VECTORIAL METHOD, MASS MOMENTS VECTORS AND ROTATOR VECTORS IN NONLINEAR HEAVY GYROROTOR DYNAMICS

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***Abstract**: By using examples of the heavy rotor, as well as gyrorotor which rotates about two, or more axes with section/s in one point or more points, or without section, the vectorial method of the kinetic parameters nonlinear dynamic analysis of the free or forced rotodynamics is presented. For that reason, the mass moments vectors for the pole in the stationary shaft bearing and for the different rotate shaft axis, as well as kinematic rotators vectors are introduced.*

For the select examples of the solutions rotate equations, the analysis of the static and dynamic equilibrium positions, as well as the structural stability of the phase portrait are presented in this lecture. By using integral equations of the phase trajectories family, as well as phase portrait, vector rotators as a functions of the generalized angle coordinate of the component rotations are analytically determined, and graphically presented. Expression of the kinetic pressures of shaft bearing are determined as a functions of vectors rotators and of the deviator part of the mass moments vectors.

The analogy between motions of heavy material point: 1 on the circle in vertical plane, 2* on the circle in vertical plane which rotate around vertical axis in the plane or out of the circle plane, and 3* on the sphere and corresponding motions case of the heavy rotor, as well as of the gyrorotor which rotates around two, or more axes with section/s in one point or more points.*

By using papers written by Ph. Holmes, as well as Smale-Birkhoff homoclinic theorem, and Hartman Grobman stable manifold theorem for fixed point about local stable and nonstable manifold on the diffeomorfism with a hyperbolic homoclinic saddle fixed point the heavy forced rotor oscillatory motion in the neighborhood around hyperbolic points was studied. This forced motion under the action of the periodic couple excitations is stohasticlike and chaoticlike oscillatory process with sensitive dependence of initial conditions. The Poincare maps are presented, as well as a Smale horse shoe maps.

It is studied nonlinear dynamics in the field of the turbulent damping for different gyrorotor system parameters. Equations of the phase trajectories family are determined, as well as special homoclinic orbits.

***Keywords:** vectorial method, mass moments vectors, derivatives of the mass moments vectors, linear momentum, angular momentum, rotor, gyrorotor, vector rotator, kinetic parameters, nonlinear vibrations, stochastikelike, chaoticlike, phase portrait, saddle point, homoclinic point, axoids, polhode, herpolhode, kinetic pressures, support, constant energy curves.*

1. INTRODUCTION

In my previous papers [4], [5], [6], [7], [20], [24], [13], [14] and [11] the mass moments vectors for the pole and the axis are introduced by definitions. By using these vectors I introduced vector method for mass moment state analysis in the referential point of the body or space. In certain papers (for example see [8], [9], [10], [12], [15], [16], [17], [18], [19], [21], [22] and [23]), I pointed out that these vectors can be used for qualitative analysis of the kinetic parameters properties of the rotors dynamic as well as of the bearing kinetic pressures of the shaft.

In papers [13], [14] [24] and [10] some knowledge about change (rate) in time and derivatives of the body mass linear moment vectors and body mass inertia moment vectors for the pole and axis for the different properties of the body are pointed out. Body is observed for following cases: a) body is rigid and when body is rotated with angular velocity around fixed axis; b) body is with rigid structure configuration but with changeable body mass in these structure configuration; c) body is with changeable structure configuration as well as with changeable body mass in these structure configuration.

2. MASS MOMENTS VECTORS

The body mass moments vectors for the pole and axis (see [[4], [5], [6], [7], [20], [24], [13], [14] and [11]) are introduced by author by using following definitions:

1* Vector $\vec{\mathrm{S}}_{\vec{n}}^{(O)}$ of the body mass linear moment for the pole O and axis oriented by unite vector $\vec{n}$ in the following form:

$$\vec{\mathrm{S}}_{\vec{n}}^{(O)} \overset{def}{=} \iiint_V [\vec{n}, \vec{\rho}]dm \qquad dm = \sigma dV \tag{1}$$

where $\vec{\rho}$ is vector position of the body mass particles, $\vec{\rho}_C$ is vector position of the body mass center in the relation of the pole O (see Fig. no. 1).

2* Vector $\vec{\mathrm{J}}_{\vec{n}}^{(O)}$ of the body mass inertia moment for the pole O and axis oriented by unite vector $\vec{n}$ in the following form:

$$\vec{\mathrm{J}}_{\vec{n}}^{(O)} \overset{def}{=} \iiint_V [\vec{\rho}, [\vec{n}, \vec{\rho}]]dm \tag{2}$$

In paper [13] the "support" vectors of the body mass linear moment as well as of the body mass inertia moment for the pole O and axis oriented by unit vector $\vec{n}$ are introduced by following definitions and expressions:

1.1* Vector $\vec{S}_{\vec{n}}^{(O)}$ is vector "support" of the body mass linear moment of the body point $N: \overrightarrow{ON} = \vec{\rho}$, for the pole in the point O and for the axis oriented by unit vector $\vec{n}$ and is define by following expression:

$$\vec{S}_{\vec{n}}^{(O)} \overset{def}{=} \frac{\partial \vec{\mathrm{S}}_{\vec{n}}^{(O)}}{\partial m} = [\vec{n}, \vec{\rho}] \tag{3}$$

2.1* Vector $\vec{\mathrm{N}}_{\vec{n}}^{(O)}$ is vector "support" of the body mass inertia moment of the body point $N: \overrightarrow{ON} = \vec{\rho}$, for the pole in the point O and for the axis oriented by unit vector $\vec{n}$ and is define by following expression:

$$\vec{\mathrm{N}}_{\vec{n}}^{(O)} \overset{def}{=} \frac{\partial \vec{\mathrm{J}}_{\vec{n}}^{(O)}}{\partial m} = [\vec{\rho}, [\vec{n}, \vec{\rho}]] \tag{4}$$

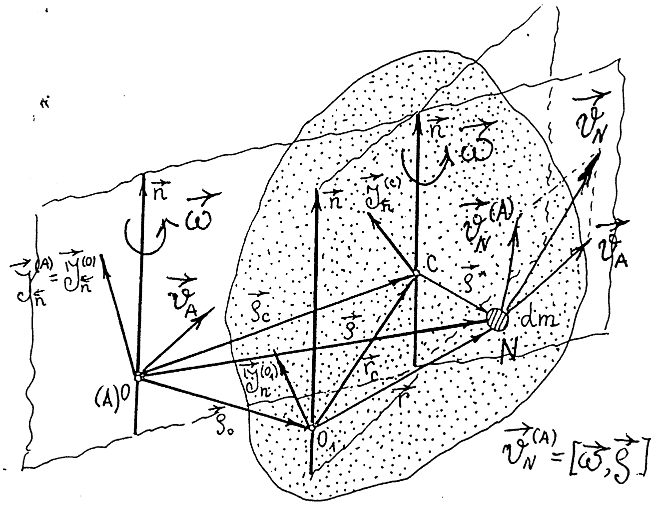


Figure No. 1

3. DERIVATIVES OF THE MASS MOMENTS VECTORS

For these defined vectors supports of the mass moments vectors we determined following derivatives [13]:

a* when rigid body is rotate around fixed axis with angular velocity $\vec{\omega}$:

$$\frac{d\vec{S}_{\vec{n}}^{(O)}}{dt} = \frac{d}{dt}\frac{\partial \vec{\mathrm{S}}_{\vec{n}}^{(O)}}{\partial m} = [\vec{n},[\vec{\omega},\vec{\rho}]] = [\vec{\omega},\vec{S}_{\vec{n}}^{(O)}] \qquad \frac{d\vec{\mathrm{S}}_{\vec{n}}^{(O)}}{dt} = [\vec{\omega},\vec{\mathrm{S}}_{\vec{n}}^{(O)}] \tag{5}$$

$$\frac{d\vec{\mathrm{N}}_{\vec{n}}^{(O)}}{dt} = [\vec{\omega},\vec{\mathrm{N}}_{\vec{n}}^{(O)}] \qquad \frac{d\vec{\mathrm{J}}_{\vec{n}}^{(O)}}{dt} = [\vec{\omega},\vec{\mathrm{J}}_{\vec{n}}^{(O)}] \tag{6}$$

b* when the body is with changeable structure configuration and with changeable body mass and when body is rotate around fixed axis with angular velocity $\vec{\omega}$ as well as around moving axis with angular velocity $\vec{\Omega}$:

$$\frac{d\vec{S}_{\vec{n}}^{(O)}}{dt} = \frac{d}{dt}\frac{\partial \vec{\mathrm{S}}_{\vec{n}}^{(O)}}{\partial m} = \left[\vec{n},\overset{*}{\vec{\rho}}\right] + [\vec{\omega},\vec{S}_{\vec{n}}^{(O)}] + \Omega[\vec{n},\vec{S}_{\vec{\Omega}_0}^{(O)}] = \left[\vec{n},\overset{*}{\vec{\rho}} + \omega\vec{S}_{\vec{n}}^{(O)} + \Omega\vec{S}_{\vec{\Omega}_0}^{(O)}\right] \tag{7}$$

$$\frac{d\vec{\mathrm{J}}_{\vec{n}}^{(O)}}{dt} = [\vec{\omega}+\vec{\Omega},\vec{\mathrm{J}}_{\vec{n}}^{(O)}] + \Omega\vec{\mathrm{J}}_{[\vec{n},\vec{\Omega}_0]}^{(O)} + 2\left[\overset{*}{\vec{\rho}},\vec{\mathrm{S}}_{\vec{n}}^{(O)}\right] + \frac{\overset{*}{d}\,\vec{\mathrm{J}}_{\vec{n}}^{(O)}}{dt} \tag{8}$$

$$\frac{d\vec{\mathrm{J}}_{\vec{u}}^{(O)}}{dt} = [\vec{\omega}+\vec{\Omega},\vec{\mathrm{J}}_{\vec{u}}^{(O)}] + \vec{\mathrm{J}}_{[\vec{u},\vec{\Omega}+\vec{\omega}]}^{(O)} + 2\left[\overset{*}{\vec{\rho}},\vec{\mathrm{S}}_{\vec{u}}^{(O)}\right] + \frac{\overset{*}{d}\,\vec{\mathrm{J}}_{\vec{u}}^{(O)}}{dt} \tag{9}$$

The expression of the derivative of the mass inertia moment tensor matrix when the body is with changeable structure configuration and with changeable body mass and when body is rotate around fixed axis with angular velocity $\vec{\omega}$ as well as around moving axis with angular velocity $\vec{\Omega}$:

$$\frac{d\mathbf{J}^{(O)}}{dt} = \mathbf{RJ}^{(O)} + \frac{\overset{*}{d}\,\mathbf{J}^{(O)}}{dt} + 2\overset{*}{\mathbf{r}}\mathbf{S}^{(O)} + \mathbf{J}^{(O)(\vec{\omega}+\hat{\Omega})} \tag{10}$$

The expression of the derivative of the vector $\vec{\mathrm{J}}_{\vec{n}}^{(O)}$ of the rigid body mass inertia moment for the pole O and the axis oriented by the unit vector $\dot{n}$ at the dimensional curvilinear coordinate system N we derived in the following form [24]:

$$\frac{d\vec{\mathrm{J}}_{\vec{n}}^{(O)}}{dt}=\iiint_V\left\{\left[\left(\Gamma_{kr,p}+\Gamma_{pr,k}\right)\vec{g}_l-\left(\Gamma_{kr,l}+\Gamma_{lr,k}\right)\vec{g}_p+\left(g_{kp}\Gamma_{lr}^s-g_{kl}\Gamma_{pr}^s\right)\vec{g}_s\right]\dot{x}^r x^k x^p\lambda^l\right\}dm+$$
$$+\iiint_V\left\{\left(g_{kp}\vec{g}_l-g_{kl}\vec{g}_p\right)\left[\left(\dot{x}^k x^p\lambda^l+x^k\dot{x}^p\lambda^l+x^k x^p\dot{\lambda}^l\right)dm+x^k x^p\lambda^l d\dot{m}\right]\right\}+$$
$$+\iiint_V\left(\frac{\partial g_{kp}}{\partial t}\vec{g}_l+g_{kp}\frac{\partial\vec{g}_l}{\partial t}-\frac{\partial g_{kl}}{\partial t}\vec{g}_p-g_{kl}\frac{\partial\vec{g}_p}{\partial t}\right)x^k x^p\lambda^l dm$$

By using mass moment vectors and their derivatives, the linear momentum and angular momentum of the rotor which rotates around one or two rotation axes are expressed simpler then the other ways, as it was shown in the paper. That fact is the main reason which for simpler qualitative analysis kinetic properties of rotor dynamic and their kinetic pressures on the shaft bearings.

It is shown that the main vector of the inertia forces and resulting couple of the inertia forces moment are determined by mass moments vectors, angular velocity and angular accelerations as well as by their derivatives. It is shown that the main vector of the reactive forces and resulting couple of the reactive forces moment of the rotor with changeable mass are determined by mass moments vectors, angular velocity and angular accelerations as well as by their derivatives, as well .

4. LINEAR MOMENTUM AND ANGULAR MOMENTUM OF THE BODY MOTION

The classic literature gives a very well known definition of the rigid body linear momentum (motion quantity) and angular momentum (motion quantity moment). We shall consider it a little with a slight modification due to the interpretation of the rigid body dynamic parameters by means of the introduced body mass moment vectors (see [11]). We are following the classic definition by using the prepositions from previous parafgaph, as well as Fig. No. 1 and 2, so that we write for the linear momentum following expression:

$$\vec{\mathrm{K}}=\iiint_V\vec{v}_N\,dm=\iiint_V\left(\vec{v}_A+\left[\vec{\omega},\vec{\rho}\right]\right)dm=\mathrm{M}\vec{v}_A+\omega\vec{\mathrm{S}}_{\vec{n}}^{(A)}\qquad(11)$$

The expression (11) of the linear momentum $\vec{\mathrm{K}}$ of the rigid body whose points have the translation velocity $\vec{v}_A$ of the referential point A and the relative velocity $\left[\vec{\omega},\vec{\rho}\right]$ due to the rotation around the axis oriented by the vector $\vec{\omega}=\omega\vec{n}$ through the point A has two parts: 1* the translatory one equal to the product of the referential point and the body mass - the linear momentum due to the translation motion with the velocity of the referential point A ; and the rotatory one equal to the product of the magnitude ω of the angular velocity $\vec{\omega}=\omega\vec{n}$ and the vector $\vec{\mathrm{S}}_{\vec{n}}^{(A)}$ of the body mass linear moment at the referential point A for the axis oriented by the unit vector $\vec{n}$.

If the pole A is the body mass center C then the linear momentum is equal only in the translatory part since the vector $\vec{\mathrm{S}}_{\vec{n}}^{(A)}$ of the body mass linear moment for the pole in the body mass center is equal to zero regardless of its orientation so that the linear momentum is equal to the product of this velocity $\vec{v}_C$ of the body mass center and the rigid body mass: $\vec{\mathrm{K}}=\mathrm{M}\vec{v}_C$. The same stands for if the pole A is not the body mass center but if the axis oriented with $\vec{\omega}=\omega\vec{n}$ trough pole A passes trough the mass center.

The second kinetic vector connected to the referential point which plays an important part (role) in the rigid body dynamics is the rigid body angular momentum (motion quantity moment) for the given pole, $\vec{\mathrm{L}}_O$. Following the classic definition according to the Ref. [29] the rigid body angular momentum is calculated by means of the following expression:

$$\vec{L}_O = \iiint_V [\vec{r}, \vec{v}_N] dm = \iiint_V [\vec{r}_A + \vec{\rho}, \vec{v}_A + [\vec{\omega}, \vec{\rho}]] dm \qquad (12)$$

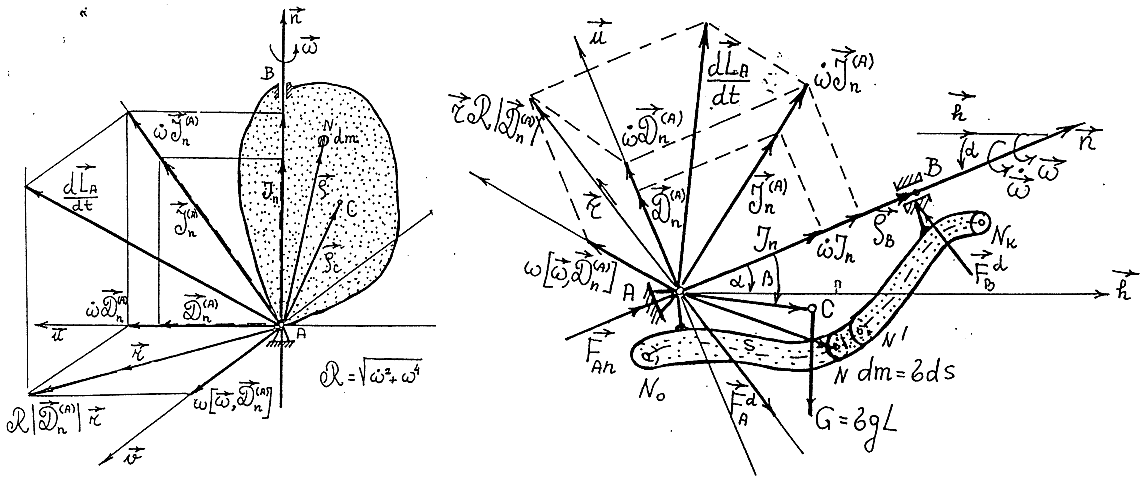


Figure No. 2 a* and b*

Following the idea of this paper that at the basis of the rigid body motion interpretation there are rigid body dynamic parameters which express the mass inertia moment properties and the kinematic parameters, translation velocity $\vec{v}_A$ of the rigid body referential point and the angular velocity $\vec{\omega}$ of the relative momentary rotation around the axis oriented with $\vec{\omega}$ and through the referential point A then the angular momentum for the point A, $\vec{L}_A$ is connected not only to the pole but to the axis oriented by the momentary angular velocity vector to which we connect the vectors $\vec{S}^{(A)}_{\vec{n}}$ and $\vec{J}^{(A)}_{\vec{n}}$ of the rigid body mass linear and inertia moments by connecting the body mass to the translation velocity of the referential point A.

If the referential point A is in the body mass center than the angular momentum for the pole O is equal to:

$$\vec{L}_O = [\vec{S}^{(O)}_{\vec{n}}, \vec{v}_C] + \omega \vec{J}^{(C)}_{\vec{n}} \qquad (13)$$

while the angular momentum for the pole in the mass center C is :

$$\vec{L}_C = \omega \vec{J}^{(C)}_{\vec{n}} \qquad (14)$$

and it is equal to the product of the magnitude of the momentary angular velocity ω and the vector $\vec{J}^{(C)}_{\vec{n}}$ of the rigid body mass inertia moment for the central axis oriented by the vector of the momentary angular velocity $\vec{\omega}$.

The Ref. [11] has the deviation center of the body for the given direction for the material particles system and the deviation load by the linear momentum analysis. Considering that we have introduced the deviation load vector by the analysis of the vector $\vec{J}^{(A)}_{\vec{n}}$ of the body mass inertia moment as its component normal to the axis for which it is determined we can see that the deviational part of the angular momentum vector proportional to the vector $\vec{D}^{(A)}_{\vec{n}}$ of the deviational load the body mass inertia moment of the axis around which the rigid body rotates since it is:

$$\vec{L}_A = [\vec{S}^{(A)}_{\vec{n}}, \vec{v}_A] + \vec{\omega}(\vec{n}, \vec{J}^{(A)}_{\vec{n}}) + \omega[\vec{n}[\vec{J}^{(A)}_{\vec{n}}, \vec{n}]] = [\vec{S}^{(A)}_{\vec{n}}, \vec{v}_A] + \vec{\omega}(\vec{n}, \vec{J}^{(A)}_{\vec{n}}) - \omega \vec{D}^{(A)}_{\vec{n}} \qquad (15)$$

If the point A is the mass center then it stands for:

$$\vec{L}_C = \vec{\omega}(\vec{n}, \vec{J}^{(C)}_{\vec{n}}) - \omega \vec{D}^{(C)}_{\vec{n}} \qquad (16)$$

If the rotation axis is the main mass inertia moment axis then the angular momentum does not have any deviational part since the rotation axis is not subjected to the deviation load by the rigid body mass inertia moment and the angular momentum vector for the mass center is collinear with the rotation axis.

5. ONE INTERPRETATION FOR THE CASE OF THE RIGID BODY ROTATION AROUND THE FIXED AXIS

The Fig. No. 2 shows the rigid body with the rotation axis around which it rotates with the angular velocity $\vec{\omega}$ which changes in time so that there appears the angular acceleration $\dot{\vec{\omega}}$. The linear momentum and angular momentum are (see Ref. [11]):

$$\vec{\mathsf{K}} = [\vec{\omega}, \vec{\rho}_C]\mathrm{M} = \omega\vec{\mathsf{S}}_{\vec{n}}^{(A)} \tag{17}$$

$$\vec{\mathsf{L}}_A = \vec{\omega}\left(\vec{n}, \vec{\mathsf{J}}_{\vec{n}}^{(A)}\right) + \omega\left[\vec{n}\left[\vec{\mathsf{J}}_{\vec{n}}^{(A)}, \vec{n}\right]\right] = \vec{\omega}\left(\vec{n}, \vec{\mathsf{J}}_{\vec{n}}^{(A)}\right) + \omega\vec{\mathsf{D}}_{\vec{n}}^{(A)} \tag{18}$$

Since the velocity $\vec{v}$ and the acceleration $\vec{a}$ of the body elementary mass at the point N are:

$$\vec{v} = [\vec{\omega}, \vec{\rho}] \qquad \vec{a} = \left[\dot{\vec{\omega}}, \vec{\rho}\right] + [\vec{\omega}, [\vec{\omega}, \vec{\rho}]] \tag{19}$$

then for the main vector $\vec{\mathrm{F}}_{\mathrm{rj}}$ the inertia force of the overall rigid body rotating around the axis with the angular velocity $\vec{\omega}$ we obtain:

$$\vec{\mathrm{F}}_{\mathrm{rj}} = -\iiint_V \vec{a}\,dm = -\dot{\omega}\vec{\mathsf{S}}_{\vec{n}}^{(A)} - \omega\left[\vec{\omega}, \vec{\mathsf{S}}_{\vec{n}}^{(A)}\right] \tag{20}$$

For the main moment of the inertia forces of the overall rigid body rotating around the axis and for the point A we calculate the following:

$$\vec{\mathsf{M}}_{\mathrm{Aj}} = \iiint_V \left[\vec{\rho}, d\,\vec{\mathrm{F}}_{\mathrm{rj}}\right] = -\dot{\omega}\vec{\mathsf{J}}_{\vec{n}}^{(A)} - \omega\left[\vec{\omega}, \vec{\mathsf{J}}_{\vec{n}}^{(A)}\right] \tag{21}$$

as well as:

$$\vec{\mathsf{M}}_{\mathrm{Aj}} = \iiint_V \left[\vec{\rho}, d\,\vec{\mathrm{F}}_{\mathrm{rj}}\right] = -\frac{\dot{\omega}}{\omega}\vec{\mathsf{L}}_A - \omega\left[\vec{\omega}, \vec{\mathsf{L}}_A\right] \tag{21*}$$

The dynamic equations of the body rotation around fixed axis can be obtained by differentiating in time the expression (17) for the *linear momentum* and expression (18) for *angular momentum* on the basis of which we obtain:

$$1^* \quad \frac{d\vec{\mathsf{K}}}{dt} = \dot{\omega}\vec{\mathsf{S}}_{\vec{n}}^{(A)} + \omega\left[\vec{\omega}, \vec{\mathsf{S}}_{\vec{n}}^{(A)}\right] = -\vec{\mathrm{F}}_{\mathrm{rj}} = \vec{\mathrm{F}}_{\mathrm{r}} \tag{22}$$

$$\frac{d\vec{\mathsf{K}}}{dt} = \left|\vec{\mathsf{S}}_{\vec{n}}^{(A)}\right|\left(\dot{\omega}\vec{u}_1 + \omega^2\,\vec{\mathrm{v}}_1\right) = \vec{\mathsf{R}}\left|\vec{\mathsf{S}}_{\vec{n}}^{(A)}\right| = \mathsf{R}\left|\vec{\mathsf{S}}_{\vec{n}}^{(A)}\right|\vec{\mathrm{r}}_1 \tag{23}$$

$$\vec{\mathsf{R}}_1 = \mathsf{R}\,\vec{\mathrm{r}}_1 \qquad \mathsf{R} = \sqrt{\dot{\omega}^2 + \omega^4} \tag{24}$$

The ***rotator*** $\vec{\mathsf{R}} = \mathsf{R}\,\vec{\mathrm{r}}_1$ is normal to the rotation axis and the gravitational plane through the axis.

The equation (17) for the linear momentum change which is equal to the main vector (resultant) of the active and reactive forces shows that the motion linear momentum changes the vector normal to the rotation axis and has two components: one due to the angular velocity change which is normal to the rotation axis and the plane which contains the body mass center and the rotation axis, and the other component which depends on the angular velocity square which is normal to the rotation axis and lie in the plane formed by rotation axis and the rigid body mass center doing rotation.

$$2^* \quad \frac{d\vec{\mathrm{L}}_A}{dt} = \dot{\omega}\vec{\mathrm{J}}_{\vec{n}}^{(A)} + \omega\left[\vec{\omega}, \vec{\mathrm{J}}_{\vec{n}}^{(A)}\right] = -\vec{\mathrm{M}}_{\mathrm{Aj}} = \vec{\mathrm{M}}_{\mathrm{A}} \qquad (25)$$

$$\frac{d\vec{\mathrm{L}}_A}{dt} = \dot{\vec{\omega}}\,\mathrm{J}_{\vec{n}}^{(A)} + \dot{\omega}\vec{\mathrm{D}}_{\vec{n}}^{(A)} + \omega\left[\vec{\omega}, \vec{\mathrm{D}}_{\vec{n}}^{(A)}\right] = \dot{\vec{\omega}}\,\mathrm{J}_{\vec{n}}^{(A)} + \left|\vec{\mathrm{D}}_{\vec{n}}^{(A)}\right|\vec{\mathrm{R}} \qquad (26)$$

$$\vec{\mathrm{R}} = \mathrm{R}\,\vec{\mathrm{r}} \qquad \mathrm{R} = \sqrt{\dot{\omega}^2 + \omega^4} \qquad (27)$$

The rotator $\vec{\mathrm{R}} = \mathrm{R}\,\vec{\mathrm{r}}$ which is rotating and increasing by the angular velocity and by the angular acceleration at the same causes the inertia forces deviation moment to increase.

The equation (26) which is written on the basis of the law of the body angular momentum change which is says that the derivative in time of the body angular momentum for a certain pole in stationary bearing, equal to the moment of the active and reactive forces acting on the body for the same pole.

This form (26) immediately shows that the first component depending on the angular acceleration is collinear with the rotation axis; the second component which also depends on the angular acceleration is normal to the rotation axis and the vector $\vec{\mathrm{J}}_{\vec{n}}^{(A)}$ of the rigid body mass inertia moment for the pole in the fixed bearing A and for the rotation axis, that is, it is proportional to the magnitude of the angular acceleration $\dot{\vec{\omega}}$ and the vector $\vec{\mathrm{D}}_{\vec{n}}^{(A)}$ of the rotation rigid body mass deviation moment of the rotation axis in the stationary bearing A and for the rotation axis; the third component is proportional to the square of the angular velocity ω^2 and to the magnitude of the vector $\vec{\mathrm{D}}_{\vec{n}}^{(A)}$ of the rotation rigid body mass deviation moment of the rotation axis in the stationary bearing A and for the rotation axis, whereas it is like a vector normal to the rotation axis and the vector $\vec{\mathrm{D}}_{\vec{n}}^{(A)}$ of the deviation load to the rotation axis which means it is normal to the deviation plane. In the case it is the rotation with a constant angular velocity the stroke derivative components in time do not appear in the deviation plane; there is only a component normal to the deviation plane $\omega\left[\vec{\omega}, \vec{\mathrm{D}}_{\vec{n}}^{(A)}\right]$.

Figure No. 2 shows the characteristic vectors, the rigid body mass moment vectors and the rigid body dynamics kinetic vectors in the rotation around fixed axis.

If we now return to the expressions (20) and (21) for the inertia force main vector and the inertia force main moment for the pole at the stationary bearing A we come to the following conclusion: 1* the expression (20) is equal to the one for the rigid body linear momentum derivative in time a changed sigh, while the expression (21) is equal to the angular momentum for the pole at the stationary bearing A, derivative in time, with a changed sigh so that the conclusions drawn to the expressions (25) and (26) also stand for the expression (22) and (23). These conclusions can also be defined in another way: we conclude from expression (21) that the inertia forces main moment for the rigid body rotation around the fixed axis has three components: the first one is purely rotatory around the rotation axis collinear with it if the angular acceleration is different from zero and it is proportional to the angular acceleration $\dot{\omega}$ and the body mass axial inertia moment for the rotation axis, $\mathrm{J}_{\vec{n}}^{(A)}$; and the second deviational component is normal to the rotation axis which also depends on the angular acceleration and the vector $\vec{\mathrm{D}}_{\vec{n}}^{(A)}$ of the deviation load of the rotation axis; and third component depending on the angular velocity squared of the rigid body rotation around the fixed axis and on the magnitude of the mass deviation moment vector of the rotation axis at the pole in the stationary bearing.

The derivative in time of the body angular momentum for a certain pole in stationary bearing normal to the rotation axis is:

$$\frac{d\vec{\mathsf{L}}_A^d}{dt} = \dot{\omega}\vec{\mathsf{D}}_{\vec{n}}^{(A)} + \omega\left[\vec{\omega}, \vec{\mathsf{D}}_{\vec{n}}^{(A)}\right] = \left|\vec{\mathsf{D}}_{\vec{n}}^{(A)}\right|\vec{\mathsf{R}} \tag{28}$$

By expressions (20), (21), (23) and (28) we can write following relations:

$$\frac{\left|\vec{\mathrm{F}}_{\mathrm{rj}}\right|}{\left|\vec{\mathsf{M}}_{A\,\mathrm{j}}\right|} = \frac{\left|\dfrac{d\vec{\mathsf{K}}}{dt}\right|}{\left|\dfrac{d\vec{\mathsf{L}}_A^d}{dt}\right|} = \frac{\left|\vec{\mathsf{S}}_{\vec{n}}^{(A)}\right|}{\left|\vec{\mathsf{D}}_{\vec{n}}^{(A)}\right|} = constant \tag{29}$$

6. INTERPRETATION OF THE KINETIC PRESSURES ON BEARING BY MEANS OF THE MASS MOMENT VECTORS FOR THE POLE AND THE AXIS

In this part the kinetic pressures of shaft bearings are expressed by means of the mass moment vectors: $\vec{\mathsf{S}}_{\vec{n}}^{(A)}$ of the body mass linear moment and $\vec{\mathsf{D}}_{\vec{n}}^{(A)}$ of the deviation load by the body mass inertia moment of the rotation axis for the pole in the stationary bearing.

The figure No. 2 shows a rigid body that can rotate around a stationary axis is like a rigid shaft without mass supported on the stationary bearing A and on the moveable sliding one along the rotation axis. In the general case let a rigid body be subjected to a system of forces $\vec{\mathrm{F}}_{\mathrm{k}}$ whose points application N_{ko} are determined by the position vectors $\vec{\rho}_k$ with respect to the pole in the stationary bearing A.

Let's denote the rotation angle of the body around the stationary axis oriented by unit vector $\vec{n}$ with $\vec{\varphi} = \varphi\vec{n}$.

Following the expressions (20) and (21), as well as expression (23) and (26) we can write the following two vector equations:

$$\frac{d\vec{\mathsf{K}}}{dt} = \left|\vec{\mathsf{S}}_{\vec{n}}^{(A)}\right|\left(\dot{\omega}\vec{u}_1 + \omega^2\,\vec{\mathrm{v}}_1\right) = \vec{\mathsf{R}}\left|\vec{\mathsf{S}}_{\vec{n}}^{(A)}\right| = \mathsf{R}\left|\vec{\mathsf{S}}_{\vec{n}}^{(A)}\right|\vec{\mathrm{r}}_1 = \sum_{k=1}^{k=N}\vec{\mathrm{F}}_{\mathrm{k}} + \vec{\mathrm{F}}_A + \vec{\mathrm{F}}_B \tag{30}$$

$$\frac{d\vec{\mathsf{L}}_A}{dt} = \dot{\vec{\omega}}\,\mathrm{J}_{\vec{n}}^{(A)} + \dot{\omega}\vec{\mathsf{D}}_{\vec{n}}^{(A)} + \omega\left[\vec{\omega}, \vec{\mathsf{D}}_{\vec{n}}^{(A)}\right] = \dot{\vec{\omega}}\,\mathrm{J}_{\vec{n}}^{(A)} + \left|\vec{\mathsf{D}}_{\vec{n}}^{(A)}\right|\vec{\mathsf{R}} = \sum_{k=1}^{k=N}\left[\vec{\rho}_k, \vec{\mathrm{F}}_{\mathrm{k}}\right] + \left[\vec{\rho}_B, \vec{\mathrm{F}}_B\right] \tag{31}$$

These two vectorial equations are kinetic equations of dynamic equilibrium in motion-rotation of the body around the stationary axis under the action of the active force system $\vec{\mathrm{F}}_{\mathrm{k}}$.

If we now multiply scalarly and vectorly these equations (30) and (31) be the unit vector $\vec{n}$ and having in mind that the $\vec{\rho}_B = \rho_B\vec{n}$, we obtain:

1* the rotation equation around the axes oriented by unit vector $\vec{n}$ in the form:

$$\left(\vec{\mathsf{J}}_{\vec{n}}^{(A)}, \dot{\vec{\omega}}\right) = \sum_{k=1}^{k=N}\left(\left[\vec{\rho}_k, \vec{\mathrm{F}}_{\mathrm{k}}\right], \vec{n}\right) \tag{32}$$

2* the equations for determining the bearings kinetic pressures, that is pressures upon the bearings , $\vec{\mathrm{F}}_{\mathrm{A}}$ and $\vec{\mathrm{F}}_{\mathrm{B}}$, that is, their components in the axis direction $\vec{n}$ and normal to the rotation axis:

$$\vec{\mathrm{F}}_{\mathrm{A}\vec{n}} = \left(\vec{\mathrm{F}}_{\mathrm{A}}, \vec{n}\right)\vec{n} = -\vec{n}\sum_{k=1}^{k=N}\left(\vec{\mathrm{F}}_k, \vec{n}\right) \tag{33}$$

$$\vec{\mathrm{F}}_{\mathrm{A}T} = -\vec{\mathrm{F}}_B + \vec{\mathsf{R}}_1\left|\vec{\mathsf{S}}_{\vec{n}}^{(A)}\right| - \sum_{k=1}^{k=N}\left[\vec{n}, \left[\vec{\mathrm{F}}_k, \vec{n}\right]\right] \tag{34}$$

$$\vec{\mathrm{F}}_{\mathrm{B}} = \frac{1}{\rho_B}\vec{\mathsf{R}}\left|\vec{\mathsf{D}}_{n}^{(A)}\right| - \frac{1}{\rho_B}\sum_{k=1}^{k=N}\left[\vec{n}, \left[\left[\vec{\rho}_k, \vec{\mathrm{F}}_k\right], \vec{n}\right]\right] \tag{35}$$

From the expression for the bearings pressures (resistance) we select a part which is the result of the action of an external active forces and the influence of which upon the bearings resistances in possible variable in time is only due to the change of their line of application as well as the point of application with respect to the configuration of the body which is rotating such as in the case when the force of the body's own weight which retains the application line direction in relation to the rotation axis, and thus its position with respect to the body configuration, although in doing this it retains the application point constantly in the body mass center which rotates around the rotation axis together with body. The body mass center describes a circle or an arc in the plane through the mass center normal to the rotation axis.

Other part of the bearing kinetic resistance (pressures) in the body rotation around the stationary axis is the result exclusively of the kinetic-inertial body properties with respect to the rotation axis and the rotation kinematics and rigid body rotation kinematics around the stationary axis. These parts appear as parameters depending on the rotator vector $\vec{\mathsf{R}}$ which in itself contains the angular velocity and the angular acceleration of the body rotation around the rotation axis and the rigid body mass moment properties with respect to the pole A at stationary bearing and the rotation axis expressed by the mass moment vectors: $\vec{\mathsf{S}}_{\vec{n}}^{(A)}$ of the body mass linear moment and $\vec{\mathsf{D}}_{\vec{n}}^{(A)}$ of the deviation load by the body mass inertia moment of the rotation axis for the pole A in the stationary bearing.

In order to discuss the rotor effect on the kinetic pressure upon the bearings in which the rigid body shaft axis is rotating it is necessary to know the angular acceleration $\dot{\vec{\omega}}$ and the angular velocity $\vec{\omega}$ and in order to do this it is necessary to solve the body rotation/oscillation equation around the axis (32), namely, to determine $\vec{\varphi}(t)$ and $\vec{\omega}(t)$ as well as $\omega(\varphi)$.

If the rotation axis is the central and main mass inertia moment axis and the for the pole in the stationary bearing then it is a rigid body which is dynamically balanced and the member in the kinetic pressures depending on the vectors $\vec{\mathsf{S}}_{\vec{n}}^{(A)}$ of the body mass linear moment and $\vec{\mathsf{D}}_{\vec{n}}^{(A)}$ of the deviation load by the body mass inertia moment of the rotation axis for the pole A in the stationary bearing are equal to zero and are not influenced by the rotator change. Then there are only the components of the bearing resistance arising from the bearings "kvazi-static" resistances in the definite position of the active forces system and the reactive forces system during the body rotation.

If the rotation axis is the axis of the mass inertia moment asymmetry for the referential point in the stationary bearing then the kinetic pressures are the greatest both on moveable and stationary bearing. Since at each point on the rigid body there are three pairs of such mutually perpendicular axes which are in pair perpendicular to one main mass inertia moment direction and they form with the others an angle of $\pi/4$ each so that the mass inertia moment asymmetry axes which are perpendicular to the second main mass inertia moment direction forming angle of $\pi/4$ each with the first and third main mass inertia moment directions as the rotation axes will be the greatest vector of the deviation load and at the same time the greatest kinetic pressures on both the bearings.

The kinetic pressure on the stationary bearing depends on the body mass center position with respect to the rotation axis and this can be adjusted by the choice of the inertia asymmetry axes in pair as well as by the choice of the moveable bearing position with respect to the stationary one on the definite axis of mass inertia moment asymmetry. The body mass inertia moment asymmetry axes should be avoided as the rotation axis in order to reduce the dynamic pressures upon the bearings.

For a pair of the mass inertia moment asymmetry axes as the rotation axes the axial mass inertia moment of the rotatory body is identical so that depending on the body mass center position with respect to one axis or another and on the choice of the moveable bearing an increase, that is, decrease of the kinetic pressure at a given constant value of the initial energy communicated to the rotating body.

There are four (that is, eight) axes through each point of the body which we have chosen as a stationary bearing for which the axial mass inertia moments are the same value and the vectors $\vec{\mathrm{D}}_{\vec{n}}^{(A)}$ of the deviation load by the body mass inertia moment of the rotation axis for the pole A in the stationary bearing are proportional to the sum of the three mass deviation load vectors by the body mass inertia moment of the mass inertia moment asymmetry axes. For these octahedral axes the dynamic pressures on both the stationary and moveable bearings are the same while the pressures on the stationary bearing are different and by choosing one of the octahedral axes minimization of maximization of their value can be performed. By displacing the moveable bearing from one to another octahedral axis through the stationary bearing the kinetic pressure on the stationary bearing can be adjusted while retaining the share in the pressure on both the bearing of the part that corresponds to the deviation load vector although the rotator is going to change as well (but this can also be adjusted). The smallest pressures would appear an octahedral axis is chosen so that the body mass center is closest to the rotation axis, that is, the most favorable of all the octahedral axes for the rotation axes is the one which body mass center is closest to.

A general conclusion would be that if we cannot select in the design way the rotation axis as the rigid body main central mass inertia moment axis when the system is dynamically balanced and analysis of the inertia moment state should be performed at each of the possible points of the stationary bearing positioning and according to the design requirements the selection should be done of both the stationary bearing and of the rotation axis according to the analysis.

These conclusions are very important if the designer cannot change the stationary bearing but if we can change the moveable one and chose it freely in the rigid body then his choose is important since the dynamic pressures should be as small as possible.

7. ON ROTATION OF A HEAVY BODY AROUND A STATIONARY AXIS IN THE FIELD WITH TURBULENT DAMPING AND KINETIC PRESSURES ON BEARING

In this part the differential equation of the rotation and/or oscillations of a heavy body rotor around a stationary axis in the Earth gravitational field as well as in the turbulent damping field (see Fig. No. 3) are studied. In the general case the rotation axis is not horizontal (see Ref. [8]) . Using the solution of this differential equation the kinetic components of the bearing reactions of the shaft are studied. From this scalar differential equation of the heavy body rotation/ oscillation the first integral is determined as well as the energy integral of the corresponding conservative system by the use of which the representational point motion character analysis is carried out on the phase trajectories in the phase plane.

The analysis of the representational point transition from one to the other curve of the energy constant is also performed together with an appropriate analysis of the constant energy curves in the phase plane.

7.1 DIFFERENTIAL EQUATION OF ROTATION OF A HEAVY BODY AROUND A STATIONARY AXIS IN THE FIELD WITH TURBULENT DAMPING

Let's assume that the system is in the turbulent damping field so that the body is under the action of the resistance moments of resulting effect $\vec{\mathrm{M}}_{wA}$:

$$\vec{\mathrm{M}}_{wA} = -b_m \vec{\omega}|\vec{\omega}| \qquad (36)$$

which are proportional to the squared angular velocity $\vec{\omega}$ of the rotation in which b_m is the resistences coupled coefficient; and the resistence forces whose resulting action can be

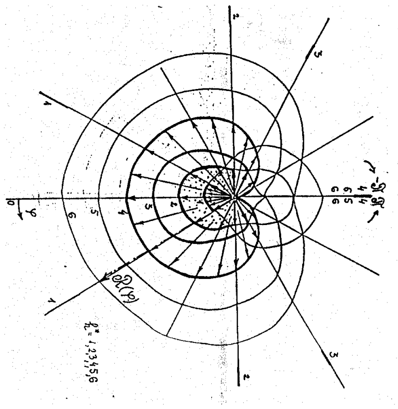


Figure No. 4 e*

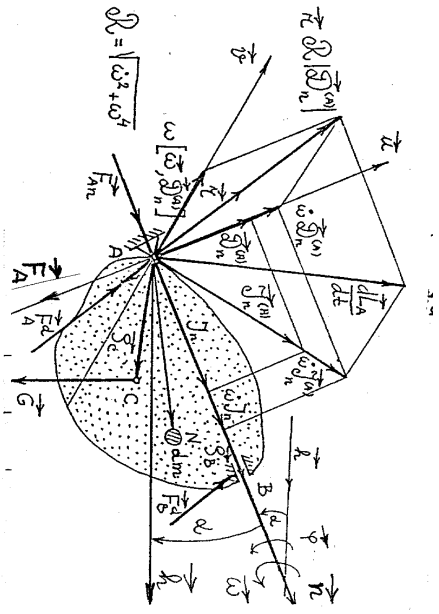


Figure No. 3

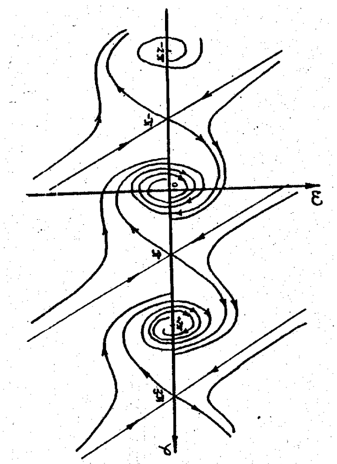


Figure No. 4 a*

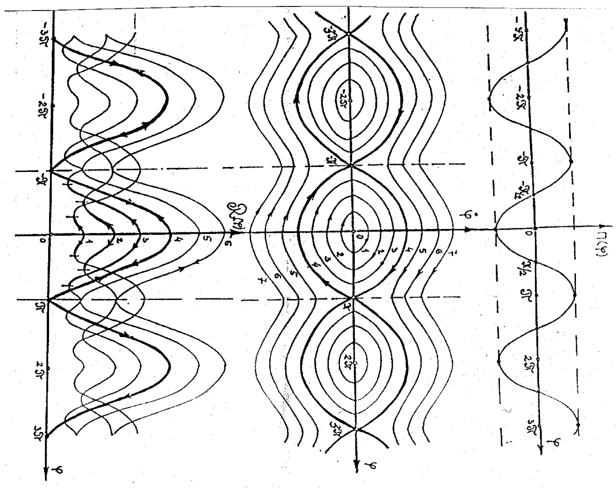


Figure No. 4 b*

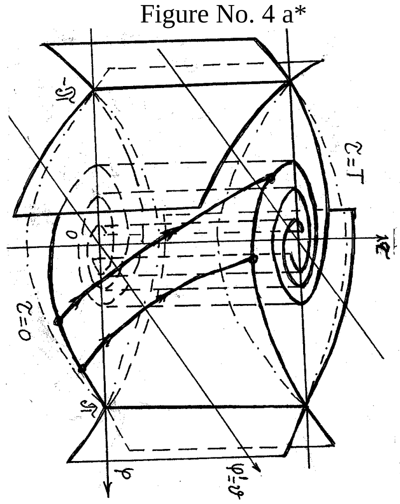


Figure No. 4 f*

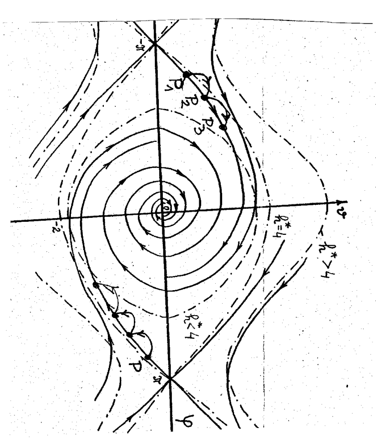


Figure No. 4 c*

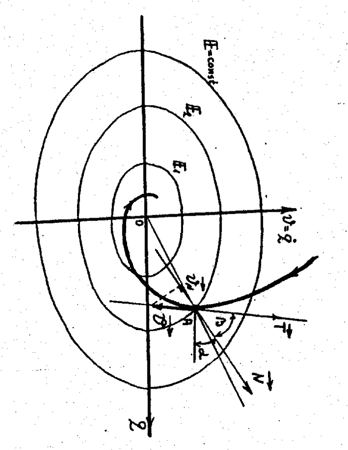


Figure No. 4 d*

expressed by means of the squared velocity $\vec{v}_C$ and whose point application is in this mass center:

$$\vec{\mathrm{F}}_w = -b_F^* \vec{v}_C \left|\vec{v}_C\right| = -b_F^* \omega^2 \vec{\mathsf{S}}_{\vec{n}}^{(A)} \left|\vec{\mathsf{S}}_{\vec{n}}^{(A)}\right| \tag{37}$$

By using the vector differential equations of a heavy body rotation (30) and (31) and the differential equation of the rotation (32) derived in the previous part here we can write the following differential equation for rotation/oscillation of the heavy body in the field with turbulent dumping:

$$\left(\vec{\mathsf{J}}_{\vec{n}}^{(A)}, \dot{\vec{\omega}}\right) + b_m\left(\vec{\omega}, \left|\vec{\omega}\right| \vec{n}\right) = \left(\left[\vec{\rho}_C, \vec{\mathrm{G}}\right], \vec{n}\right) + \sum_{k=1}^{k=N}\left(\left[\vec{\rho}_k, \vec{\mathrm{F}}_k\right], \vec{n}\right) - b_F \omega^2 \left(\left[\vec{\rho}_C, \vec{\mathsf{S}}_{\vec{n}}^{(A)}\right], \vec{n}\right)\left|\vec{\mathsf{S}}_{\vec{n}}^{(A)}\right| \tag{38}$$

For determining of the bearings pressures of the shaft we can use the expressions from (30) to (31) from previous part with different values of the vector rotator $\vec{\mathsf{R}}$ according to the solution of the differential equation (38) and with additional part of the resistance forces:

$$\vec{\mathrm{F}}_{Bw} = -\vec{\mathrm{F}}_{Aw} = -\frac{1}{\rho_B} b_F \left[\vec{\omega}, \left[\left[\vec{\rho}_C, \vec{\mathsf{S}}_{\vec{n}}^{(A)}\right], \vec{\omega}\right]\right] \left|\vec{\mathsf{S}}_{\vec{n}}^{(A)}\right| \tag{39}$$

From the expressions for the bearings resistances we select parts which are the directly result of the kinetic properties of the body with respect to the rotation axis. These parts appear as a parameters depending on the rotor vector $\vec{\mathsf{R}}$ which in itself comparises the angular velocity $\vec{\omega}$ and the angular acceleration $\dot{\vec{\omega}}$ of the body rotation around the rotation axis as well as on the rigid body mass moment deviational properties with respect to the pole in the stationary bearing and to the rotation axis expressed by the mass moment vectors $\vec{\mathsf{S}}_{\vec{n}}^{(A)}$ and $\vec{\mathsf{D}}_{\vec{n}}^{(A)}$.

Further we concentrate on the rigid body motion consideration by the solution of the differential equation (38). In the case the rotation/oscillation differential equation comes to the form:

$$\ddot{\varphi} + \frac{b_m + b_F \rho_C^3 \sin^3\beta}{\mathrm{J}_n^{(A)}} \dot{\varphi}\left|\dot{\varphi}\right| + \frac{\mathrm{M} g \rho_C \sin\beta \cos\alpha}{\mathrm{J}_n^{(A)}} \sin\varphi = 0 \tag{40}$$

If we denote the following expression with Ω^2 and 2δ:

$$\Omega^2 = \frac{\mathrm{M} g \rho_C \sin\beta \cos\alpha}{\mathrm{J}_n^{(A)}} = \frac{g}{\ell_r}, \qquad 2\delta = \frac{b_m + b_F \rho_C^3 \sin^3\beta}{\mathrm{J}_n^{(A)}} = 2\varsigma\Omega \tag{41}$$

in which ℓ_r denotes the expression of the form same as the expression (415), then the motion differential equation (40) comes to the form:

$$\ddot{\varphi} + 2\delta\dot{\varphi}\left|\dot{\varphi}\right| + \Omega^2 \sin\varphi = 0 \tag{42}$$

which is known in the Reference [30] as a mathematical model for a heavy material point motion along the circle of the radius ℓ_r in the vertical plane and in the turbulent damping field.

This motion differential equation (42) is nonlinear and by the substitution $v = \dfrac{\dot{\varphi}}{\Omega}$ it can be reduced to the form:

$$v\frac{dv}{d\varphi} = -2\delta v\left|v\right| - \sin\varphi \tag{43}$$

or it can be written as:

$$\frac{dv^2}{d\varphi} \pm 4\delta v^2 = -2\sin\varphi \tag{44}$$

The substitution of the variable is carried out on the basis of:

$$v = \dot{\varphi}\sqrt{\frac{\mathrm{J}_{\vec{n}}^{(A)}}{\mathrm{M}\, g\rho_C \sin\beta \cos\alpha}} \qquad (45)$$

The previous differential equation (44) is a common linear one with respect to the unknown and its solution is easy to find in the form:

$$v^2 = \frac{2}{16\delta^2 + 1}\left[\cos\varphi \mp 4\delta \sin\varphi\right] + \mathrm{C}_{1,2}\, e^{\mp 4\delta\varphi} \quad \mp\; for\, v < 0\, or\; for\, v > 0 \qquad (46)$$

In the previous differential equations (46) constants $\mathrm{C}_1 and \mathrm{C}_2$ are the integrational ones and they depend on the initial conditions, on the communicated initial angular velocity ω_o and on the initial elongation φ_o at the initial moment of the body motion. This means that these constants are determined from the total system energy communicated to the rigid body: kinetic energy via the angular velocity ω_o and the potential energy communicated to the system by the initial elongation φ_o of the angle measured from the stable position of the body. By the values of the integrational constants $\mathrm{C}_1 and \mathrm{C}_2$ the motion character of the rigid body is determined in its rotation.

By means of the equations (46) and by solving with respect to t we obtain the following integral:

$$t = \frac{1}{\Omega}\int_0^{\varphi} \frac{d\varphi}{\sqrt{\frac{2}{16\delta^2+1}\left(\cos\varphi \pm 4\delta\sin\varphi\right) + \mathrm{C}_{1,2}\, e^{\mp 4\delta\varphi}}} \qquad (47)$$

and for the $\delta = 0$ this integral comes to the case considered in the previous part when there is no turbulent damping, that is, when the rigid body only moves in the conservative field of the Earth gravity.

7.2. PHASE PORTRAIT ANALYSIS AND CONSTANT ENERGY CURVES ANALYSIS IN THE PHASE PLANE

On the basis of the numerical analysis of the equations (46) by changing the values of the integrational constants $\mathrm{C}_1 and \mathrm{C}_2$ a family of integral curves in obtained whose scheme is shown in the Figure No. 4 a* and which are compatible with the ones known in the Reference /48/ for a heavy material particle motion along the circle (gravitational pendulum) in the turbulent damping field.

From the phase trajectories in the Figure No. 4 we conclude that there are two type of the singularity: stable focus and unstable saddle point. To the stable focus $\varphi = 2k\pi, k = 0,\pm1,\pm2,\pm3,..$ the position of the stable equilibrium corresponds in which the system has a minimal potential energy, whereas to the unstable saddle points $\varphi = (2k\pm1)\pi, k = 0,\pm1,\pm2,\pm3,..$ the unstable equilibrium position corresponds in the system has a maximal value of the potential energy. In this position the mass center is at the highest level.

Depending on the system initial energy, that is, on the constants $\mathrm{C}_1 and \mathrm{C}_2$ there are three kinds of the phase trajectories, that is of the motion:

1* Oscillatory descending motion for small values of the initial angular velocity ω_o and of the initial elongation φ_o at the initial moment of the body motion when the body oscillates around the low equilibrium position. In this case the amplitudes and the velocities interchange, that is descend;

2* Motion along the separating curve passing through $\varphi = (2k\pm1)\pi, k = 0,\pm1,\pm2,\pm3,..$ depending on the angular velocity sense; it can be done as asymptotic towards $\varphi = \pi, \dot{\varphi} = 0$, or as oscillatory descending until its stabilization around the low balanced equilibrium position;

3* Progressive periodic motion of the total body revolutions around the rotation axis and its continuation as the oscillatory descending motion around the low stable equilibrium position until its stabilization.

The question of the time interval in which these motions are performed is not going to be analyzed here for each particular case but some estimations can be done on the basis of the motion time analysis for a gravitational pendulum without damping (presented in the previous part) as well as on the basis of the formula (47).

The constant energy curves, presented on Fig. No. 4 b*, are formed on the basis of the supposition-condition that the summa of the kinetic and potential energy is constant. The $\mathrm{E}_o = \mathrm{E}(\dot{\varphi}_o, \varphi_o)$ is the total system energy which is equal to the energy that is communicated to the rotor body at the initial moment of the time measurement by means of the initial angular velocity ω_o and of the initial elongation φ_o at the initial moment of the body motion and has the form done by expression from Ref. [8] whereas $\mathrm{E}(\dot{\varphi}, \varphi)$ is total system energy at the certain moment of the body motion and it has the following value:

$$\mathrm{E}(\dot{\varphi}, \varphi) = \frac{1}{2} \mathrm{J}_{\tilde{n}}^{(A)} \dot{\varphi}^2 + \mathrm{M} g \rho_C \sin\beta \cos\alpha (1 - \cos\varphi) = \frac{1}{2} \mathrm{J}_{\tilde{n}}^{(A)} h \qquad (48)$$

The equation of the constant energy curves has the form:

$$h = \dot{\varphi}^2 + 2\Omega^2 \cos\alpha (1 - \cos\varphi) = const \qquad (49)$$

If we change the constant values we obtain a family of the constant energy curves in the phase plane. The Fig. No. 4 c* shows the families of the phase trajectories and of the constant energy curves. On the basis of the sections of the phase trajectories and of the constant energy curves we conclude that the total system energy decreases during the representational point motion along the phase trajectories. Thus the total system energy is the greatest at the initial moment of the body motion and it tends to a certain definite value that corresponds to the singular point of the saddle type or to the zero that corresponds to the singular point of the focus type. A special case of the representational point motion is along the separatrice branch that enter the singular point of the saddle type and when it is taken into consideration that the time needed for the representational point to get to the saddle extends to infinity this means that system energy drops slowly and that it tends to the energy value that corresponds to the energy curves of the separatrice type, that is the singular point of the saddle type.

If the body is communicated a small initial angular velocity and/or a small elongation such as $\dot{\varphi}_o = \omega_o(small)\, and\, \varphi = 0$, and if it is a sufficiently small value of the rigid body will behave as a physical pendulum oscillating around the rotation/oscillation axis and around the stable equilibrium position. The total system energy is the greatest at the initial moment and it decreases in time so that the body will stabilize itself in the stable balanced position after a sufficient amount of time in which the system has a minimal value of the energy that corresponds to the potential energy minimum.

If the body is communicated a great amount of initial energy by means of the great initial velocity and/or elongation such as $\dot{\varphi}_o = \omega_o(great) and \varphi = 0$, the body will make one or more revolutions around the suspension/rotation axis before it starts oscillating around the stable balances position $\varphi = 2k\pi, k = 0, \pm1, \pm2, \pm3, ..$ In this case the total system energy is the greatest at the initial moment and during the motion it constantly decreases tending zero or to the minimal value of the potential energy when the motion turns into the oscillatory descending motion. In this oscillatory motion the system potential energy oscillates around the minimal value that correspond to the potential energy of the body resting in a stable balanced position whereas the kinetic energy changes in the oscillatory descending way and it tends zero.

If the representational point moves along the separating curves if it descends towards one of the singularities then the rigid body gets an appropriate amount of energy at the initial moment, that is, the amount which is determined by the initial angular velocity and/or elongation. For instance: let $\dot{\varphi}_o = \omega_o and \varphi = 0$, then ω_o has to be chosen in this way so that the representational point moves towards the separatrice so that it descends towards one saddle at $\varphi = (2k-1)\pi$ when the body makes $(k-1)$ revolution and it asymptotically approaches the upper unstable position or it revolves around one focus $\varphi = 2k\pi$, when the body makes k revolutions and then it oscillatory approaches the stable equilibrium position.

A question of determining the scope of the initial angular velocities ω_o arises within the representational point will reach the front singularity while moving along a corresponding phase curve $(\varphi, \dot{\varphi})$. This practically means that we are looking for the initial angular velocity for which the rigid heavy body will make a certain number of k full revolutions around the rotation axis in the turbulent damping conditions and then it will oscillatory and pseudo-periodically tend to the equilibrium position $\varphi = 2k\pi$. Also we may ask if there is a boundary-bifurcational value of the initial conditions in which the representational point along the separatrice -separating phase trajectory tends to the unstable equilibrium position $\varphi = (2k-1)\pi$ after $(k-1)$ revolutions.

In order to answer these questions it is necessary to return along the separatrice (See Fig. No. 4 c*) to the ordinate axis $\dot{\varphi}_o = \omega_o \; for \varphi = 0$ and to read on them two values $\dot{\varphi}_0^{(s)} = \omega_{ok}$ and $\dot{\varphi}_0^{(N)} = \omega_{o(k+2)}$ between ω_0 : $\omega_{ok} \le \omega_o \le \omega_{o(k+1)}$ is initial angular velocity that has to be communicated to the body so that after k revolutions of the body starts to behave oscillatory until its final stabilizing at the defined stable balanced position.

This value ω_{ok} can be determined explicitly from expression (46) by determining first the integrational constants $\mathrm{C}_1 and \mathrm{C}_2$ from the condition that the phase trajectories should be respective separatrices that pass through the given singularities. The integral constant C_1 is determined so that for $v = 0, \varphi = (2k-1)\pi$, whereas for $\varphi = 0, v_o = \dfrac{\omega_o}{\Omega}$ so that it follows:

a^o Integrational constant $\mathrm{C}_{1(2k-1)}$:

$$v = 0 \varphi = (2k-1)\pi n = 2k-1$$

$$v_0 = \frac{\omega_o}{\Omega} \varphi = 0 \mathrm{C}_{1n} = \mathrm{C}_{1(2k-1)} = \frac{2}{16\delta^2 + 1} e^{4\delta(2k-1)\pi} \tag{50}$$

while the initial angular velocity that must be communicated to the body so that the representational point should move along the separatrice is:

$$[\dot{\varphi}(0)]^2 = \omega_{on}^2 = \omega_{o(2k-1)}^2 = \frac{2\Omega^2}{16\delta^2+1}\left[e^{4\delta(2k-1)\pi} + 1\right] = \omega_o^2 \tag{51}$$

b^o For the following separatrice which passes trough a singular point of the saddle type $v = 0, \varphi = (2k+1)\pi$ the integration constant

$$v = 0 \varphi = (2k+1)\pi n = 2k+1$$

$$v_0 = \frac{\omega_o}{\Omega} \varphi = 0 \mathrm{C}_{1(n+2)} = \mathrm{C}_{1(2k+3)} = \frac{2}{16\delta^2 + 1} e^{4\delta(2k+1)\pi} \tag{52}$$

whereas the initial angular velocity that must be communicated to the rigid body rotor so that the representational point should move along the separatrice and tend to the singularity of the saddle type point $v = 0, \varphi = (2k+1)\pi$ is:

$$[\dot{\varphi}((2k+1)\pi)]^2 = \omega_{o(2+n)}^2 = \omega_{o(2k+1)}^2 = \frac{2\Omega^2}{16\delta^2+1}\left[e^{4\delta(2k+1)\pi} + 1\right] = \omega_o^{*2} \tag{53}$$

In order to control the rigid body motion in the Earth gravitational field and in the turbulent damping so that it makes k full revolutions around the rotation axis and then it starts to move oscillatory descendingly around the equilibrium position it is necessary to communicate to it at the equilibrium position the angular velocity from the interval $\omega^2_{o(2k-1)} \le \omega^2_o \le \omega^2_{o(2k+1)}$ in which the interval boundaries are determined by the expressions (51) and (53).

Since for nonconservative systems it stands that the dissipation function is the measure of the system energy drop (degradation) it stands that: $\frac{d}{dt}\left(\mathrm{E}_k + \mathrm{E}_p\right) = -2\Phi$.

The figure No. 4 d* shows that when the representational point moves along any phase curve it does not move along the constant energy curve as well because the phase trajectories cut across the constant energy curves. Since the phase trajectories cut across the constant energy it is necessary to determine what kind of selection that is.

Let $\vec{N}$ be the unit vector of the normal to the constant energy line in the direction of the scalar $\mathrm{E} = \mathrm{E}_k + \mathrm{E}_p$ increase and let it form the angles α and β with the coordinate axes and its cosines are:

$$\cos\alpha = \frac{1}{\mathrm{A}}\frac{\partial \mathrm{E}}{\partial \varphi}, \cos\beta = \frac{1}{\mathrm{A}}\frac{\partial \mathrm{E}}{\partial \dot{\varphi}}, \mathrm{A} = \sqrt{\left(\frac{\partial \mathrm{E}}{\partial \varphi}\right)^2 + \left(\frac{\partial \mathrm{E}}{\partial \dot{\varphi}}\right)^2}, \qquad (54)$$

The phase velocity determines the phase trajectories sense, that is, the sense of the representational point P motion along the phase trajectory and its coordinate are: $\mathrm{V}_\varphi = \dot{\varphi}, \mathrm{V}_{\dot{\varphi}} = \ddot{\varphi},$. Since the derivative of the total system energy is: $\frac{d}{dt}\left(\mathrm{E}_k + \mathrm{E}_p\right) = \mathrm{A}\,\mathrm{V}_\mathrm{N}$, where V_N is the projection of the representational point phase velocity to the direction of the normal $\vec{N}$ of the constant energy curve. Having in view previous expression we conclude that the phase velocity projection to the normal direction to the constant energy line: $\mathrm{V}_\mathrm{N} = -\frac{2\Phi}{\mathrm{A}}$ and it is negative which shows that the phase trajectory sections the constant energy curve externally -internally, which means that in this dissipation system energy decreases in time and it asymptotically tends to the system potential energy in the stable balanced position.

By comparing the phase trajectories in the case of the rigid heavy body rotation around the axis at an angle with respect to the horizon without any resistance forces and the case with the turbulent damping field in this paper we conclude that the phase trajectories in the quoted References only represent the constant energy curves for the second case. The phase velocity is: $\mathrm{V} = \sqrt{\dot{\varphi}^2 + \left[2\delta\dot{\varphi}|\dot{\varphi}| + \Omega^2 \sin\varphi\right]^2}$. The dissipation function is $\Phi(\dot{\varphi}) = \delta\,\mathrm{M}\,\ell_r^2 \dot{\varphi}^2 |\dot{\varphi}|$, whereas $\mathrm{A} = \sqrt{\left(\mathrm{M}\,g\ell_r \sin\varphi\right)^2 + \left(\mathrm{M}\,\ell_r^2\dot{\varphi}\right)^2}$, so that the phase velocity projection to the direction of the normal to the constant energy line is: $\mathrm{V}_\mathrm{N} = -\frac{2\Phi}{\mathrm{A}} = -\frac{2\delta\dot{\varphi}^2|\dot{\varphi}|}{\sqrt{\left(\Omega^2 \sin\varphi\right)^2 + \dot{\varphi}^2}}$, and it is negative which means that the phase trajectory cuts across the constant energy line externally-internally and it asymptotically approximates the position of the stable or unstable equilibrium.

The extreme values of the total system energy at which asymptotic motion of the rigid body appears its rotation around a stationary axis, that is, at which the representational point moves along the separating phase trajectory is the one which corresponds to the extreme values of the energy constant. This analysis is same as the analysis from previous part.

7. 3. ROTATOR

In the expressions for the kinetic pressures there is a rotator vector $\vec{\mathsf{R}}$ which we have named rotator whose squared intensity is in the form of the function of the generalized coordinate φ :

$$\mathsf{R}(\varphi)^2 = \dot{\omega}^2 + \omega^4 = \left[2\delta\dot{\varphi}|\dot{\varphi}| + \Omega^2 \sin\varphi\right]^2 + \Omega^4 \left\{\frac{2}{16\delta^2 + 1}\left[\cos\varphi \pm 4\delta \sin\varphi\right] + \mathrm{C}_{1,2}\, e^{\pm 4\delta\varphi}\right\}^2 \tag{55}$$

where $\dot{\varphi}$ is done by solution (44). Rotator depends on the initial conditions which determine the constants $\mathrm{C}_1 and \mathrm{C}_2$. For bifurational values the initial conditions are done by expressions (50) and (51) and the constants $\mathrm{C}_1 and \mathrm{C}_2$ are determined by the expressions (52) and (53) .

For the case when there are no resistance, no turbulence damping, forces in the previous part the value of this rotator is analyzed and the graphic representations of its intensity are given as the functions of the angle (See Figure No. 4 e*) . This can be used for comparison.

A part of the kinetic pressures which depends on the turbulent damping directly proportional to the squared angular velocity, to the reduced resistance forces coefficient and to the body mass moment properties with respect to the pole in the stationary bearing and to the rotation axis and they do not change during the rotation. Only the squared angular velocity changes and together with it a part of the kinetic pressures due to the turbulent damping. If the axis is passing through the center of gravity then this part of the pressures is equal to zero.

8. DYNAMICS OF A HEAVY ROTOR IN THE FIELD OF LINEAR AND TURBULENT DAMPING EXCITED BY A ONE FREQUENCY COUPLE

We will now consider a heavy rigid body that performs a motion and rotation around an inclined stationary axis in the field of gravitate in the case of a given turbulent damping of the motion excited by one-frequency couple with constant amplitude and frequency. In this case, the kinetic equation (42) can be rewritten in the following form [8] and [25]:

$$\ddot{\varphi} + 2\delta\dot{\varphi}|\dot{\varphi}| + \bar{\gamma}\dot{\varphi} + \Omega^2 \sin\varphi = \bar{\sigma}\cos\bar{\nu}t \tag{56}$$

where the following notations are used (see Figure 3):

$$\Omega^2 = \frac{Mg\rho_C \sin\beta\cos\alpha}{J_n^{(A)}}; \quad 2\delta = \frac{b_m + b_F\,\rho_C^3 \sin^3\beta}{J_n^{(A)}} = 2\zeta\Omega; \quad \bar{\gamma} = \frac{b_L}{J_n^{(A)}}; \quad \bar{\sigma} = \frac{\mathrm{M}_o}{J_n^{(A)}}; \tag{57}$$

Making the change of the time variable $\tau = \Omega t$, differential equation (56) can be written as:

$$\varphi'' + 2\delta\varphi'|\varphi'| + \gamma\varphi' + \sin\varphi = \sigma\cos\nu\tau \tag{58}$$

inhere

$$\frac{d}{d\tau} = (\ )'; \qquad \frac{d^2}{d\tau^2} = (\ )''; \qquad \tau = \Omega t = t\sqrt{\frac{Mg\rho_C \sin\beta\cos\alpha}{J_n^{(A)}}};$$

$$\gamma = \frac{\bar{\gamma}}{\Omega} = \frac{b_L}{J_n^{(A)}\Omega}; \qquad \sigma = \frac{\bar{\sigma}}{\Omega^2} = \frac{\mathrm{M}_o}{J_n^{(A)}\Omega^2}; \qquad \nu = \frac{\bar{\nu}}{\Omega} \tag{59}$$

In the Reference [8] the kinetic pressures on the shaft bearing of the rotor, for the unpeterbed motion, are studied, and corresponding analytical expressions are obtained. Graphical presentations of the vector rotator with the phase portrait and energy curves are obtained. Especially, the case where the bifurcation value of the systems parameter corresponds to the homoclinic orbit is studied..

In the paper [8] a special case of a self rotation in the field of turbulent damping is studied. The corresponding expressions for kinetic pressures as well as an expression for the vector rotator are determined. By using the expression of the determined vector rotator, the dynamics of the change of the deviation couple and the kinetic pressures change are study. In the quoted cases, for the free rotation, influences of the initial are studied. In these cases, the influence of the initial conditions on the free rotation is analyzed. A vector and geometrical interpretation with graphical presentation is given for these cases.

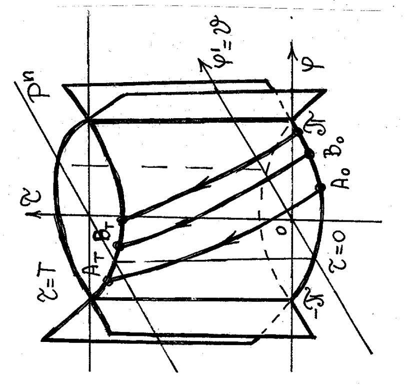


Figure No. 5 a*

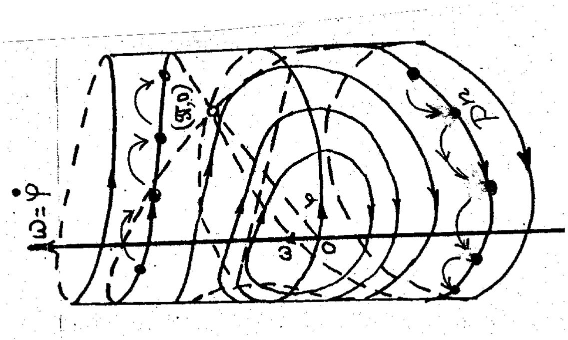


Figure No. 5 b*

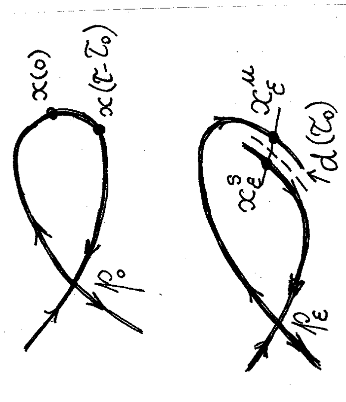


Figure No. 5 c*

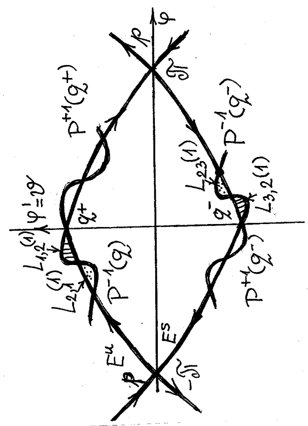


Figure No. 5 d*

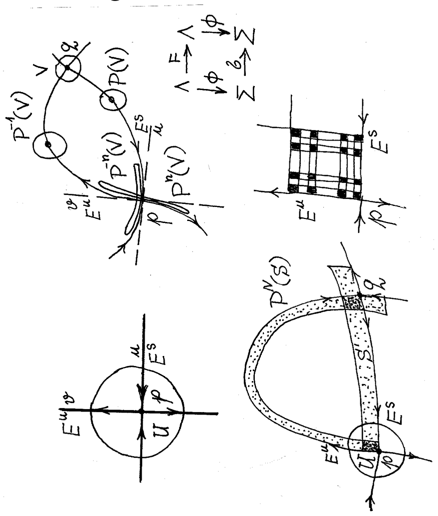


Figure No. 5 e*

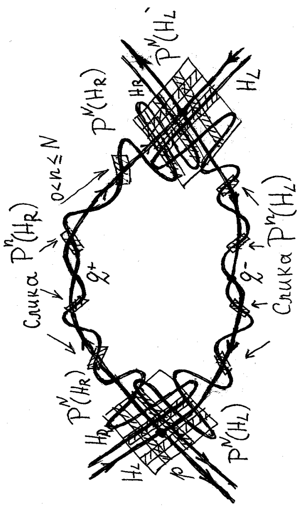


Figure No. 5 f*

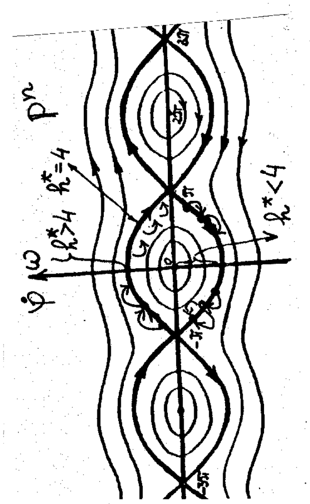


Figure No. 5 g*

In the present paper following Ph. Holmes [27], the oscillatory phenomenon is analyzed for a forced rotation of the rotor described by differential equations (56) or (58), with the use of a new knowledge in nonlinear mechanics and the dynamical systems theory. The results are compared with resuls of K. Hedrih and B. Pavlov [26].

At the beginning, we construct an analytical expression for constant energy curves. Following references [8], [9] and [25]. the sum of kinetic and potential energy of the studied rotor motion we can be expressed as:

$$\mathbf{E}=\mathbf{E}_{\mathbf{k}}+\mathbf{E}_{\mathbf{p}}=\frac{1}{2}J_n^{(A)}\Omega^2\left[\varphi'^2+2(1-\cos\varphi)\right]=\frac{1}{2}J_n^{(A)}\Omega^2 h(\varphi,\varphi') \qquad (60)$$

Differential equation (60) can be written as the system of the three first order differential equations:

$$\begin{aligned}&\varphi'=\mathrm{v}\\&\mathrm{v}'=-\sin\varphi-2\delta\mathrm{v}|\mathrm{v}|-\gamma\mathrm{v}+\sigma\mathbf{S}(\tau)\\&\tau'=1\end{aligned} \qquad (61)$$

We regard the time as a (trivially evolving) third dependent variable and treat it as a rheonmic coordinate [25].

8.1 SOME FACTS ABOUT POINCARE MAPS APPLIED TO ROTOR DYNAMICS IN A NEIGHBORHOOD OF THE UPPER EQUILIBRIUM

L.R. Devaney [2] and Ph. Holmes [27], whose ideas we follow, give some basic facts with an emphasis on the one- and two-dimensional cases of the Poincare maps.

For the unpeterbed case of the rotor dynamics (61), the fixed points are: $(\varphi,\mathrm{v})=(0,0)$, a stable point of the type center and other ones at $(\varphi,\mathrm{v})=(\pm\pi,0)$, which are clearly saddle points.

The existence of the hyperbolic point allows, on the basis of *Hartman-Grobman theorem* (see Ref. [3] and [27]) , to describe the behavior of the linearization of a totally nonlinear map in a neighborhood u of the fixed point p.

In the example of the unpeterbed rotation of rotor, the fixed point $(\varphi,\mathrm{v})=(\pi,0)$ of the map $\mathbf{P}=\mathbf{P_o}$ is clearly a saddle point. In fact, the linearized map can be obtained by integrating the linearized differential equations at $(\varphi,\mathrm{v})=(\pi,0)$, by which are entries of the matrix of the linearization of the map.

The linearized Poincare map corresponding to one period $\mathbf{T}$ is realized by following maps:

$$\mathbf{DP_o}(\pi,0)\begin{Bmatrix}\varphi\\ \mathrm{v}\end{Bmatrix}=\begin{bmatrix}Ch\overline{\tau} & Sh\overline{\tau}\\ Sh\overline{\tau} & Ch\overline{\tau}\end{bmatrix}\begin{Bmatrix}\varphi_1\\ \mathrm{v}_1\end{Bmatrix} \qquad (62)$$

We conclude that singular points $(\varphi,\mathrm{v})=(\pm\pi,0)$ of the saddle type have the eigen values that lie inside and outside the unit circle.

Both of these singular points correspond to an equilibrium of the rotor if the center of mass of the rotor lies in the upper equilibrium position. It is possible to prove by a simple using the implicit function theorem that, for small values of the amplitude of forced couple, σ, and the coefficient of linear damping $\gamma=\boldsymbol{O}\ (\varepsilon)$, map $\mathbf{P}=\mathbf{P_o}(\pi,0)$ perturbs to a close map $\mathbf{P}_\varepsilon=\mathbf{P_o}(\pi,0)+\boldsymbol{O}\ (\varepsilon)$, which has fixed points $(\varphi,\mathrm{v})=(\pm\pi,0)+\boldsymbol{O}(\varepsilon)$ with eigen values $\lambda_1=e^{-\mathbf{T}}+\boldsymbol{O}(\varepsilon)$, $\lambda_2=e^{+\mathbf{T}}+\boldsymbol{O}(\varepsilon)$.

By using the eigen values λ_1 and λ_2, the linearized matrix of the operator can be diagonalized, so that the linear map is uncoupled $u\to\lambda_1 u$, $v\to\lambda_2 v$ and two axes $v=0$, $u=0$ are then the invariants, the stable subspace $\mathbf{E}^{\mathbf{stable}}=\mathbf{E}^{\mathrm{s}}$ and $\mathbf{E}^{\mathbf{unstable}}=\mathbf{E}^{\mathbf{u}}$ unstable subspace. A geometric interpretation is shown in Figure 2.

In the example of a heavy rotor, by stable manifold theorem [3] shows that at local scale in a neighborhood U of the saddle point p , the structure of the nonlinear system

$x \to \mathbf{P}(x)$ is qualitatively similar to the linearized system. More precisely, in a neighborhood U of the saddle point p there exist local stable and unstable manifolds $\mathbf{W}^{\mathbf{s}}_{\mathbf{loc}}(p)$, $\mathbf{W}^{\mathbf{u}}_{\mathbf{loc}}(p)$ that are tangent to the corresponding eigen subspases $\mathbf{E}^{\mathbf{s}}_{\bar{x}}$ i $\mathbf{E}^{\mathbf{u}}_{\bar{x}}$ of the matrix of the linearized operator $\mathbf{DG}(\bar{x})$ at fixed point p and have the degree of smootness equal to the smoothness class of the map $\mathbf{P}$. The Figure 3 and 4 show these invariant manifolds for nonlinear map $x \to \mathbf{P}(x)$ and their transversal intersection at homoclinic point q. Following the idea of Holmes [27] we assert that the rotor may perform "*chaotic motions*" in a neighborhood of the upper equilibrium position corresponding to the saddle point, since, although the local structure is nice, the global structure need not be, and herein lies much of the reason for the "chaotic motions" as we shall see. Following the terminology of Poincaré (from 1899), Holmes calls the point $q \in \mathbf{W}^{\mathbf{u}}(p) \cap \mathbf{W}^{s}(p)$ a homoclinic point. By definition, the orbit $\{\mathbf{P}^{n}(q)\}_{n=-\infty}^{\infty}$ of point q is both forward and backward asymptotic to the saddle point p. If the manifolds, stable $\mathbf{W}^{s}(p)$ and unstable $\mathbf{W}^{\mathbf{u}}(p)$ intersect transversely at point q, then iterations of the maps of a small region $\mathbf{V}$, which contains intersection point q causes $\mathbf{P}^{n}(\mathbf{V})$ and $\mathbf{P}^{-n}(\mathbf{V})$ to "*pile up*" on manifolds $\mathbf{W}^{\mathbf{u}}(p)$ and $\mathbf{W}^{s}(p)$ respectively as $n \to \infty$. It can be seen from the Picture .

8.2. APPLICATION OF SMALE'S HORSESHOE MAPS TO THE ROTOR DYNAMICS IN A NEIGHBORHOOD OF THE UPPER EQUILIBRIUM POSITION

As it was shown by Poincare (1890) (see Ref. [27]), the presence of homoclinic points and homoclinic orbits in the family of the phase trajectories can vastly complicate the dynamical behavior of the rotor. However, since, the existence of homoclinic points and homoclinic orbit implies the existence of a series of recurrent motions, it becomes possible to use Smale's horseshoe maps for the analysis of a series of iterations.

Consider the effect of the Poincare map $\mathbf{P}$, which has a transverse homoclinic point q applied to a hyperbolic saddle point p, and defined a "*rectangular* " strip S that contains p and q on its boundary (Figure No. 5). As the number of iteration, n, increases, the image of the strip $\mathbf{P}^{n}(S)$, contracts horizontally and expands vertically until the strip image $\mathbf{P}^{N}(S)$ of the N-th iteration loops around and intersects S and $\mathbf{P}$ in a "*horseshoe*" shape. It is shown on the Figure 4.

By using properties of Smale's horseshoe map (see Ref. [27] and [2]) and applying the dynamics of the horseshoe map to the phase homoclinic orbits, we can state the following:

* The invariant set Λ of the horseshoe contains: (1) a countable infinity of periodic orbits, including orbits of arbitrary high period ($(\cong 2^{k}/k)$ orbits of each period k);

* an uncountable infinity of non-periodic orbits, including countable many homoclinic and heteroclinic orbits, and

* a dense orbit.

Since, the map is a contraction in one direction and an expansion in another direction, we can conclude that the point in which the orbits intersect is an unstable point of the saddle type. In fact, all the orbits in Λ have exponentially strong unstable manifolds associated with them, and thus almost all pair of points in Λ separate exponentially fast as the number of iterations increase. This sensitive dependence on initial conditions leads to what is popularly call "chaos". More strikingly, since every bi-infinite sequence, which corresponds to a homoclinic orbit represent a forced dynamics equivalent to a random process. It is very important to point out that the set is structurally stable set. To prove the existence of such a set, it needs not necessarily be the linear map model as in the Smale's model.

Birkhoff (1977) (see Ref. [3]) has already proved the existence of countable many periodic points in any neighborhood of a homoclinic point. Smale's construction of horseshoe map provided a more complicated picture. This construction can also be used in a many dimensional case, together with a different version of the homoclinic theorem.

For sufficiently small values of the amplitude σ of excitations couple Poincaré map acting in the cross section (see Figure 5 a* and b*) has upper points in $(\varphi, \text{v}) = (0, \text{v}^-) \equiv q^-$ with $\text{v}^+ = -\text{v}^- \geq 0$, and precisely two "fringes" between q^+ and $\mathbf{P}^{-1}(q^+)$ which are denoted on the picture together with fringes, and has precisely two "fringes" between q^- and $\mathbf{P}^{-1}(q^-)$ which are pointed out on the Figure 5 and 6. To prove these statements, it is necessary to obtain *Melnikov*'s function (see [3] and [27]).

9. VECTORIAL METHOD OF THE KINETIC PARAMETERS ANALYSIS OF THE ROTOR WITH MANY AXES AND NONLINEAR DYNAMICS

By using examples of the rotor system (see Figure No. 6) which rotates about two axes with section or without section, we build the vectorial method of the kinetic parameters analysis of the rotors with many axes (see Ref. [12], [15], [19], [21] and [23]). The vectors connected for the pole and the axis (see Ref. [11]) are used for the analysis of kinetic parameters, by the use of which the rotation properties of the mass configurations are interpreted introducing the mass moment vectors for the pole and the axis, as well as the kinematic vectors rotators. Expressions for the corresponding linear momentum and angular momentum, as well as their derivatives in time are derived. By these expressions vectorial equations of the rotor system dynamics are derived, as well as the expression for the kinetic pressures on the rotor system bearings.

By using vectorial equations, we composed two scalar differential equations of the heavy rotor system nonlinear dynamic. For the case when one rotation about axis is controlled by constant angular velocity the nonlinear dynamics of the rotation about other axis is studied. Nonlinear rotor system dynamics are presented by phase portrait in the phase plane, with trigger of the singularities, as well as with homoclinic orbits and homoclinic points of the nonstable type saddle. For the case rotor system dynamics under the action of the perturbed couple the sensitive dependence in the vicinity of the equilibrium nonstable position which corresponds to homoclinic point of the type nonstable saddle, the possibility of the chaotic character behavior is pointed out.

Using these body mass moments vectors (1) and (2), nonlinear dynamics of rotor which rotate around one axis as well as kinetic pressures on bearing are studied by very simple way (see ref. [9], [10] and [11]). At the same time we see that the expression of the kinetic pressures on bearing rotor shaft are expressed by product of deviation component of the introduced mass moment vectors and pure kinematic component rotator. These knowledges (see ref. from [4] to [25]) introduced us in results of this paper. Following ideas of previous papers and by using body mass moments vectors (1) and (2), the linear momentum and angular momentum of gyro-rotor with many rotation axes are expressed. In this way the vector method of the kinetic parameters analysis of the gyro-rotor with many rotation axes is created.

This vector method is favorable and suitable because there are the separbilities in the expressions of kinetic pressures components with pure geometrical and masses properties (mass density, mass, mass moments) as well as kinematical properties (angular velocity and angular acceleration).

9.1. BASIC EQUATIONS

Heavy gyro-rotor with many rotation axes is a heavy body with the own rotor shaft positioned on the bearings in the support-rotor with shaft bearings on the next support-rotor in

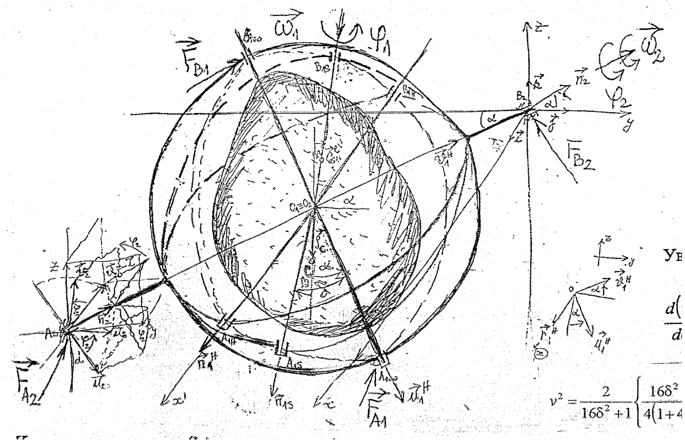


Figure No. 6

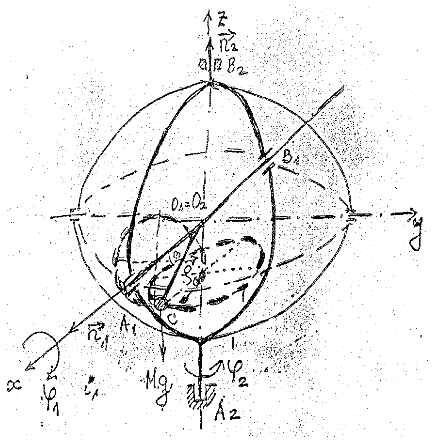


Figure No. 7

the support-rotor chain. Rotor axes of the heavy body and supports-rotor chain can be with section as well as without sections.

For presentation of vector method results in this paper, the one heavy gyro-rotor with two shaft axes is chosen in the cases with axes section as well without axes section. Stationary shaft axis (2) of light support with stationary spherical bearing A_2 and cylindrical bearing B_2 on the length l is fixed. Shaft axis (1) of the own rotation of the heavy rotor body with the spherical bearing A_1 and cylindrical B_1 on the length a are on the support (2). Rotation of the support is determined by generalized coordinate φ_2 , as well as own rotation of the rotor is determined by generalized coordinate φ_1. We introduced the following assumptions: the axes of rotation are orthogonal and on the orthogonal length $\vec{d}_1$. Poles O_1 and O_2 are points on the rotor axes, which are on the orthogonal line between these axes. $\vec{\rho}$ is vector position of the elementary body mass particle with respect to the pole O_1 on the axis (1). Velocity of the elementary mass particle is $\vec{v} = [\vec{\omega}_2, \vec{d}_1] + [\vec{\omega}_1 + \vec{\omega}_2, \vec{\rho}]$, when $\vec{\omega}_i = \omega_i \vec{n}_i = \dot{\varphi}_i \vec{n}_i, i = 1,2$. For this case the expressions of the linear momentum $\vec{\mathsf{K}}$ and of the angular momentum $\vec{\mathsf{L}}_{O_1}$ of the gyro-rotor system are:

$$\vec{\mathsf{K}} = [\vec{\omega}_2, \vec{d}_1]\mathrm{M} + \omega_1 \vec{\mathsf{S}}^{(O_1)}_{\vec{n}_1} + \omega_2 \vec{\mathsf{S}}^{(O_1)}_{\vec{n}_2} \tag{62}$$

$$\vec{\mathsf{L}}_{O_2} = \omega_1 \vec{\mathsf{J}}^{(O_1)}_{\vec{n}_1} + \omega_2 \vec{\mathsf{J}}^{(O_1)}_{\vec{n}_2} + \omega_1 [\vec{d}_1, \vec{\mathsf{S}}^{(O_1)}_{\vec{n}_1}] + \omega_2 [\vec{d}_1, \vec{\mathsf{S}}^{(O_1)}_{\vec{n}_{21}}] + \omega_2 \vec{\mathsf{J}}^{(O_2 \to O_1)}_{\vec{n}_2} + \omega_2 [\vec{\rho}_C, \vec{\mathsf{S}}^{(O_2 \to O_1)}_{\vec{n}_1}]$$

For special case of the gyro-rotor with many shaft rotor axes with one section O these expression of the linear momentum and of the angular momentum are very simple:

$$\vec{\mathsf{K}} = \sum_{i=1}^{i=p} |\vec{\omega}_i| \vec{\mathsf{S}}^{(O)}_{\vec{n}_i} \qquad \vec{\mathsf{L}}_O = \sum_{i=1}^{i=p} |\vec{\omega}_i| \vec{\mathsf{J}}^{(O)}_{\vec{n}_i} \tag{63}$$

In the future application defined vector method, the gyro-rotor with two orthogonal rotation axes with section is studied. For this gyro-rotor the time derivatives of the linear momentum and of the angular momentum can be written in the following vectorial form:

$$\frac{d\vec{\mathsf{K}}}{dt} = \dot{\omega}_1 \vec{\mathsf{S}}^{(O_1)}_{\vec{n}_1} + \omega_1^2 [\vec{n}_1, \vec{\mathsf{S}}^{(O_1)}_{\vec{n}_1}] + \dot{\omega}_2 \vec{\mathsf{S}}^{(O_1)}_{\vec{n}_2} + \omega_2^2 [\vec{n}_2, \vec{\mathsf{S}}^{(O_1)}_{\vec{n}_2}] +$$

$$+ \omega_1 \omega_2 [\vec{n}_2, \vec{\mathsf{S}}^{(O_1)}_{\vec{n}_1}] + \omega_1 \omega_2 [\vec{n}_1, \vec{\mathsf{S}}^{(O_1)}_{\vec{n}_2}] + \omega_1 \omega_2 \vec{\mathsf{S}}^{(O_1)}_{[\vec{n}_2, \vec{n}_1]}$$

$$\frac{d\vec{\mathsf{K}}}{dt} = \vec{\mathsf{R}}_1 \left|\vec{\mathsf{S}}^{(O_1)}_{\vec{n}_1}\right| + \vec{\mathsf{R}}_2 \left|\vec{\mathsf{S}}^{(O_1)}_{\vec{n}_2}\right| + \vec{\mathsf{R}}_{21} \left|\vec{\mathsf{S}}^{(O_1)}_{\vec{n}_1}\right| + \vec{\mathsf{R}}_{12} \left|\vec{\mathsf{S}}^{(O_1)}_{\vec{n}_2}\right| + \vec{\mathsf{R}}_3 \left|\vec{\mathsf{S}}^{(O_1)}_{[\vec{n}_2, \vec{n}_1]}\right| \tag{64}$$

$$\frac{d\vec{\mathsf{L}}_{O_2}}{dt} = \dot{\omega}_1 \vec{\mathsf{J}}^{(O_1)}_{\vec{n}_1} + \omega_1^2 \left[\vec{n}_1, \vec{\mathsf{J}}^{(O_1)}_{\vec{n}_1}\right] + \dot{\omega}_2 \vec{\mathsf{J}}^{(O_1)}_{\vec{n}_2} + \omega_2^2 \left[\vec{n}_2, \vec{\mathsf{J}}^{(O_1)}_{\vec{n}_2}\right] +$$
$$+ \omega_1\omega_2 \left[\vec{n}_2, \vec{\mathsf{J}}^{(O_1)}_{\vec{n}_1}\right] + \omega_1\omega_2 \left[\vec{n}_1, \vec{\mathsf{J}}^{(O_1)}_{\vec{n}_2}\right] + \omega_1\omega_2 \vec{\mathsf{J}}^{(O_1)}_{[\vec{n}_2,\vec{n}_1]}$$
$$\frac{d\vec{\mathsf{L}}_{O_2}}{dt} = \dot{\omega}_1 \mathsf{J}^{(O_1)}_{\vec{n}_1} \vec{n}_1 + \dot{\omega}_2 \mathsf{J}^{(O_1)}_{\vec{n}_2} \vec{n}_2 + \vec{\mathsf{R}}_1 \left|\vec{\mathsf{D}}^{(O_1)}_{\vec{n}_1}\right| + \vec{\mathsf{R}}_2 \left|\vec{\mathsf{D}}^{(O_1)}_{\vec{n}_2}\right| + \qquad (65)$$
$$+ \vec{\mathsf{R}}_{21} \left|\vec{\mathsf{J}}^{(O_1)}_{\vec{n}_1}\right| + \vec{\mathsf{R}}_{12} \left|\vec{\mathsf{J}}^{(O_1)}_{\vec{n}_2}\right| + \vec{\mathsf{R}}_3 \left|\vec{\mathsf{J}}^{(O_1)}_{[\vec{n}_2,\vec{n}_1]}\right|$$

where the kinematic vectors rotator are introduced in following form:

$$\vec{\mathsf{R}}_1 = \dot{\omega}_1 \vec{u}_1 + \omega_1^2 \vec{v}_1; \mathsf{R}_1 = \sqrt{\dot{\omega}_1^2 + \omega_1^4} \; ; \qquad \vec{\mathsf{R}}_2 = \dot{\omega}_2 \vec{u}_2 + \omega_2^2 \vec{v}_2; \mathsf{R}_2 = \sqrt{\dot{\omega}_2^2 + \omega_2^4}$$
$$\vec{\mathsf{R}}_{21} = \omega_1\omega_2 [\vec{n}_2, \vec{u}_1]; \mathsf{R}_{21} = \omega_1\omega_2 \sin\varphi_1; \quad \vec{\mathsf{R}}_{12} = \omega_1\omega_2 [\vec{n}_1, \vec{u}_2]; \mathsf{R}_{12} = \omega_1\omega_2 \sin\gamma (=) 0$$
$$\vec{\mathsf{R}}_3 = \omega_1\omega_2 \vec{u}_3; \mathsf{R}_3 = \omega_1\omega_2 . \qquad (65)$$

In the previous expression (64) $\vec{\mathsf{D}}^{(O)}_{\vec{n}}$ is deviation part of the corresponding mass moment vector for the pole and axis, as well as $\mathsf{J}^{(O)}_{\vec{n}}$ is axial part - axial mass inertia moment.

By using the mass moments vectors, the vector kinetic equations of the gyro-rotor dynamics (Figure N0. 6), which is rotating around two rotation axis with section, can be derived in the following form:

$$\dot{\omega}_1 \left(\vec{n}_1, \vec{\mathsf{J}}^{(O_1)}_{\vec{n}_1}\right) + \dot{\omega}_2 \left(\vec{n}_1, \vec{\mathsf{J}}^{(O_1)}_{\vec{n}_2}\right) + \omega_2^2 \left(\vec{n}_1, \left[\vec{n}_2, \vec{\mathsf{D}}^{(O_1)}_{\vec{n}_2}\right]\right) +$$
$$+ \omega_1\omega_2 \left(\vec{n}_1, \left[\vec{n}_2, \vec{\mathsf{J}}^{(O_1)}_{\vec{n}_1}\right]\right) + \omega_1\omega_2 \left(\vec{n}_1, \vec{\mathsf{J}}^{(O_1)}_{[\vec{n}_2,\vec{n}_1]}\right) = \left(\vec{n}_1, \left[\vec{\rho}_C, \vec{\mathrm{G}}\right]\right) + \sum_{k=1}^{k=N} \left(\vec{n}_1, \left[\vec{\rho}_k, \vec{\mathrm{F}}_k\right]\right) \qquad (66)$$
$$\dot{\omega}_1 \left(\vec{n}_2, \vec{\mathsf{J}}^{(O_1)}_{\vec{n}_1}\right) + \dot{\omega}_2 \left(\vec{n}_2, \vec{\mathsf{J}}^{(O_1)}_{\vec{n}_2}\right) + \omega_1^2 \left(\vec{n}_2, \left[\vec{n}_1, \vec{\mathsf{D}}^{(O_1)}_{\vec{n}_1}\right]\right) +$$
$$+ \omega_1\omega_2 \left(\vec{n}_2, \left[\vec{n}_1, \vec{\mathsf{J}}^{(O_1)}_{\vec{n}_2}\right]\right) + \omega_1\omega_2 \left(\vec{n}_2, \vec{\mathsf{J}}^{(O_1)}_{[\vec{n}_2,\vec{n}_1]}\right) = \left(\vec{n}_2, \left[\vec{\rho}_C, \vec{\mathrm{G}}\right]\right) + \sum_{k=1}^{k=N} \left(\vec{n}_2, \left[\vec{\rho}_k, \vec{\mathrm{F}}_k\right]\right) \qquad (67)$$

Kinetic pressures on bearings are expressed by introduced mass moments vectors (1) and (2) as well as by vector rotators (65) but they cannot presented in this boundary length paper.

9.2. NONLINEAR DYNAMICS ANALYSIS OF THE GYRO-ROTOR

For the special case (see Figure No.. 7) when the support shaft axis is vertical and rotor shaft axis is horizontal the system differential equations of the rotation around these axis are derived. When the shaft support is rotating with constant angular velocity the differential equation of the rotation around rotor shaft and moment of the support motion are determined in following forms:

$$\ddot{\varphi}_1 + 2\delta\dot{\varphi}_1|\dot{\varphi}_1| + 2\delta_1\dot{\varphi}_1 + \Omega^2(\lambda - \cos\varphi_1)\sin\varphi_1 = \mathsf{M} \; ; \qquad \mathsf{M}^*_{k2} = \ddot{\varphi}_1 \sin\varphi_1 + \dot{\varphi}_1^2 \sin\varphi_1 \qquad (68)$$

When the gyro-rotor dynamic is without damping as well as without external couple on the self shaft the expression of the phase trajectories as well as of the curves of the constant energy are determined by following formula:

$$\dot{\varphi}_1 = \pm\sqrt{\dot{\varphi}_{1o}^2 + 2\Omega^2\left[\lambda(\cos\varphi_1 - \cos\varphi_{1o}) + \frac{1}{2}\left(\sin^2\varphi_1 - \sin^2\varphi_{1o}\right)\right]} \qquad (69)$$

This gyro-rotor system with two axes has two statique equilibrium positions, but it may have four dynamic equilibrium possible positions: 1* $\dot{\varphi}_1 = 0, \varphi_1 = 0$; 2* $\dot{\varphi}_1 = 0, \varphi_1 = \pi$; i 3* $\dot{\varphi}_1 = 0, \varphi_1 = \pm\arccos\lambda$. On the basis of the qualitative analysis, it can be seen that the statique equilibrium position $\dot{\varphi}_1 = 0, \varphi_1 = 0$ in dynamical conditions lose stability and changes character from stable center to the unstable saddle homoklinic point. By using qualitative analysis the phase trajectories portrait (see Figure No. 8) is studied for the different dynamic parameters of the rotor system and the phase portrait are composed using sketches from ref. [1]..

Homoclinic orbits of the phase trajectories as well of the constant energy curves are determined (Figure No. 8). One of the homoclinic orbits is in the form of the figure "eight" passing trough saddle homoclinic point

$$\dot{\varphi}_1 = 0, \varphi_1 = 0 : \quad \dot{\varphi}_1 = \pm\Omega\sqrt{\sin^2\varphi_1 + 2\lambda(\cos\varphi_1 - 1)} \tag{70}$$

as well as other which is passing trough the other saddle homoclinic point

$$\dot{\varphi}_1 = 0, \varphi_1 = \pi : \qquad \dot{\varphi}_1 = \pm\Omega\sqrt{2\lambda(1+\cos\varphi_1) + \sin^2\varphi_1} \tag{71}$$

For moving of the representative point during the homoclinic trajectory trough the singular homoclinic point $\dot{\varphi}_1 = 0, \varphi_1 = \pi$, and noted that is $\tau = \Omega t$ we can fined following solutions:

$$\varphi = \pm 2\arcsin\frac{\sqrt{\lambda}Sh\tau\sqrt{1+\lambda}}{\sqrt{\lambda Ch^2\tau\sqrt{1+\lambda}+1}}; \ \dot{\varphi} = \Omega v = \Omega\varphi'_\tau = \pm 2\Omega\sqrt{\lambda}(1+\lambda)\frac{Ch\tau\sqrt{1+\lambda}}{\lambda Ch^2\tau\sqrt{1+\lambda}+1}. \tag{72}$$

For moving of the representative point during the homoclinic trajectory trough the singular homoclinic point $\dot{\varphi}_1 = 0, \varphi_1 = 0$ we can fined following solutions

$$\varphi = \pm 2\arccos\frac{\sqrt{\lambda}Ch\tau\sqrt{1-\lambda}}{\sqrt{1+\lambda Sh^2\tau\sqrt{1-\lambda}}}; \ \dot{\varphi} = \Omega v = \Omega\varphi'_\tau = \pm 2\Omega\sqrt{\lambda}(1-\lambda)\frac{Sh\tau\sqrt{1-\lambda}}{\lambda Sh^2\tau\sqrt{1-\lambda}+1}. \tag{73}$$

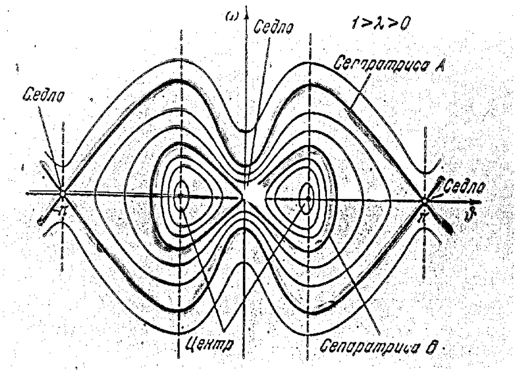


Figure No. 8 a*

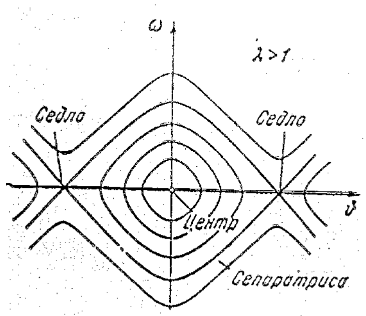


Figure No. 8 d*

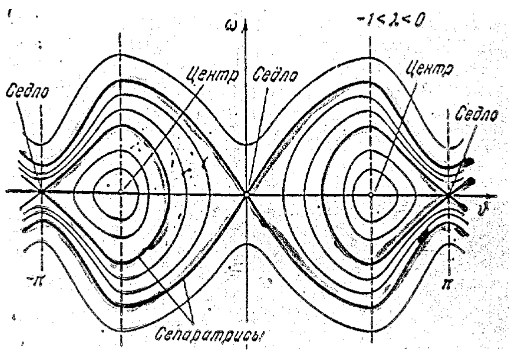


Figure No. 8 b*

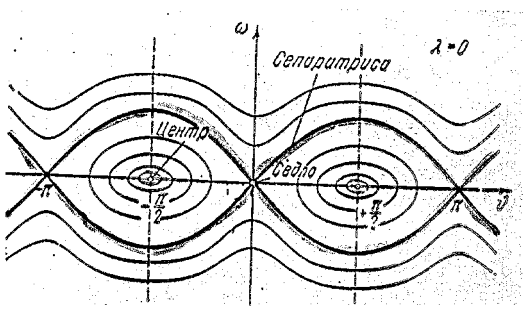


Figure No. 8 c*

10. AXOIDS (CONES) IN THE NONLINEAR DYNAMICS OF THE HEAVY ROTORS WITH MANY AXES OF THE ROTATION

The study results of the heavy rotors nonlinear dynamics, with many rotation axes with cross section (gyro-rotor system) are pointed out in this part of the paper.

By example of the heavy disc, eccentrically and skewlly positioned on the easy, neglected mass, shaft, with bearings on the easy, neglected mass support which is rotated with constant angular velocity on the vertical stationary shaft axis, the axoids-cones - herpolhode and polhode are studied using trajectories of the rotor dynamics.

Rotation axes have cross sections and the rotor dynamics of the eccentrically and skewlly positioned heavy disc can be considered as a rotation about fixed point in the cross section of axes.

By using the phase trajectories the axoids (cones) - herpolhode- space cone and polhode - body cone are constructed. Instantaneous axis of resulting rotation of the heavy disc, eccentrically and skewlly positioned on the easy, neglected mass, shaft with bearings on the easy, neglected mass support which is rotated with constant angular velocity on the vertical stationary shaft axis generates a cone about fixed shaft vertical axis which we called the space cone or herpolhode-cone, as well as instantaneous axis of resulting rotation of the heavy disc also generates a cone about proper shaft axis we called body cone or polhode-cone. These cones for this particular examples are not steady, and will not be right-circular cones. The body (disc) cone (polhode) rolls on the space cone (herpolhode) without slipping.

By means of these axoids - cones we will consider motion of the heavy eccentrically and skewlly positioned disc in rotation around two rotation axes with cross section as rolling motion without slipping of the body cone on a space cone when the axes of the cones are axes of the shafts of the gyro-rotor system.

For geometrical presentation of the axoids (cones) we introduced the following prepositions: height of the herpolhode cone is constant unit, as well as the radius of the cone polhode is constant unit. By using phase trajectories for the defined motion energy, for the herpolhode cone with fixed support shaft axis as well with unit constant height, the radius of the basis is define as a function of the relative self rotation angle as a generalized motion coordinate.

By using phase trajectories for the defined motion energy, for the polhode cone with self shaft rotation axis as well as with unit constant radius of the basis, the height as a function of the relative self rotation angle as a generalized motion coordinate.

The instantaneous axis of the disc motion is the touching generated axis of the cones in the rolling motion of the body cone (polhode) on a space cone (herpolhode).

For the different cases of the eccentricity of the heavy disc as well as of the angle of skewlly disc, the phase trajectories portrait is studied as well as graphically presented. By means of these phase trajectories we can read radius of the herpolhode space cone, as well as height of the polhode - body cone.

10.1. AXIODS

The nonlinear dynamics of the gyro-rotor with two rotation axes with section is rotating about fixed point in the axes section. The axoids-cones of the nonlinear dynamics of the heavy gyro-rotor with two axes of rotations can be obtained (determined) by phase curves in phase plane. In this way the phase plane method is introduced for explanation of the axoids-cones in nonlinear dynamics of the heavy gyro rotor with many axes of rotations.

By using the phase trajectories the axoids (cones) - herpolhode- space cone and polhode - body cone are constructed. Instantaneous axis of resulting rotation of the heavy body rotor positioned on the easy, neglected mass, shaft with bearings on the easy, neglected mass support which is rotated with constant angular velocity on the vertical stationary shaft axis, generates a cone about fixed shaft vertical axis which we called the space cone or herpolhode-cone, as well as instantaneous axis of resulting rotation of the heavy body also generates a cone about proper shaft axis we called body cone or polhode-cone. These cones for this particular examples are not steady, and will not be right-circular cones. The body cone (herpolhode) rolls on the space cone (polhode) without slipping (see Reference No. [23]).

For geometrical presentation of the axoids (cones) we introduced the following prepositions: height of the herpolhode cone is constant unit, as well as the radius of the cone polhode is constant unit. By using phase trajectories for the defined motion energy, for the herpolhode cone with fixed support shaft axis as well with unit constant height, the radius $R_2(\varphi_1,\Omega)$ of the basis is defined as a function of the relative self rotation angle as a generalized motion coordinate:

$$R_2(\varphi_1,\Omega)=H_1(\varphi_1,\Omega)=\frac{1}{\Omega}\sqrt{\dot{\varphi}_{1o}^2+2\Omega^2\left[\lambda(\cos\varphi_1-\cos\varphi_{1o})+\frac{1}{2}(\sin^2\varphi_1-\sin^2\varphi_{1o})\right]} \tag{74}$$

By using phase trajectories for the defined motion energy, for the polhode cone with self shaft rotation axis as well as with unit constant radius of the basis, the height $H_1(\varphi_1,\Omega)$ as a function (74) of the relative self rotation angle is a generalized motion coordinate.

The instantaneous axis of the rotor body motion is the touching generated axis of the cones in the rolling motion of the body cone (polhode) on a space cone (herpolhode). By means of these phase trajectories we can read radius of the herpolhode space cone, as well as height of the polhode - body cone.

10.2. GEOMETRICAL PRESENTATIONS

In the general case the heavy rotor with two shafts of the rotation, one support shaft with axis $\vec{n}_2$, and proper rotation shaft with axis $\vec{n}_1$, bearing of which is rotated about axis $\vec{n}_2$, is presented on the Fig. No. 6.

The rotor with two shafts, which consists on the heavy material point, eccentrically positioned on the easy, neglected mass, shaft, with bearings on the easy, neglected mass support which is rotated with constant angular velocity on the vertical stationary shaft axis, is presented on n the Fig. No.7. The support shaft (2) is vertically positioned and shaft of proper rotor rotation (1) is moveable in the horizontal plane.

The phase portraits of the heavy rotor nonlinear dynamics from Fig. No. 7, are presented on the Figures No. 8 a*, b*, c* and d* for the different rotor parameters, as well as for the different initial energy conditions. These phase portrait are analogous equal as phase portraits from Ref. [1] by *Andronov, A. A., Vitt, A. A., Haykin, S.E.*

For the heavy disc, eccentrically and skewlly positioned on the easy, neglected mass, shaft, with bearings on the easy, neglected mass support which is rotated with constant angular velocity on the vertical stationary shaft axis, the phase portraits are equal type as a phase portraits on the previous Figures No. 8 a*, b*, c* and d* .

Using these trajectory from Figures No. 8 a*, b*, c* and d* for the corresponding heavy rotor parameters, the geometrical presentations of functional relations between base radius of herpolhode axoid $R_2(\varphi_1)$ and generalized coordinate - proper rotation angle φ_1 around axis (1) are presented on the Figures No. 9 a* and b*, as well as height of the cone polhode $H_1(\varphi_1)$.

The axoids for initial conditions corresponding to the homoclinic orbit passing through two homoclinic points saddle type $(0, \pm\pi)$ are presented on the Fog. No. 10 a*. These saddle

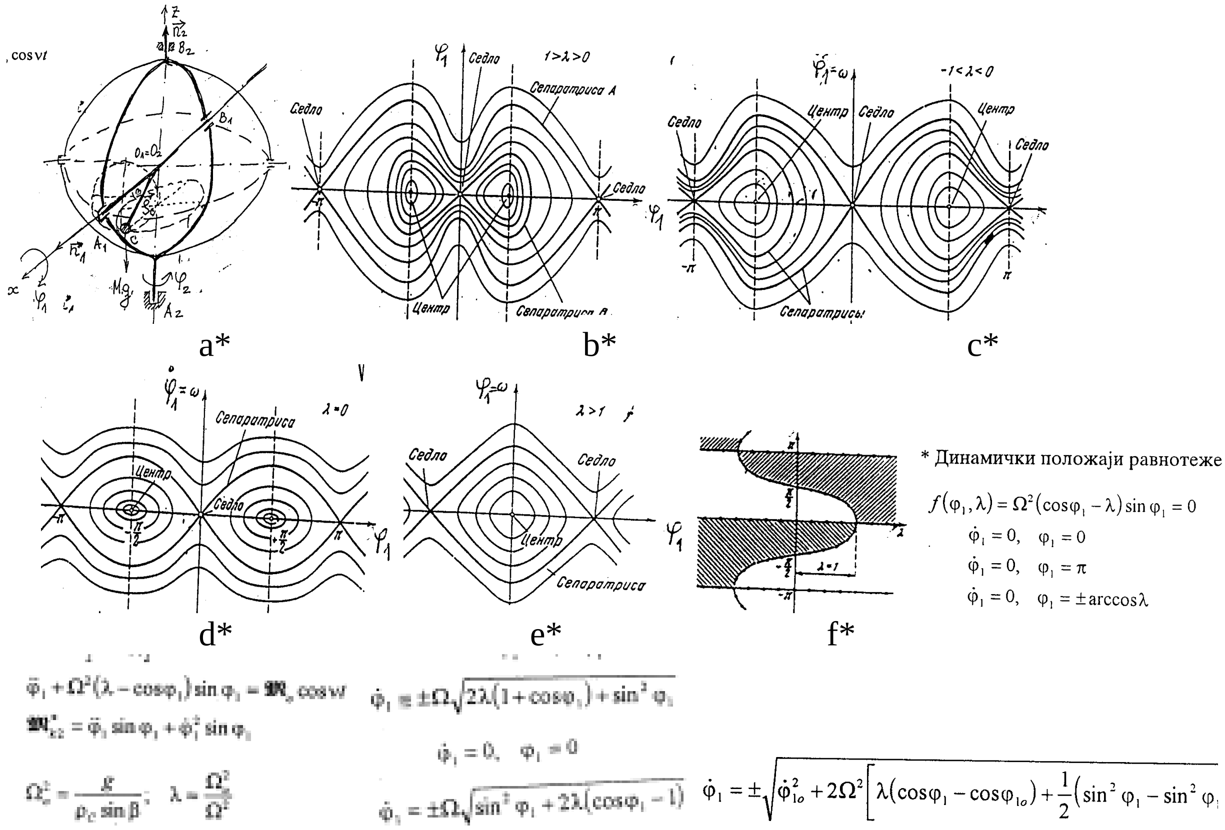
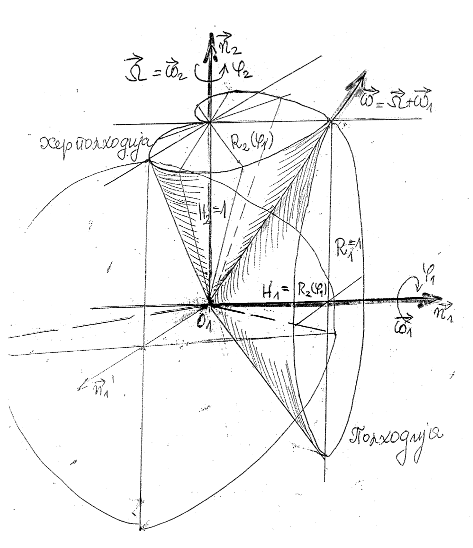


Figure 8. 1. a* Model Gyro-rotror, **b*, c*, d*, e*** Phase portraits, **f*** Potential energy curve

Figure No. 9 a*

Figure No. 9 b*



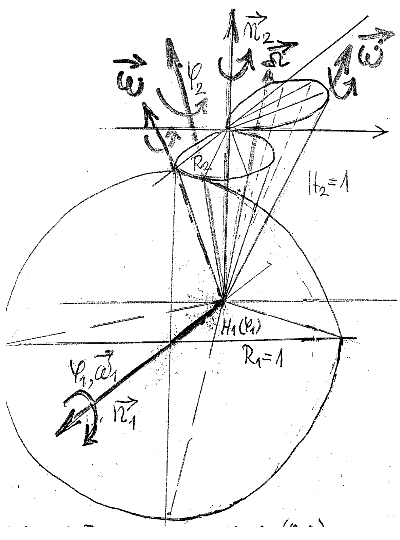


Figure No. 10 a*

Figure No. 10 b*

points correspond to the equilibrium position, when disc mass center is on the highest level and is nonstable.

For this case, the radius of the herpolhode axoid base in the function of the proper rotation angle is:

$$R_2(\varphi_1) = 2\cos\frac{\varphi_1}{2}\sqrt{\lambda + \sin^2\frac{\varphi_1}{2}}, 0 < \lambda < 1 \qquad (75)$$

and in relation on the time:

$$R_2(\tau) = 2\sqrt{1 - \left[\frac{\sqrt{\tau}Sh\sqrt{1+\lambda}\tau}{\sqrt{\lambda Ch^2\sqrt{1+\lambda}\tau + 1}}\right]^2}\sqrt{\lambda + \left[\frac{\sqrt{\tau}Sh\sqrt{1+\lambda}\tau}{\sqrt{\lambda Ch^2\sqrt{1+\lambda}\tau + 1}}\right]^2}, \tau = \Omega t, \lambda = \frac{\Omega_0^2}{\Omega^2}. \qquad (76)$$

The axoids for initial conditions corresponding to the homoclinic orbit passing through the homoclinic point saddle type $(0,0)$ are presented on the Fog. No. 10 b*. This saddle point corresponds to the equilibrium position, when disc mass center is on the lowest level of the equilibrium position, and is nonstable. This phase trajectory is homoclinic, and in the form of "*eight*" with the homoclinic point saddle type $(0,0)$.

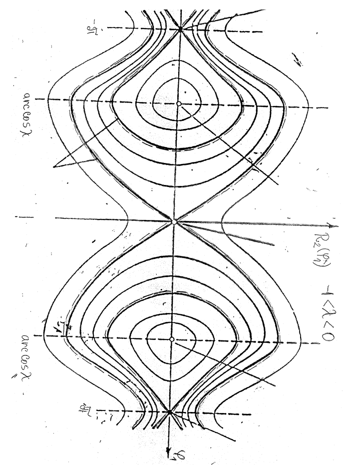
For this case, the radius of the herpolhode axoid base in the function of the proper rotation angle is:

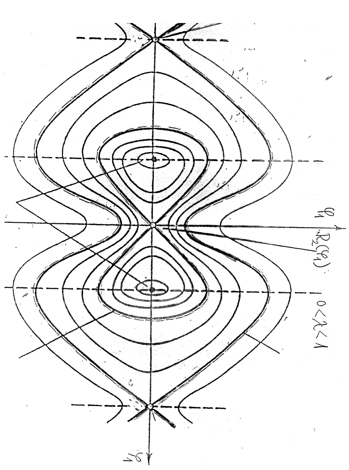
$$R_2(\varphi_1) = 2\sin\frac{\varphi_1}{2}\sqrt{\cos^2\frac{\varphi_1}{2} - \lambda}, 0 < \lambda < 1 \qquad (77)$$

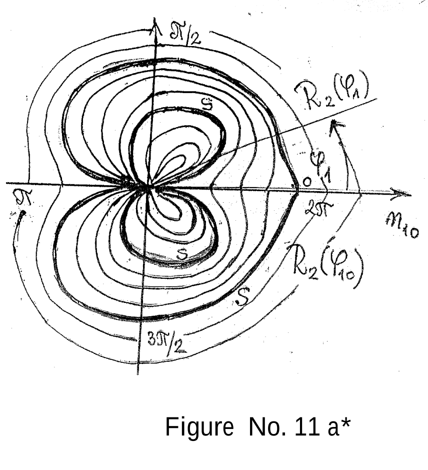
and in relation on the time:

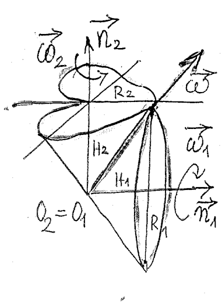
$$R_2(\tau) = 2\sqrt{1 - \left[\frac{\sqrt{\tau}Ch\sqrt{1-\lambda}\tau}{\sqrt{\lambda Sh^2\sqrt{1-\lambda}\tau + 1}}\right]^2}\sqrt{\left[\frac{\sqrt{\tau}Ch\sqrt{1-\lambda}\tau}{\sqrt{\lambda Sh^2\sqrt{1-\lambda}\tau + 1}}\right]^2 - \lambda}, \tau = \Omega t, \lambda = \frac{\Omega_0^2}{\Omega^2} \qquad (78)$$

The base portrait of the herpolhode axoid in the function of proper rotation angle φ_1 , in the polar coordinate system for unit axoide Height is presented on the Fig. No. 11. The discrete forms of the polhode axoide cone, with unit base radius and variable height $H_1(\varphi_1)$ in the function of the proper rotation angle φ_1 are presented on the Fig. No. 12.









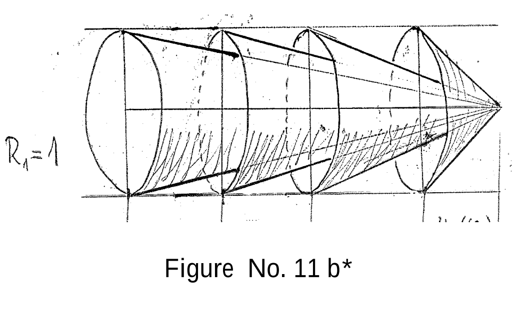


Figure No. 11 a*

Figure No. 11 b*

10.3.. CONCLUSION

From the obtained generalized differential equation of the gyro-rotor dynamics we can conclude the following: In the case when the support shaft axis is vertical, and rotor self axis is in the horizontal plane, as well as the support rotation is with constant angular velocity, the resulting motion of the disc is free autonomous, and method of the phase trajectories portrait is suitable to use for explanation of the disc nonlinear dynamic, as well as for obtaining axoids - cones in the rolling motion.

When the support shaft fixed axis is not vertical, and the rotor self axis is not in the horizontal plane, the support precession rotation with constant angular velocity introduced in the motion of the gyro-rotor system forced excitation as well as forced nonlinear dynamics.

For the case of the eccentrically, skewly positioned disc on the shaft rotor support in the field with and without turbulent damping, the equations of the phase trajectories family can be obtained, as well as the equations of the special homoclinic trajectories through homoclinic unstable points, which correspond to the unstable dynamic equilibrium positions of the gyro-rotor disc.

For the case of the eccentrically, skewly positioned disc on the shaft rotor support in the field with turbulent damping, the equations of the phase trajectories family can be obtained in the following way. By introducing the following $v=\frac{\dot{\varphi}_1}{\Omega}$, the first equation of the previous system (10) is transformed into the following form:

$$\frac{d\left(v^2\right)}{d\varphi_1} \pm 4\delta v^2 = -2\left(\lambda-\cos\varphi_1\right)\sin\varphi_1 \tag{79}$$

Solution of this differential equation is:

$$v^2=\frac{2}{16\delta^2+1}\left\{\frac{16\delta^2+1}{4\left(1+4\delta^2\right)}\left[\sin^2\varphi_1-\cos^2\varphi_1 \pm 4\delta\cos\varphi_1\sin\varphi_1\right]+\lambda\left[\cos\varphi_1 \mp 4\delta\sin\varphi_1\right]+C_{1,2}e^{\mp 4\delta\varphi_1}\right\} \tag{80}$$

This rotor system with two axes has two statique equilibrium positions, but it may have four dynamic equilibrium possible positions: 1* $\dot{\varphi}_1=0,\varphi_1=0$; 2* $\dot{\varphi}_1=0,\varphi_1=\pi$; i 3* $\dot{\varphi}_1=0,\varphi_1=\pm\arccos\lambda$. On the basis of the qualitative analysis, it can be seen that the statique equilibrium position $\dot{\varphi}_1=0,\varphi_1=0$ in dynamical conditions lose stability and changes character from stable center to an unstable saddle homoklinic point. By using qualitative analysis the phase trajectories portrait is studied for the different dynamic parameters of the rotor system and the phase portrait are composed.

By using properties that the homoclinic separatrice orbits are passing through homoclinic saddle points $\dot{\varphi}_1=0,\varphi_1=\pi$ i $\dot{\varphi}_1=0,\varphi_1=0$ the integral constant are determined. For example, homoclinic orbits passing through saddle point $\dot{\varphi}_1=0,\varphi_1=\pi$, for $v>0$ are in the following form:

$$\begin{aligned} v^2 = &\frac{2}{16\delta^2+1}\left\{\frac{16\delta^2+1}{4\left(1+4\delta^2\right)}\left[\sin^2\varphi_1-\cos^2\varphi_1+4\delta\cos\varphi_1\sin\varphi_1\right]+\lambda\left[\cos\varphi_1-4\delta\sin\varphi_1\right]\right\}+ \\ &+\left[\frac{1}{2\left(1+4\delta^2\right)}+\frac{2\lambda}{1+16\delta^2}\right]e^{-4\delta(\varphi_1-\pi)} \end{aligned} \tag{81}$$

Initial angular velocity at initial moment of the heavy rotor rotation "during" the homoclinic orbit is:

$$\left[\dot{\varphi}_1(0)\right]^2=\frac{\Omega^2}{2\left(1+4\delta^2\right)}\left[\frac{4\lambda\left(1+4\delta^2\right)}{16\delta^2+1}-1+\left(1+\lambda\right)e^{4\delta\pi}\right] \quad \text{for} \quad \varphi_1(0)=0 \tag{82}$$

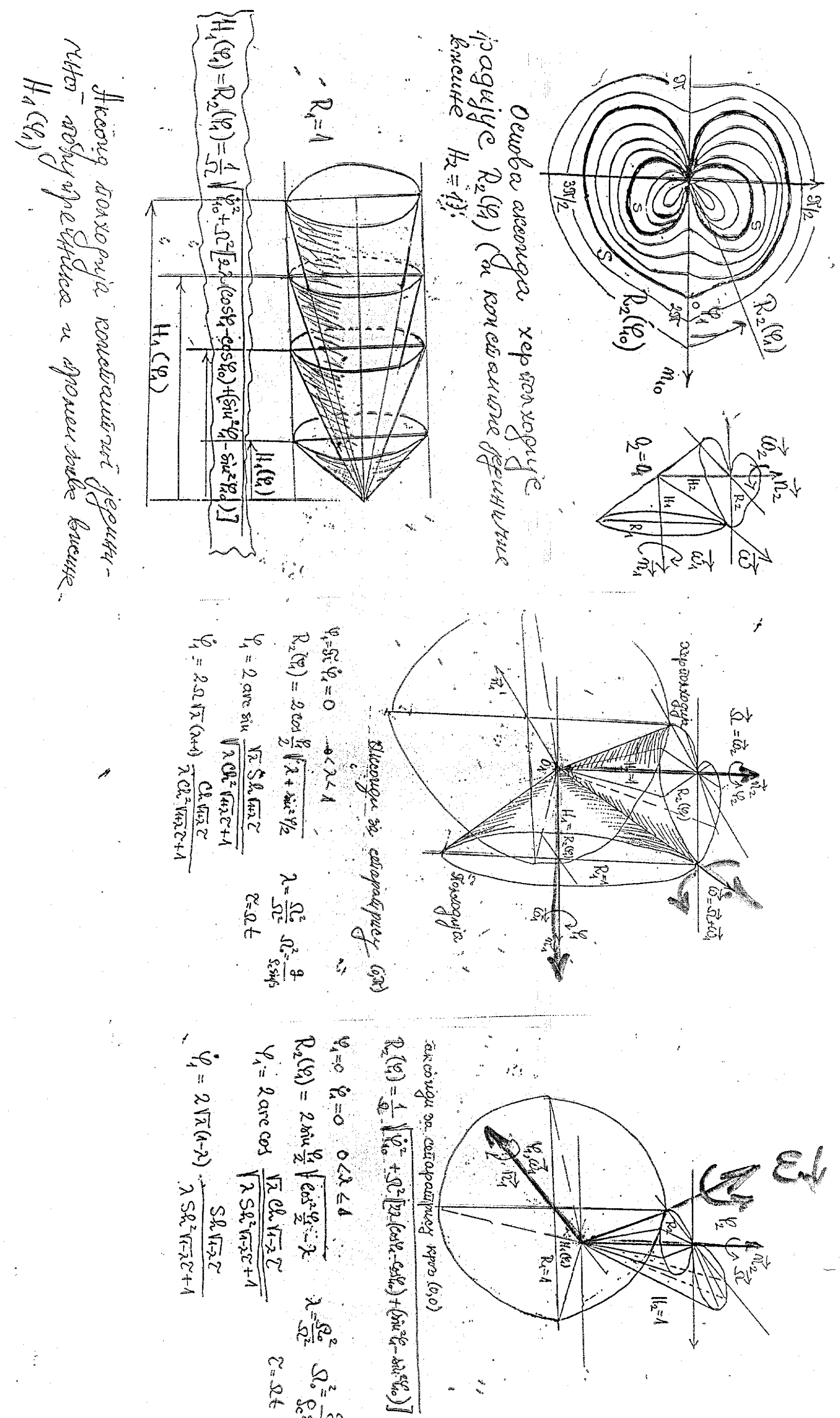


Figure No.12

By using the phase trajectories the axoids (cones) - herpolhode- space cone and polhode - body cone are constructed. These cones for this particular examples are not steady, and will not be right-circular cones. The body cone (polhode) rolls on the space cone (herpolhode) without slipping. The herpolhode cone is open.

For geometrical presentation of the axoids (cones) we introduced the following prepositions: height of the herpolhode cone is constant unit, as well as the radius of the cone polhode is constant unit. By using phase trajectories for the defined motion energy, for the herpolhode cone with fixed support shaft axis as well with unit constant height, the radius

$R_2(\varphi_1,\Omega)$ of the basis is defined as a function of the relative self rotation angle as a generalized motion coordinate:

$$R_2^2(\varphi_1)=\frac{1}{\Omega^2}\frac{2}{16\delta^2+1}\left\{\frac{16\delta^2+1}{4(1+4\delta^2)}\left[\sin^2\varphi_1-\cos^2\varphi_1\pm 4\delta\cos\varphi_1\sin\varphi_1\right]+\lambda\left[\cos\varphi_1\mp 4\delta\sin\varphi_1\right]+C_{1,2}e^{\mp 4\delta\varphi_1}\right\} \tag{83}$$

and is open.

For the case when the rotor is rotated around many shafts with axes with cross section in one point then rotor rotates around fixed point, and we can present this rotation by using one axoid herpolhode and corresponding axoids - polhodes. These axoids - polhodes correspond to a shaft axes of the girorotor. Axoid polhode which corresponds to the axis of proper rotation roll, without slipping, on the next axoid - polhode - support in the chain. Last axoid - polhode - support rolls, without slipping, on the axoid - herpolhode which corresponds to the stationary shaft axis of the right support.

11. SENSITIVE DEPENDENCE ON INITIAL CONDITIONS OF FORCED VIBRATION MOTION OF A HEAVY ROTOR WITH TWO ROTATION AXES

By using ideas of Ph. Holmes from ref. [27] in the condition that the shaft of own gyro-rotor dynamics is excited by single-frequency couple in the field with damping in the closeness (neighborhood) of the homoclinic point we can be rewrite the differential equation

$$\varphi''+2\delta\varphi'|\varphi'|+\gamma\varphi'+(\lambda-\cos\varphi)\sin\varphi=\sigma\cos\nu\tau \tag{84}$$

in the following system:

$$\begin{aligned}&\varphi'=\mathrm{v}\\&\mathrm{v}'=-(\lambda-\cos\varphi)\sin\varphi-2\delta\mathrm{v}|\mathrm{v}|-\gamma\mathrm{v}+\sigma\mathbf{S}(\tau)\\&\tau'=1\end{aligned} \tag{85}$$

where $\frac{d}{d\tau}=(\ )'$; $\frac{d^2}{d\tau^2}=(\ )''$; $\tau=\Omega t$; $\nu=\frac{\bar{\nu}}{\Omega}$. As well as $\tau=\Omega t=\varphi_2$ in this way is the introduced rheonomic coordinate. The state of the gyro-rotor system is uniquely specified by the pair $\left(\varphi,\varphi'=\frac{d\varphi}{d\tau}\right)$, angular position and velocity. For introducing Poincare map we can use a system of first order equations (85) and treat rheonomic coordinate as third dependent variable. The Poincare cross section is: $\mathbf{P}(\varphi_o,\mathrm{v}_0)=(\varphi(\mathbf{T},\varphi_o,\mathrm{v}_0),\mathrm{v}(\mathbf{T},\varphi_o,\mathrm{v}_0))$ where $\varphi(\tau,\varphi_o,\mathrm{v}_0)$, $\mathrm{v}(\tau,\varphi_o,\mathrm{v}_0)$, $\tau=\tau$ is the solution to the differential equations (84) based at (φ_o,v_0) .

As well as the position $(0,0)$ of the dynamic equilibrium correspond to saddle homoclinic point, elementary analysis shows that the fundamental solution matrix to this system can be obtained by integrating the linearised differential equation (84) at $(0,0)$, and hence for the linearized map may be written:

$$\mathbf{DP_o}(0,0)\begin{Bmatrix}\varphi\\ \mathrm{v}\end{Bmatrix}=\begin{bmatrix}Ch\bar{\tau} & Sh\bar{\tau}\\ Sh\bar{\tau} & Ch\bar{\tau}\end{bmatrix}\begin{Bmatrix}\varphi_1\\ \mathrm{v}_1\end{Bmatrix} \tag{86}$$

By using paper [27] written by Ph. Holmes, as well as Smale-Birkhoff homoclinic Theorem, and as well as Hartman-Grobman stable manifold theorem for fixed point about local stable and nonstable manifold on the diffeomorphism with a hyperbolic saddle fixed point we can conclude that in the defined interval of the support angular velocity, the heavy rotor forced motion in the neighborhood around hyperbolic point is stochasticlike and chaoticlike process motion. In this case gyro-rotor motion is sensitive dependent on initial condition under the action of the periodic couple excitation.

12. CONCLUDING REMARKS

When support shaft axis is not vertical, the rotor motion has the character of forced nonlinear motion excited by harmonic forced action, and opposite - when support shaft axis are vertical the rotor motion has the character of own nonlinear motion if there isn't an external couple on the shaft of own heavy gyro-rotor rotation.

From previous parts of this paper, an analogy between models of nonlinear dynamics of heavy material point which rotate along vertical circle and rotor in gravitational and turbulent filed is vosible.

Analogy between motion models of the heavy material particle along the circle in vertical plane (see Ref. [28], [29] and [30]) and a rotor with inclined axis in the respect to the horizon in the turbulent fields (see Ref. [6] and [7]) is visible by using a phase portrait with homoclinic points and orbits, as well as by using the portrait of the constant energy curves.

Analogy between motion models of the heavy material particle along the circle (see Ref. [1]) in vertical plane which rotate around vertical axis and a gyro-rotor (see Ref. [22]and [23]) which rotate around two axes with cross section in the turbulent fields is visible by using a phase portrait with homoclinic points and orbits, as well as by the portrait of the constant energy curves. The appearance of sensitive dependence of nonlinear forced dynamics with chaotic behavior in the vicinity of homoclinic points is visible.

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