

# Vibration Impact Processes in Systems with Viscous-Elastic Limiters

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## Abstract

Some models of viscous-elastic limiters from polymeric materials are examined. It is concluded that the rheological models of hereditary mechanics are preferable for the description of the behavior of similar limiters at impact. The rheological parameters of the offered models of the limiters are shown depending on the deformation of the material and their geometrical characteristics. Developed hereditary models allow simulating the behavior of the limiters with sufficient accuracy that makes it possible to investigate the vibrating impact processes in systems with viscous-elastic limiters correctly.

## 1 Introduction

At the present moment the electromagnetic vibration exciters with armature movement viscous-elastic limiters are widely used in various fields of industries. Presence of the limiters allows changing the operation dynamics and parameters of generated oscillations. The distinctive property of the exciters with the movement limiters is exciting the vibration impact modes. In comparison with unimpact modes, the impact ones are more dynamically stable and conduce to create large working efforts, to increase the power of the vibration exciter with the same properties of the elastic system, and to make mechanical adjustment of armature movement parameters by relative displacement of the limiters. The said advantages promote wide use of such exciters in diverse vibration machines, especially in hand-operated equipment and products of medical and household engineering.

However, notwithstanding the wide spectrum of possible use of these vibration exciters, in many cases important problems, related with its practical calculations, arise. As a matter of fact, on projection and creation of similar exciters, as a rule, calculation method is used, based upon the simplified model of the impact limiters as a sum of elastic and viscous elements. Use of similar simplified models leads towards significant errors on calculations by virtue of ignoring the real properties of material, such as tangential elasticity, relaxation etc. At last stage it leads to the selection of nonoptimal parameters of element constructions of the exciters and the realization of their work modes with insufficient effectiveness.

One of possible ways of increasing the quality of calculation, in our opinion, is considering the hereditary mechanics rheological models of the limiters.

## 2 Actual contents

In the projection of various models of vibration exciters for the description of the behavior of the limiters, made from polymer filled materials, as a rule, this general model is used:

$$\sigma = E\varepsilon + \eta\dot{\varepsilon}, \quad (1)$$

where  $\sigma$  - deformation stress;

$E$  - modulus of elasticity of material;  
 $\varepsilon$  - strain (longitudinal elongation or shortening);  
 $\eta$  - coefficient of viscosity.

However, by this description, series of existing rheological properties of the material of the limiters is not considered. It has been agreed that the impact effect of interaction of moving parts of the electromagnetic exciter with the limiters might be written as the model (1). It has been assumed that the values of  $E$  (modulus of elasticity) and  $\eta$  (coefficient of viscosity) are independent of deformation and deformational velocity. Using this model for numerical modeling of viscous elastic impact limiters introduces large errors into the calculations.

On the compression of rubber material related to the filled polymers group, it is noted that nonlinear characteristics of the reaction force takes place (fig.1).

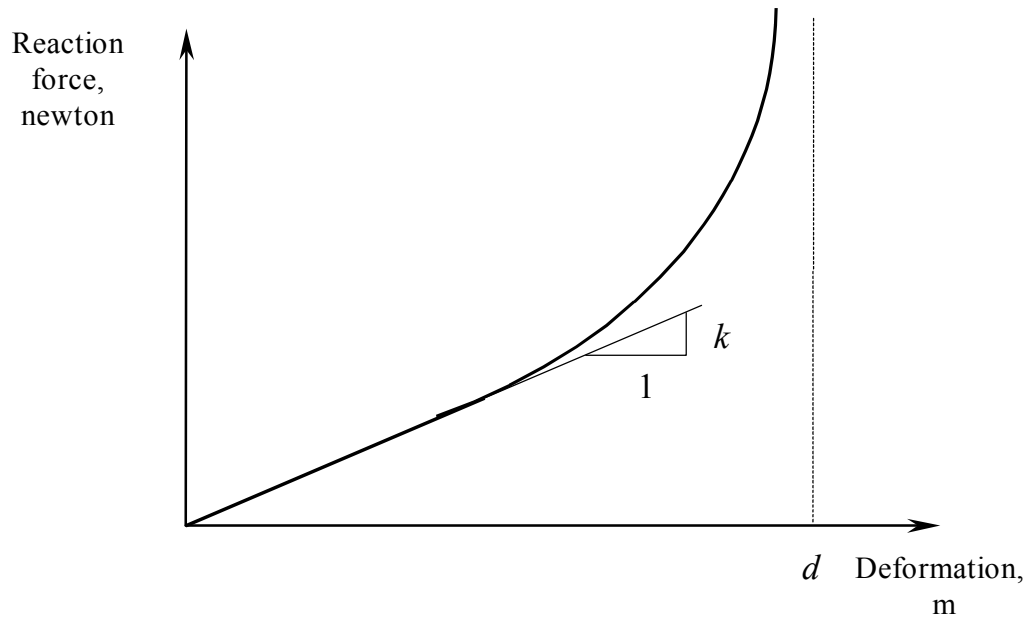


Figure.1. Tangential elasticity of rubber element

This characteristic can be written by the expression [1]:

$$F(\varepsilon) = \frac{2kd}{\pi} \operatorname{tg} \frac{\pi\varepsilon}{2d}, \quad (2)$$

where  $k$  - slope of the curve on initial part,  
 $d$  - deformation determining a vertical asymptote.

This way, relation (2) for the determination of the reaction force of nonlinear viscous elastic element can be written in the form:

$$F(\varepsilon) = \frac{2kd}{\pi} \operatorname{tg} \frac{\pi\varepsilon}{2d} + \eta\dot{\varepsilon} \quad (3)$$

Experimental research on the deformation of the test specimens from filled polymer materials reveals that relation between stress and deformation essentially depends on the history, method and deformational velocity of the specimen. With the help of the models (1-3) these particulars cannot be revealed in the defining the limiters.

The theory of hereditary mechanics, based on linear operators of Volterra, can be used in the developing the mathematical models of the behavior of viscous elastic elements from these materials at different types of deformations. This theory has received further developments in works [2,3].

Principal law of hereditary theory of elasticity for one-dimensional case can be shown in the following form [2]:

$$\sigma = E(1 - \Gamma)\varepsilon, \quad (4)$$

where  $\Gamma$  is named as nucleus of relaxation.

According to this theory, if in the experiment on the relaxation the constant value of deformation is held and if the deformation  $\varepsilon_0$  is imparted instantaneously to the specimens then:

$$\sigma(t) = E(1 - \Gamma)\varepsilon_0 \quad (5)$$

where the curve of  $\sigma(t)$  relation at  $\varepsilon = \varepsilon_0 = \text{const}$  is named as the curve of relaxation.

For one-dimensional stress condition the stress arising in the viscous-elastic material can be determined by the formula [4]:

$$\sigma(t) = E_0 \left[ \varepsilon - \int_0^t \Gamma(t - \tau) \varepsilon(\tau) d\tau \right], \quad (6)$$

where  $E_0$  - instantaneous modulus of elasticity;

$\int_0^t \Gamma(t - \tau) \varepsilon(\tau) d\tau$  - hereditary part of stress accounts for the influence of  $\varepsilon(\tau)$ , taking place in interval of time  $[\tau, \tau + d\tau]$ , belonging to the previous time and multiplying on the oblivious function, or nucleus of relaxation  $\Gamma(t - \tau)$ .

Other method of determining the stress, arising in the material by one-dimension stress condition is represented in [2], if the specimen is suddenly subjected to compression and its deformation  $\varepsilon_0$  is fixed then we can write:

$$\sigma(t) = \varepsilon_0 R(t) \quad (7)$$

and the curve of relaxation is the graph showing the function of relaxation.

The conducted research [4] shows that use in calculations the nucleus  $\Gamma$  instead of function of relaxation possesses some advantages in those cases under which experimental data without their analytical approximation form the basis of calculation. For direct determination of nucleus  $\Gamma$  it is necessary to differentiate the relaxation function determined experimentally and this operation involves errors which may be significant. Similar difficulties do not arise if the analytical structure of nucleus is given earlier and only formula's constants are determined from experiments.

For the function of relaxation in literature [3] a series of approximation's relations in the form of exponential nucleus, nucleus of Abel, fractional exponential and more ones is proposed. In the study of reinforced rubber creeping it is shown [5] that the functions of relaxation can be satisfactorily written as nucleus of Rjanicin:

$$R(t - \tau) = a(t - \tau)^{\alpha - 1} \varepsilon^{-\beta(t - \tau)}, \quad (8)$$

where  $a, \alpha, \beta$  - rheological parameters.

The conducted experimental research on instantaneous static deformation of viscous elastic impact limiters from filled polymer materials allows to determine parameters of models (3, 6, 7, 8) for each type of the limiters. Some of the rheological parameters of the nucleus (8) were found as nonlinear and, as an example, for a test specimen of the impact limiter can be written in the following forms:

$$a(\varepsilon) = -0.32\varepsilon + 0.86 \quad (9)$$

$$\alpha(\varepsilon) = -0.046\varepsilon^2 + 0.96 \quad (10)$$

$$\beta(\varepsilon) = -0.01\varepsilon^2 + 0.006 \quad (11)$$

The graphs of these relations are presented in fig.2,3.

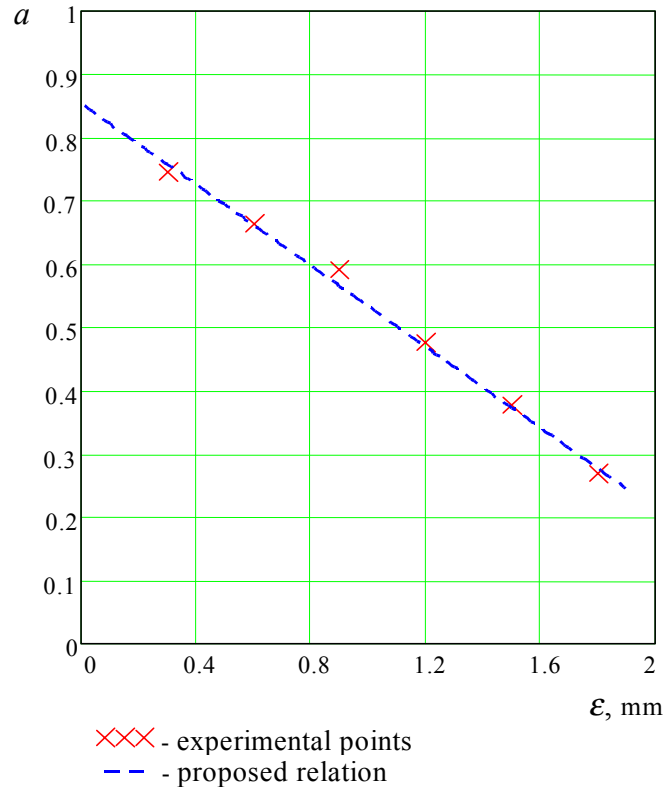


Figure 2. The relation of rheological parameter  $a$  against deformation  $\varepsilon$

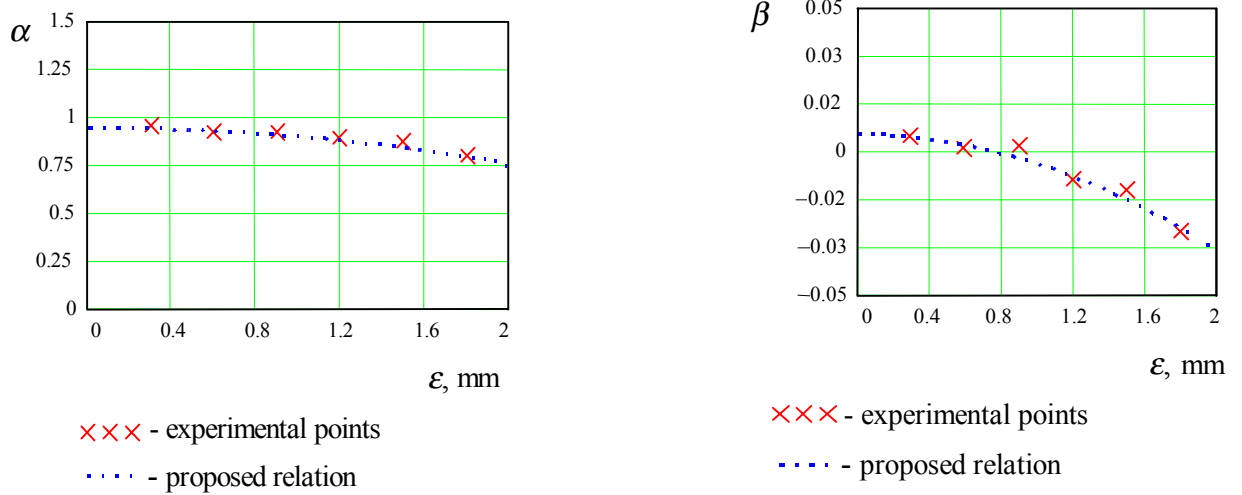


Figure 3. The relations of rheological parameter  $\alpha$  and  $\beta$  against deformation  $\varepsilon$

Considering the relations (9-11), the approximation of experimental points of deformation of limiter's test specimen at different deformation levels with help of the model (8) is shown in fig.4.

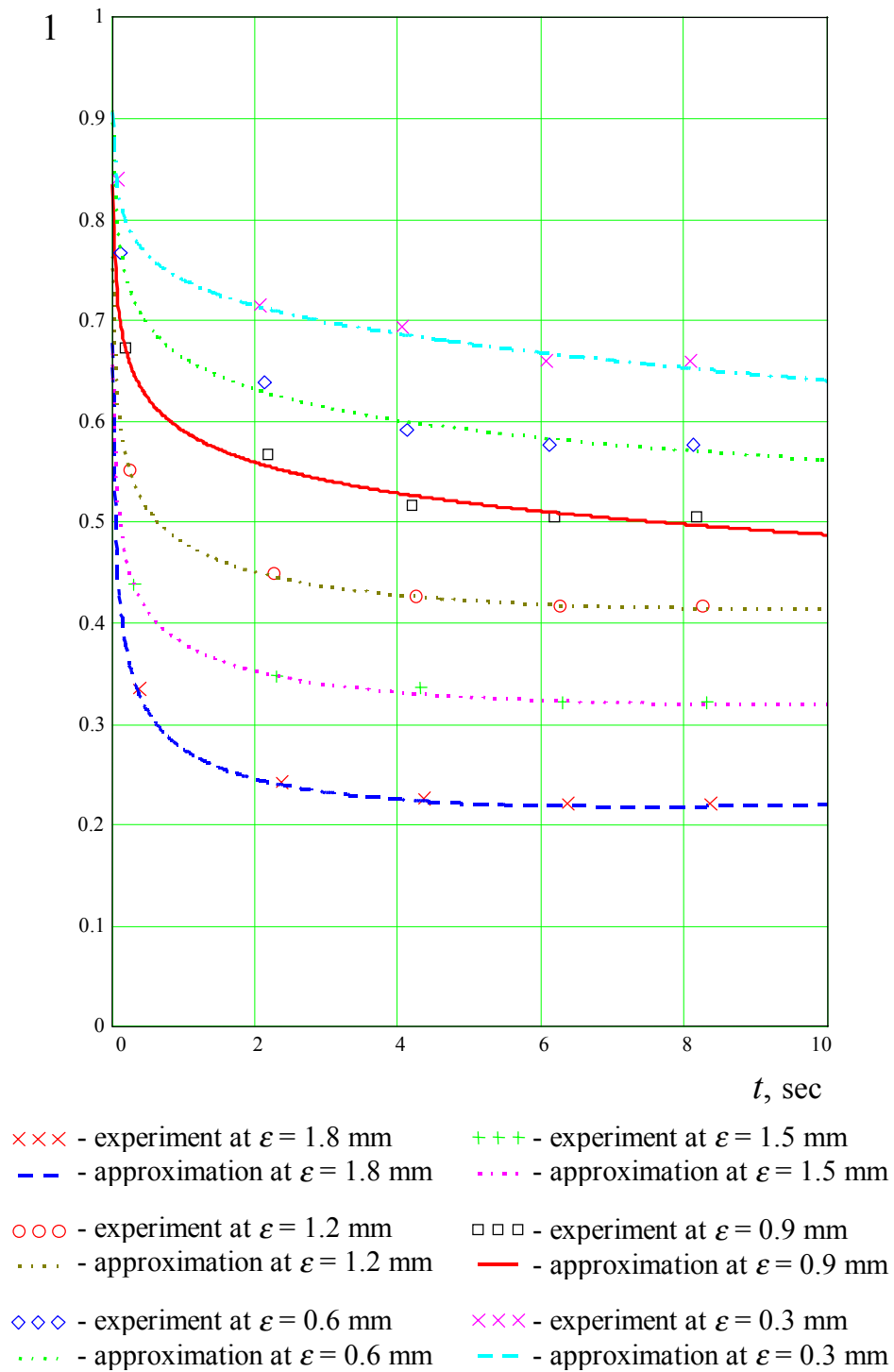


Figure 4. The approximation of experimental points of deformation of limiter's test specimen at different deformation levels with help of the model (8)

The numerical modeling the behavior of the limiters by received models on various methods of deformation was conducted. In fig.5 the comparison between three methods of simulation of viscous elastic limiter reaction at near instantaneous deformation in terms of hereditary theory of elasticity and by model (3) is shown for test specimen of limiter.

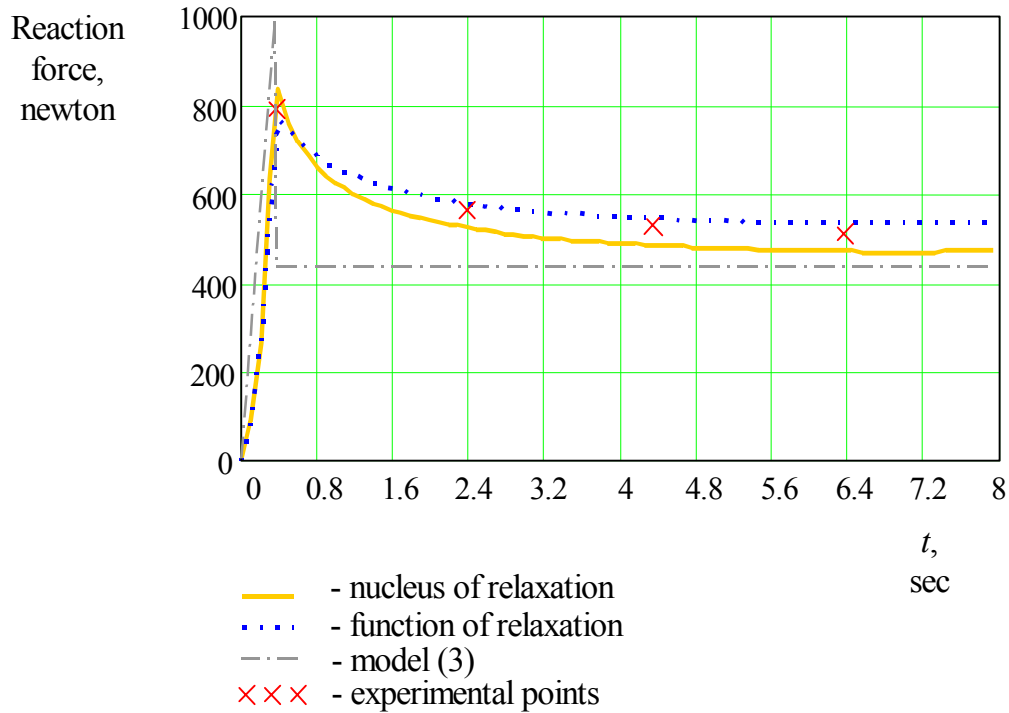


Figure 5. Numerical modeling the limiter reaction force on static instantaneous deformation by the model (3) and nucleus and function methods of hereditary mechanics.

### 3 Conclusion

Results of comparison between numerical and real experiments at different types of deformations have allowed determining several conclusions.

- Rigidity of the limiters essentially depends on deformation, velocity of deformation, law of deformation of material and geometrical parameters of the limiters.
- Field of possible uses of model (3), as applied to viscous elastic elements, is modeling the process of static deformation only on part of starter instantaneous deformation without considering relaxation processes with the use of parameters  $k$ ,  $d$ ,  $\eta$  expression (3), experimentally determined on the velocity of deformation of test specimens of the order of viscous elastic elements' deformation velocity of the simulation process.
- For the simulation of the reaction force of viscous elastic impact limiters, it is necessary to use the method of relaxation nucleus of hereditary mechanics. This method allows using the model (6) with rheological parameters singly determined on the basis of experiment for each type of the limiters in wide range of amount, velocity and the law of their deformation.
- Use of relaxation function of hereditary mechanics in model (7) allows simulating the behavior of similar limiters with higher accuracy only for instantaneous static method of deformation.

▪ As a result of close analysis of experimental data of the instantaneous static deformation of viscous elastic limiters it has been found that rheological parameters of the model of deformation and instant rigidity of the limiter essentially depends on the level of deformation and geometrical parameters of the limiter. These relationships have nonlinear character.

▪ Curve of relaxation, received by the result of experimental deformation of viscous elastic limiters, allows receiving rheological parameters used in nucleus of relaxation and also parameters of relation between instant rigidity of the limiters and deformation.

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