oscillations with very high amplitudes in mechanical part. Oscillations of current y_2 do not increase, but their sinusoidal form changes into the nearly rectangular form. The beginning of this jump from low to very high amplitudes of mechanical oscillation y_1 and the change of form y_2 are seen in Fig. 7a in the time history records. The same jumps are represented in phase plane in Fig. 7b, both at frequency η = 1.263.



a) b)

Fig. 7

We can see that the current limitation is of the great significance for the stability of rotor supported by AMB, particularly if the retainer bearing is absent or has a very low stiffness, (e. g. $0 \le k_n < 2$)
Let us see now on the properties of AMB at higher stiffness of the safety retainer bearing. This is one of the possibilities how to stabilise the rotor motion.
If stiffness of bearing after impact increases to $k_n = 5$, its influence is strong enough to suppress the negative properties of current limitation. Results of solution for this stiffness and for $y_{2m} = 3$ are given in Fig. 8. Response curves are smooth both in electrical y_2, η and in mechanical y_1, η subsystem.

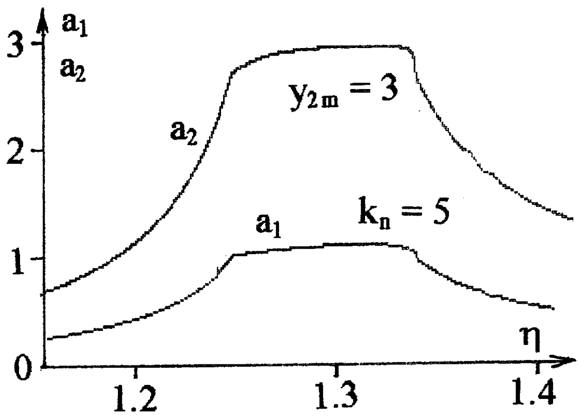


Fig.8

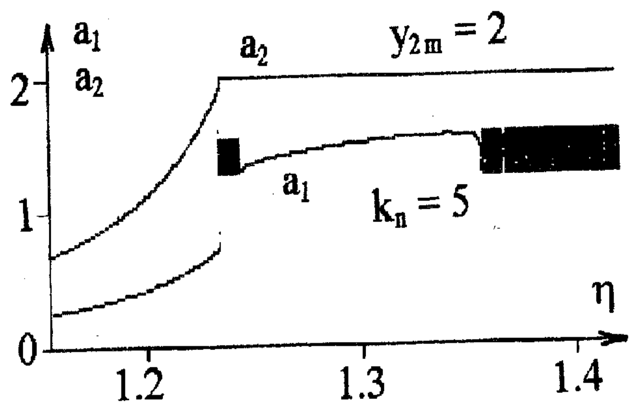
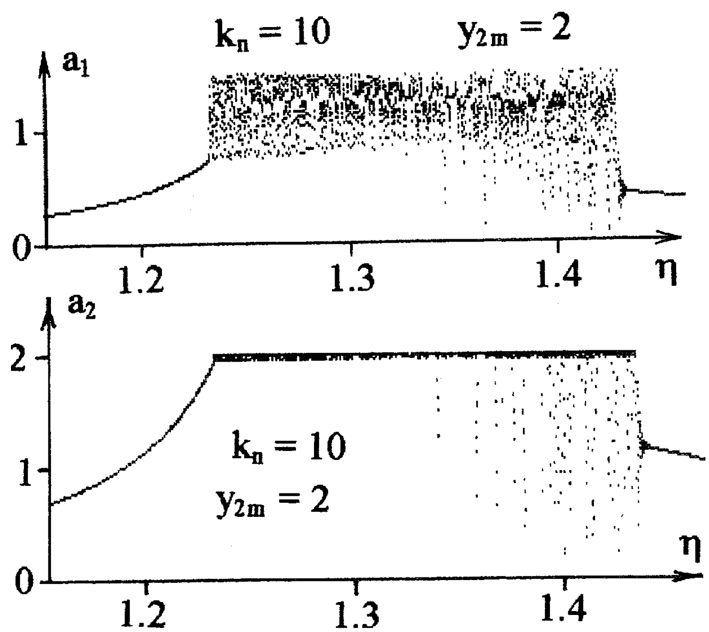


Fig.9

But further decrease of current limitation from $y_{2m} = 3$ to $y_{2m} = 2$ change again considerably the properties of motion, as can be seen from Figs. 9. Although the resonance amplitudes are limited in the high, the frequency range of resonant amplitudes enlarges to very high frequencies. In addition some intervals of quasiperiodic oscillations occur in a_1, η response curve. These quasiperiodic oscillations are very stabile and it is almost impossible to go back to the low-amplitude impact-less motion.
To restrict this resonance peak expansion, the strongest stiffness k_n has to be used.
Fig. 10a corresponds to the values $k_n = 10$; $y_{2m} = 2$. The resonance peak is narrower, but instead of quasiperiodic, the chaotic oscillations [8,9] with greater modulation occur. The phase trajectories and Poincaré mapping of these oscillations are shown in Fig. 10b,c.



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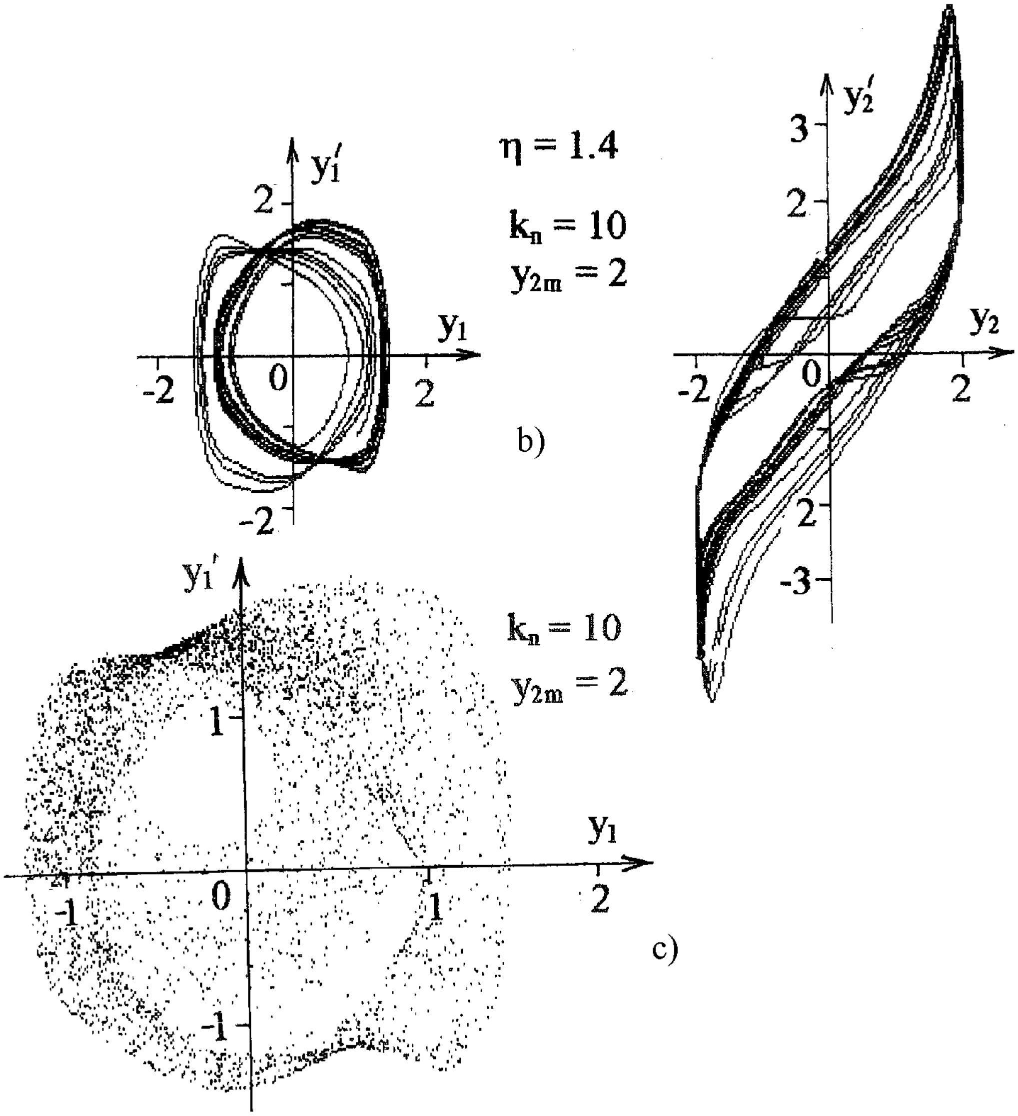
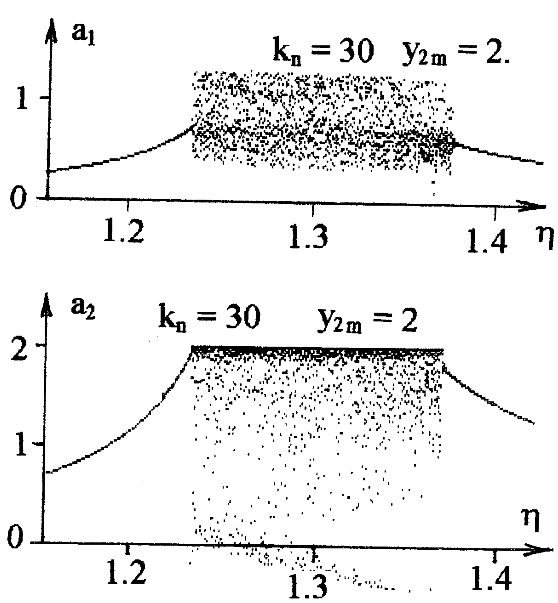


Fig. 10

Further increase of safety bearing stiffness narrows the resonance zone, but the intensity (modulation) of chaotic oscillations is greater. Response curves for the values $k_{n.} = 30$ and $y_{2m} = 2$ are in the upper half of Fig. 11, phase trajectories and Poincaré mapping are Fig. 11 below.



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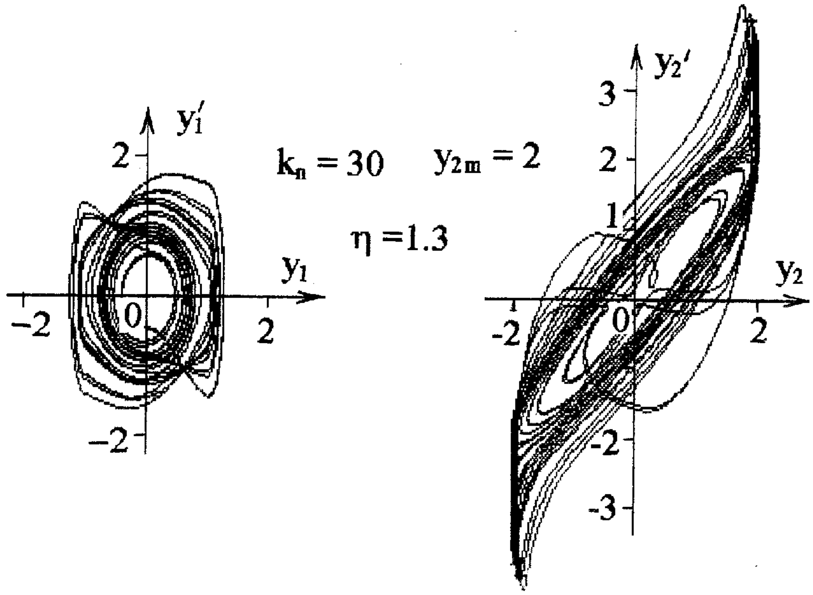
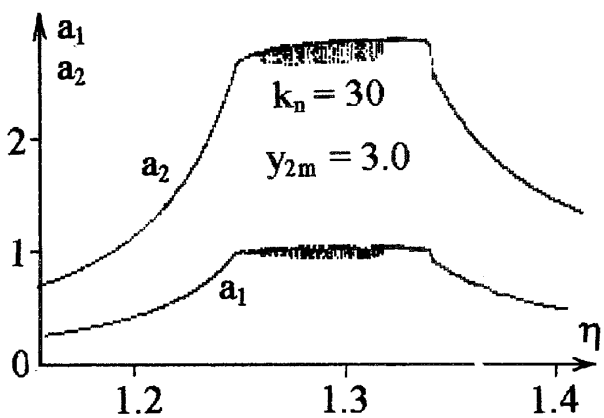


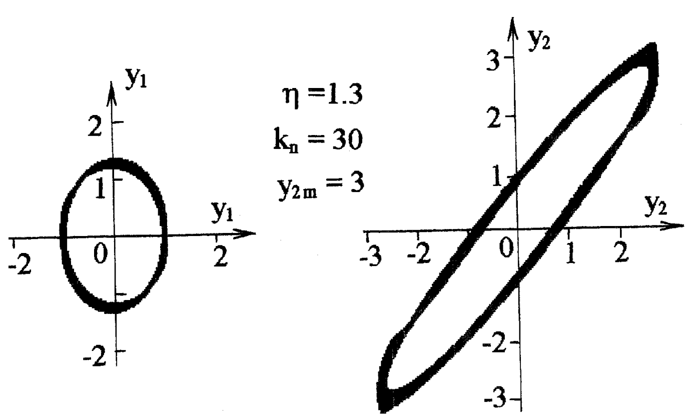
b)

Fig. 11

Let us go back to the higher limit of current i.e. to the more powerful amplifier with $y_{2m} = 3.$ at the same stiffness of safety bearing $k_n = 30$. Response curves (Fig.12) are now restricted to small oscillations, resonance peaks are low and are of minimum wide. The chaotic oscillations exist there as well, but their modulation is small and the form of oscillation approaches to the harmonic motion.



a)



b)

Fig. 12

Acknowledgement

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Conclusion

Mechatronic system of active magnetic bearing, in which nonlinear elements with both progressive and degressive (hardening and softening) characteristics exist, was studied analytically and by numerical simulation.

Analytical method of equivalent linearisation enables to ascertain, even in this case of two different nonlinearities, response curves in first approximation, but without determination of their stability.

By means of numerical simulation, the forms of response curves, their stability and properties in the instability regions were studied for various combinations of non-linear characteristics.

It was shown that the influence of safety retainer bearing with progressive characteristic has positive influence on the stabilisation of rotor motion.

On the contrary, the degressive characteristic caused by the small power of feedback amplifier with low current limitation results in instability of rotor motion and in emergence of chaotic oscillations. The using of safety retainer bearing is therefore very important.

By suitable combination of these both nonlinearities we can find the optimal properties of active magnetic bearing and of other similar systems with two nonlinear elements.

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