Nonlinear Structural Vibrations Control

Florian Dignath and Werner Schiehlen

Institute B of Mechanics, University of Stuttgart,
Pfaffenwaldring 9, 70550 Stuttgart, Germany
Phone +49-711-6856388, Fax +49-711-6856400
[fd,wos]@mechb.uni-stuttgart.de

Abstract

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An approach for damping structural vibrations using optimization techniques is presented. There are three different concepts of optimization: local optimization of structural or control parameters, respectively, and global optimization including all parameters of the controlled structure. The formulation of the optimization problem for multibody systems is presented and algorithms for its solution are briefly described.

As an engineering application a tethered satellite system is investigated applying the local controller optimization only. Spacecrafts and orbiting systems are subject to structural vibrations. Moreover, tethered satellite systems show large displacements and require active or passive damping mechanisms, respectively, see Beletsky and Levin [3] and Steiner et al. [30]. In this paper a tethered satellite system is modelled by the method of multibody systems using symbolic equations of motion undergoing large displacements. Active damping is provided by an actuator between the main body and the tether. As performance criteria the energy decay and the displacement of the main body are applied. First results obtained by Dignath [9] are extended to a nonlinear analysis. Then, a simple elastic pendulum serves as a benchmark for comparison of global and local optimization, i.e. structural or controller optimization, respectively. This benchmark shows clearly the advantages of the global optimization which will also be transferred to the tethered satellite system.

1 Introduction

The paper uses three different optimization concepts for damping of structural vibrations: local structural optimization, local control optimization and global optimization. This means that parameters of the passive structure and of the active parts, such as controller gains or sensor positions, respectively, are simultaneously adapted for improved overall damping. This is of particular interest for adaptive structures where active and passive elements cannot clearly be separated as illustrated by Hanselka [14]. Furthermore, the technique of global optimization does not involve any linearizations as many standard methods of control theory and can therefore be applied to highly nonlinear systems.

The optimization technique is based on the optimization of multibody systems as described by Bestle [4] and Eberhard [10]. Modeling mechanical structures as multibody systems is a well established approach, if large motions and small deformations appear, see [24]. For optimization purposes the equations of motion of such models should be derived in symbolical form. Thereby parameters can be varied without a new derivation of these differential equations. This can be done automatically by computer codes, for example by NEWEUL [17]. The optimization process itself requires algorithms for searching of a minimum, for the integration of the equations of motion and possibly for a sensitivity analysis, which are composed in the program AIMS [5].

A simple elastic pendulum is considered for the application of global optimization in comparance to local structural and control optimization, respectively. As an engineering application control optimization is applied to a tethered satellite system in order to damp its structural vibrations.

A tethered satellite system consists of two or more satellites attached to each other by a long string. Such systems show a high dynamic potential with various applications as presented by Beletsky and Levin [3]. In this paper the damping of structural vibrations is considered during the tethered deployment of a reentry capsule from the international space station as described by Messerschmid et al. [18].

2 Optimization Process

An optimization process consists of two phases. In the first phase the optimization problem is formulated, consisting of the mathematical description of the dynamic system, the parameter space and the performance criteria. In the second phase the mathematically formulated problem is solved. Since the solution can only in special cases be found analytically, numerical methods are applied in order to find the optimum.

2.1 Formulation of the Optimization Problem

Before the actual optimization problem can be defined, a mathematical model of the considered system has to be created. In the present paper, this is done by the method of multibody systems, see Schiehlen [23] or Shabana [27], which yields the equations of motion in the standard form

$$\dot{\boldsymbol{y}} = \boldsymbol{v}(t, \boldsymbol{y}, \boldsymbol{z}) \ , \tag{1}$$

$$\boldsymbol{M}(t, \boldsymbol{y})\dot{\boldsymbol{z}} + \boldsymbol{k}(t, \boldsymbol{y}, \boldsymbol{z}) = \boldsymbol{q}(t, \boldsymbol{y}, \boldsymbol{z}) \ . \tag{2}$$

where $\boldsymbol{y} \in \mathbb{R}^f$ is the vector of generalized coordinates, $\boldsymbol{z} \in \mathbb{R}^g$ the vector of generalized velocities, $\boldsymbol{M} \in \mathbb{R}^{g \times g}$ the symmetric, positive definite mass matrix, $\boldsymbol{k} \in \mathbb{R}^g$ the vector of generalized centrifugal, Coriolis and gyroscopic forces and $\boldsymbol{q} \in \mathbb{R}^g$ the vector of generalized applied forces. For holonomic systems, $f \equiv g$ and the kinematics function $\boldsymbol{v} \in \mathbb{R}^f$ is typically $\boldsymbol{v} = \boldsymbol{z}$. Moreover, if adaptive structures are considered, additional differential equations for sensors or controllers have to be added to equation (1) or (2) and additional force terms for the actuators are introduced.

Together with the initial conditions $\boldsymbol{y}(0)$ and $\boldsymbol{z}(0)$ the differential equations (1) and (2) define the dynamical system. For the optimization additionally a final condition is necessary that can be written in implicit form

$$H_{end}(t_{end}, \boldsymbol{y}_{end}, \boldsymbol{z}_{end}) = 0 \ . \tag{3}$$

Now a set of so–called design variables can be chosen. These parameters are varied during the optimization process in order to improve the dynamical behaviour of the system. The design variables are summarized in the vector $\boldsymbol{p} \in \mathbb{R}^h$ combining structural or controller parameters or sensor and actuator positions if appropriate. Usually, the design variables have to fulfill some constraints

$$\boldsymbol{h}(\boldsymbol{p}) \geq 0 \ , \tag{4}$$

$$\boldsymbol{g}(\boldsymbol{p}) = 0 \ . \tag{5}$$

This can be either inequality constraints $\boldsymbol{h}$, limiting the parameter space, or equality constraints $\boldsymbol{g}$, reducing its dimension, respectively.

Secondly, the n optimization criteria ψ_i are to be formulated. If the criteria are continuous, differentiable functions, they can be written in the standard form

$$\psi_i = G_i(t_{end}, \boldsymbol{y}_{end}, \boldsymbol{z}_{end}, \boldsymbol{p}) + \int_{t_0}^{t_{end}} F_i(t, \boldsymbol{y}, \boldsymbol{z}, \dot{\boldsymbol{z}}, \boldsymbol{p}) dt \ , \ i = 1(1)n \ , \tag{6}$$

where G_i is a function only depending on design variables and final states, while $\int F_i dt$ evaluates the system states over some time period.

Sometimes, it is of advantage to choose non-differentiable optimization criteria such as maximum–value or threshold functions in order to describe the optimization goals, see [11].

If several criteria are considered, a multicriteria problem is given, resulting in an extended scalar optimization criterion ψ_{res}. Usually, the simple consideration of weighting factors leads not to the best results and more sophisticated methods, such as hierarchical methods or goal programming, are applied, see Stadler [29] and Eberhard [10].

The mathematical problem reads as

$$\min \psi_{res}(\boldsymbol{p}) \ , \ \text{with } \boldsymbol{p} \in \mathbb{P} \ ,$$

$$\mathbb{P} = \{\boldsymbol{p} \in \mathbb{R}^h \mid \boldsymbol{g}(\boldsymbol{p}) = \boldsymbol{0} \wedge \boldsymbol{h}(\boldsymbol{p}) \geq \boldsymbol{0}\} \ . \tag{7}$$

2.2 Solving the Optimization Problem

Only in special cases the optimization problem can be solved analytically, therefore an numerical iteration process is usually applied. A scalar optimization algorithm chooses new sets of design variables $\boldsymbol{p}$ based on the information about the criterion ψ_{res} and possibly its sensitivities $d\psi_{res}/d\boldsymbol{p}^T$ until convergence is reached. The required scalar criterion ψ_{res} and the corresponding sensitivities are evaluated by the multicriteria method from the original vector of criteria $\boldsymbol{\psi}$ and the corresponding matrix of sensitivities.

The iteration process implies large numerical computations. In particular, the computation of criteria and sensitivities are very time consuming since an repeated integration of the differential equations (1) and (2) in each iteration step is required. It is therefore important to address some attention to the choice of the scalar optimization algorithm.

2.3 Classification of Algorithms

The algorithms can be classified into deterministic algorithms that use gradient information to find a profitable search direction towards the optimum and stochastic algorithms that use a random number generator combined with a stochastic strategy based only on the information about the criterion values.

Deterministic algorithms locate a minimum of the overall criterion within a certain number of iteration steps by proceeding along a search direction towards the minimal point. Modern algorithms are based on Quasi–Newton–Methods using gradient information about the first derivatives $d\psi_{res}/d\boldsymbol{p}^T$ and an approximation for the Hesse–Matrix, i.e. the second derivatives. If the design variables have to fulfill some constraints (4) or (5) the process is called constrained optimization and the algorithm has to consider additional equations. For this constrained optimization there are, for example, methods of sequential quadratic programming (SQP) available, see Fletcher [12]. They are well suited for optimization with smooth optimization criteria such as presented in this paper, if the first derivatives, i.e. sensitivities can be provided. For all optimizations presented in this paper the SQP–algorithm of Schittkowski [25] is used.

Stochastic algorithms minimize the optimization criterion based only on information about the criterion ψ_{res} itself. They need several hundred function evaluations in order to apply stochastic strategies to the optimization problem. Examples are Adaptive Simulated Annealing, see Ingber [15], and Evolutionary Algorithms, see Schwefel [26] and Goldberg [13]. They are well suited for optimization problems with non–smooth criterion functions where sensitivities cannot always be supplied to the algorithm or a criterion with many local minima.

2.4 Sensitivity Analysis

When using deterministic optimization algorithms as in this paper, sensitivities must be supplied to the algorithm. Since, in general,

$$\psi_{res} = \psi_{res}(\boldsymbol{y}(\boldsymbol{p}), \boldsymbol{z}(\boldsymbol{p})) \ , \tag{8}$$

this requires the computation of gradients and poses a great numerical effort. Therefore, it is important to use an efficient algorithm for the sensitivity analysis. In this paper the method of *Adjoint Variables*, see Bestle and Eberhard [6], is used for the example in the next section. This method provides gradients with high accuracy and efficiency but has the disadvantage that it needs storage of the trajectories over the complete time range. For the tethered satellite system presented in section 3 which involves long integration times it is therefore not suited and the method of *Automatic Differentiation (AD)* is used instead. Automatic differentiation provides exact gradients (apart from integration errors) with a computational effort of about $h \times t_{int}$, where h is the number of design variables and t_{int} is the integration time for the criteria computation. For more information on automatic differentiation see Bischof et al. [7] and for the application in combination with an SQP–algorithm see [11].

3 Tethered Satellite System (TSS)

The idea of using a system of two or more satellites connected by a long thin tether dates several decades back, see for example Rupp and Laue [21] or Stabekis and Bainum [28]. The most realistic applications suggested are probably the creation of artificial gravity by two satellites encircling each other, the gravitational stabilization of a two body system, the launching of small satellites and the deorbiting of a reentry capsule. An overview over the suggested applications for tethered satellite systems is given in the book by Beletsky and Levin [3] that might be considered the standard work about Space Tether Systems. This book treats the dynamical effects of massless and massive elastic tethers in various configurations and missions. The deorbit maneuver is treated with respect to aeronautical and spacecraft engineering by Messerschmid et al. [18] and Zimmermann et al. [31].

Concerning the dynamics of tethered satellite systems a great number of papers were published during the last years. In particular, the investigations of Modi et al. [19], [20], Bainum and Kumar [2] and Steiner et al. [30] deal with the control of tether vibrations and the paper of Kim and Vadali [16] presents a suitable mechanical model. Further the dynamical analysis of tethered satellite systems by Buchholz and Troger [8] has to be mentioned.

3.1 Considered Mission

In this paper, the tethered deorbit of a reentry capsule without a rocket propulsion system is considered, see Zimmermann [31].

There is either a static or a dynamic release possible as shown in Figure 1. In the first case, the capsule is slowly lowered radial to the Earth resulting in smaller orbit velocity. In the second case, the tether is deployed faster which leads to a lateral offset due to the Coriolis force. When the tether deployment is stopped, the capsule swings back towards the local vertical, which corresponds to a breaking of the capsule. The advantage of a dynamical release is that the required tether length is only about one half of the length necessary for a static release.

In this paper, the back–swing of the capsule during the dynamic release is considered and the damping of the resulting structural vibrations by active control

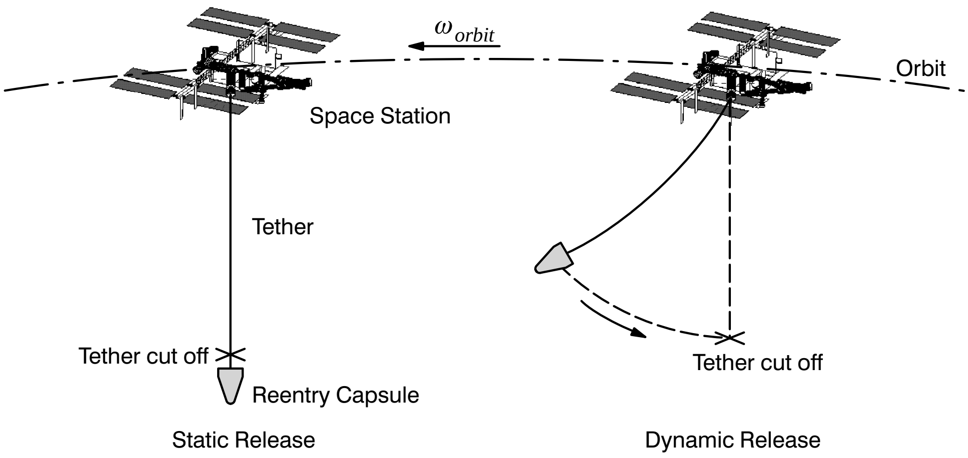

Figure 1: Static and Dynamic Release of a Reentry Capsule

is investigated. For this mission, the tether will typically have a length of 20 km with a diameter of only 0.5 mm. The mass of the return capsule is 170 kg, which corresponds to the prototype reentry capsule, see Messerschmid et al. [18]. The parameters of the station are taken from the International Space Station which is planned to fly with an overall mass of 415 t at an altitude of 400 km.

3.2 Mechanical Model

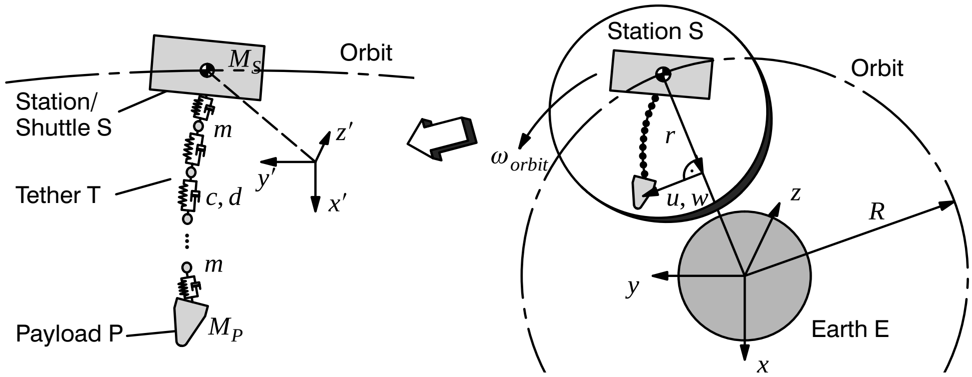

Figure 2: Multibody System Model of the Tethered Satellite System

The tethered satellite system is modelled as a multibody system as shown in Figure 2. It consists of two rigid bodies in free space, representing the station S and the return capsule, payload P. These two bodies are connected by a chain of n particles which are interconnected by spring–damper combinations representing an elastic tether. This model is similar to the bead model of Kim and Vadali [16] but differs with respect to the two end bodies, which may move completely free in space.

The parameters of the particles and spring damper combinations are chosen such that they represent an equivalent element of the tether with length l.

$$m = \rho_T A_T l \ , \tag{9}$$

$$c = \frac{E_T A_T}{l} \ , \tag{10}$$

$$d = \frac{D_T}{l} \ , \tag{11}$$

where ρ_T is the density, A_T the cross section, E_T the Young's Modulus and D_T the damping constant of the tether material. For a very long thin tether usually Kevlar or Dyneema are considered as appropriate materials, see Sabath [22]. The length l of the tether elements depends on the discretization

$$l = \frac{L_T}{n} \tag{12}$$

where L_T is the overall length of the tether. For validation purposes $n = 50$ particles were used. In the optimization process $n = 20$ since the trajectories showed no significant differences.

The equations of motion are comparatively simple if all generalized coordinates are chosen as absolute coordinates given in the non-rotating Earth fixed coordinate system.

$$\boldsymbol{y} = \left[\begin{array}{cccccc:cccccc:ccc} x_S & y_S & z_S & \alpha_S & \beta_S & \gamma_S & x_1 & y_1 & z_1 & \cdots & z_n & x_P & \cdots & \gamma_P \end{array}\right] \tag{13}$$

The equations of motion (1) and (2) then read as

$$\begin{aligned} \dot{\boldsymbol{y}} &= \boldsymbol{z} \\ \boldsymbol{M}\dot{\boldsymbol{z}} + \boldsymbol{k} &= \boldsymbol{q} \end{aligned} \tag{14}$$

with

$$\boldsymbol{M} = \left[\begin{array}{ccc:ccccc:ccc} * & \cdots & * & & & & & & & & \\ \vdots & \ddots & \vdots & & & & & \mathbf{0} & & & \\ * & \cdots & * & & & & & & & & \\ \hdashline & & & m & & & & & & & \\ & & & & m & & & & & & \\ & & & & & \ddots & & & & & \\ & & & & & & m & & & & \\ \hdashline & & & & & & & * & \cdots & * \\ & \mathbf{0} & & & & & & \vdots & \ddots & \vdots \\ & & & & & & & * & \cdots & * \end{array}\right] \tag{15}$$

$$\boldsymbol{k} = \left[\begin{array}{ccc:ccccc:ccc} * & \cdots & * & 0 & 0 & \cdots\cdots & 0 & * & \cdots & * \end{array}\right]^T \tag{16}$$

$$\boldsymbol{q} = \left[\begin{array}{ccc:ccccc:ccc} * & \cdots & * & \cdots & F_{xi} & F_{yi} & F_{zi} & \cdots & * & \cdots & * \end{array}\right]^T \tag{17}$$

where the symbol * stands for the complex terms of the two rigid bodies. It can be seen that the motions of the particles are coupled only via the vector $\boldsymbol{q}$. For the x–direction these read as

$$F_{xi} = F_{G_{xi}} + F_{C_{xi}} + F_{D_{xi}} - F_{C_{xi-1}} - F_{D_{xi-1}} \ . \tag{18}$$

where $F_{G_{xi}}$ is the gravitational force on the i-th particle, $F_{C_{xi}}$ and $F_{D_{xi}}$ the spring and damper forces respectively between particle $i+1$ and i and $F_{C_{xi-1}}$ and $F_{D_{xi-1}}$ the spring and damper forces between particle i and $i-1$. With m_E representing the mass of the Earth, γ the gravitational constant and l_0 the unstretched spring length, one obtains

$$F_{G_{xi}} = -m_E \gamma m \frac{x_i}{R_i^3} \,, \qquad \text{with } R_i = \sqrt{x_i^2 + y_i^2 + z_i^2} \,, \tag{19}$$

$$F_{C_{xi}} = c(l_i - l_0) \frac{x_{i+1} - x_i}{l_i} \,, \quad \text{with } l_i = \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2 + (z_{i+1} - z_i)^2} \,, \tag{20}$$

$$F_{D_{xi}} = d \frac{\dot{\boldsymbol{l}}_i^T \boldsymbol{l}_i}{l_i^2} (x_{i+1} - x_i) \,, \quad \text{with } \boldsymbol{l}_i = \begin{bmatrix} x_{i+1} - x_i \\ y_{i+1} - y_i \\ z_{i+1} - z_i \end{bmatrix} \text{ and } \dot{\boldsymbol{l}}_i = \frac{d\boldsymbol{l}_i}{dt} \,. \tag{21}$$

It turns out that coupling between the directions occurs only via nonlinear terms. A linearization about the quasi–static equilibrium position

$$z_i = z_{i0} + dz_i, \quad z_{i0} = 0, \quad \dot{z}_{i0} = 0, \quad \ddot{z}_{i0} = 0 \tag{22}$$

leads to a decoupling of in–plane and out–of–plane motions, see Steiner et al. [30]. Since the coupling is an important effect when investigating the damping of a tethered satellite system, the complete nonlinear equations are considered in this paper.

To reduce the vibrations by active damping control of the tether winch is applied. This is represented in the model by a force actuator at the space station S. Therefore, the actuator force F_{act} is added to the applied forces to vector $\boldsymbol{q}$ in equation (17), e.g. the term $F_{act} \frac{x_S - x_1}{l_S}$ is added to F_{xi}, $i = 1$, in equation (18). As control laws for the actuator several linear relations can be found in literature, see [2]. For the longitudinal control it is recommended

$$F_{act} = k_1 l + k_2 \dot{l} \tag{23}$$

where $l \equiv l_S$ is the distance between the station and the uppermost particle. For the lateral control the linear law

$$F_{act} = k_3 \theta + k_4 \dot{\theta} \tag{24}$$

where θ is the in–plane angle of the tether measured at the station, may be applied. Furthermore, nonlinear relations can be found, see Modi et al. [19]

$$F_{act} = k_5 \dot{\theta}^2 \,. \tag{25}$$

To reduce the influence of high frequency oscillations on the controller a PT_1 filter is used between the sensor and the controller with a cutoff frequency of $\omega_E = 1\ s^{-1}$. This frequency is significantly larger than the vibration frequency.

3.3 Motion of the Tethered Satellite System

The dynamics of the tethered satellite system are investigated by simulations. The first case describes vibrations in the tether during stationkeeping. The second case describes the motion during a three–dimensional transversal swing of the tether where the effects of the nonlinear coupling terms can be observed, too.

In both cases the station moves around the Earth as described in section 3.1, but the orbit is not an ideal circle due to small disturbances in the initial conditions and the structural vibrations.

3.3.1 Stationkeeping

In the stationkeeping operation the tether is hanging straight down from the station with the payload pointing towards the centre of the Earth while the complete system is moving on a nearly circular orbit. The parameters are chosen as by Modi et al. [20] in order to compare the simulation results,

$$L_T = 100 \text{ km},\ \rho_T = 5.76\ \frac{\text{kg}}{\text{km}},\ M_P = 500 \text{ kg},\ EA_T = 2.8 * 10^5 \text{ N}\ . \tag{26}$$

Additionally a small internal damping of $D = 2500$ Ns as reported by Sabath [22] is taken into account. Initially, the system is experiencing a large longitudinal vibration at no transversal offset.

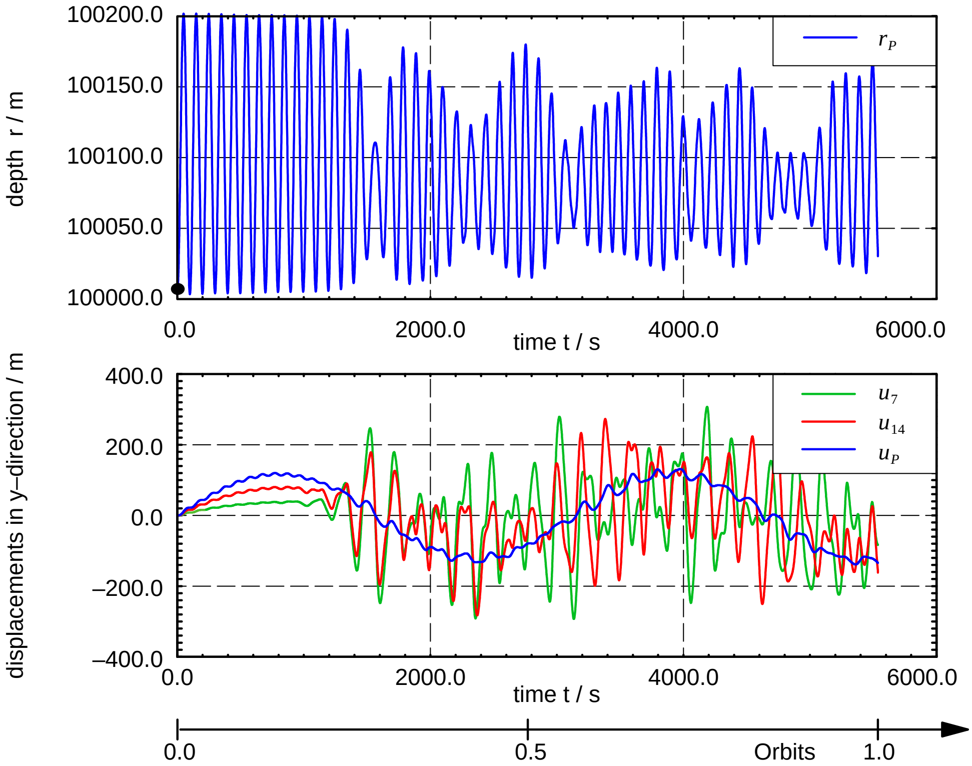

Figure 3: Simulation of stationkeeping with a tether of length $L_T = 100$km

The results of the simulation are shown in Figure 3, where r_P is the depth of the payload under the station, always pointing towards the centre of the Earth, and u_i is the transversal in–plane motion of the i–th particle or the payload P, respectively, perpendicular to r. It can be seen that the longitudinal vibration also leads to a transversal vibration of the tether and a small librational motion of the payload due to the Coriolis force. Obviously, energy initially stored in the longitudinal vibration is being transferred into the transversal in–plane motions.

Frequencies ω_0	Modi et al.	Present Study
librational	1.732	1.758
1st longitudinal	54.3	55.7
1st transversal	59.6	57.7

Table 1: Comparance of eigenfrequencies (EF)

The frequencies of the first modes are shown in Table 1 compared to the frequencies that Modi et al. [20] obtained with their linear model. They are shown as normalized frequencies, i.e.

$$\omega_0 = \frac{\omega}{\omega_{orbit}} \tag{27}$$

The frequencies agree quite well. The small differences may well be explained by the nonlinear effects. For a simplified and completely linearized model with small displacements, the librational frequency of $\sqrt{3} * \omega_{orbit}$ can be calculated analytically, see Arnold [1]. The difference of 1.5% between the analytically calculated librational frequency and the simulation can be explained by the nonlinear librational motion and the slightly elliptic orbit of the station.

3.3.2 Three-Dimensional Swing

The second case shows a librational swing of the tethered satellite. The tethered satellite has initially a large transversal offset from the quasistatic equilibrium position in both the in–plane and out–of–plane direction. The parameters are chosen for the mission also described in section 3.1, as

$$L_T = 20\ \text{km},\ \rho_T = 0.3\ \frac{\text{kg}}{\text{km}},\ M_P = 170\ \text{kg},\ EA_T = 31548.6\ \text{N},\ D = 2500\ \text{Ns}\,. \tag{28}$$

The results of the simulation are shown in Figure 4 where r is again the depth of the payload under the station, u_i the transversal in–plane motion and w_i the transversal out–of–plane motion. The angles of the payload β_P and γ_P about the y– and z–axis are measured in the body fixed coordinate system. Apart from the motion of the payload, the transversal motion of the 7th and 14th particle are shown. The small longitudinal vibration has no considerable influence on the large scale transversal motion. However, the longitudinal motion of the tether excites a tumbling motion of the payload via nonlinear coupling. When the librational swing finishes one half period even an overturn of the payload occurs. This is clearly a very undesirable effect, especially, when the payload is to be cut off and to enter the atmosphere on a specified path. Therefore, an additional damping mechanism in order to control the payload motion is necessary. This simulation shows that the payload cannot in general be modelled as a pointmass. It is necessary to use the rigid body model with rotational degrees of freedom.

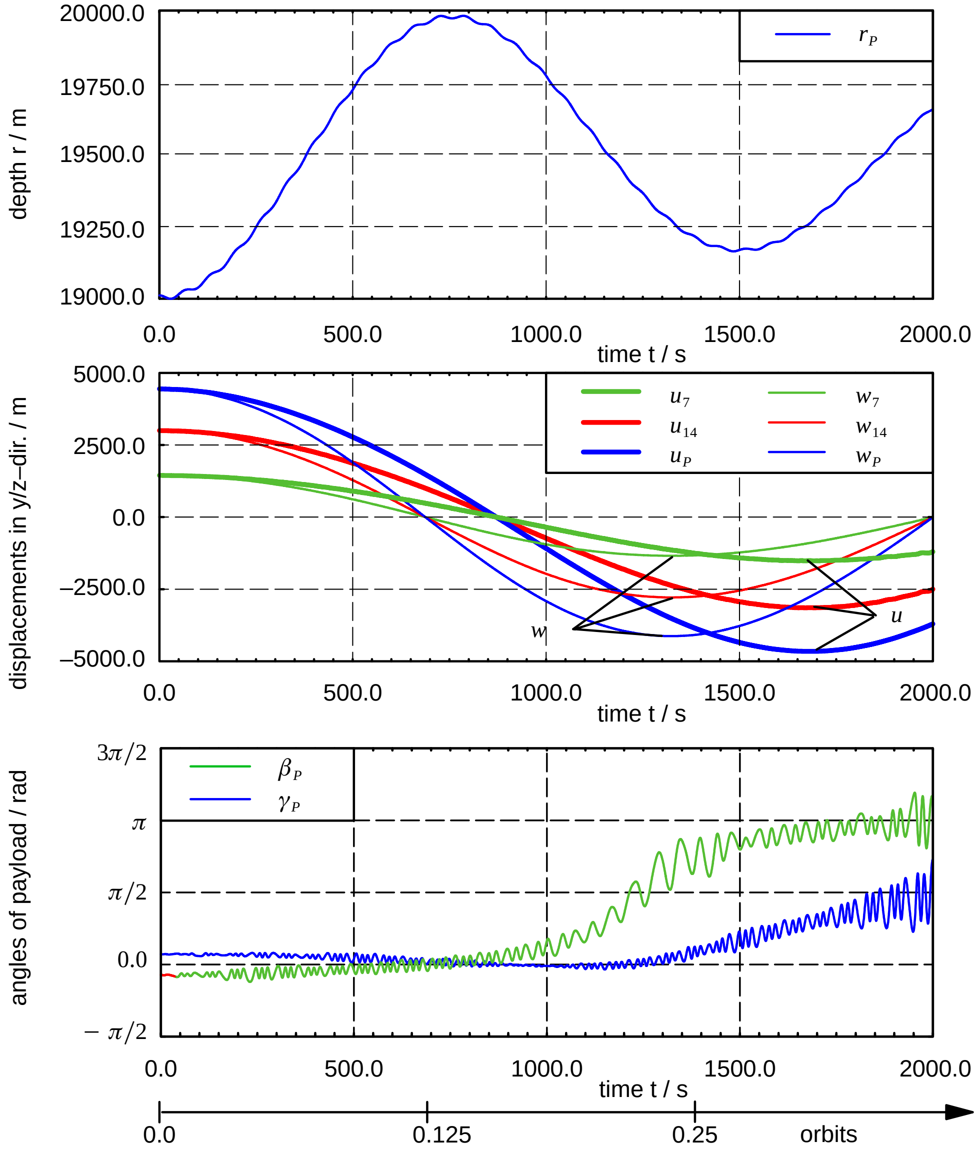

Figure 4: Simulation of three–dimensional swing with a tether of length $L_T = 20\text{km}$

3.4 Optimization

To reduce the vibrations of the tether the active damping by the tether winch is applied. The controller gains and the structural parameters are adapted to each other by an optimization process using the described SQP-algorithm in combination with Automatic Differentiation.

As a reference for evaluating the dynamical behaviour of the tethered satellite system, the back–swing of the payload as described in section 3.1 is considered using the above given parameters. The initial conditions for the system of differential equations (14) follow from a tether that is hanging in a straight line with a large lateral in–plane offset. Additionally, a small longitudinal offset is assumed, so that longitudinal and lateral oscillations occur.

If structural parameters, such as the tether stiffness are varied during the optimization process, it is recommended to define the initial conditions in a way that the initial energy is independent from these design variables, e.g. that the energy stored in the stiffness is constant. This is of particular importance, when the energy of the vibrations is taken as an optimization criterion. Other possible criteria are displacements of the payload or the tether, i.e. displacements of some particles.

In this paper an optimization with respect to the longitudinal tether vibrations is considered using above described energy criterion.

3.4.1 Optimization with Respect to Longitudinal Vibrations

To reduce the longitudinal vibrations, the control law (23) for the actuator is simply chosen as

$$F_{act} = k_1 l + k_2 \dot{l} \tag{29}$$

where the gains k_1 and k_2 are design variables. The optimization criterion is the energy criterion

$$\psi = \int_0^T \left(\frac{1}{2} M_P (v_P - v_{ref})^2 + \frac{1}{2} c \Delta l_P^2 \right) dt \tag{30}$$

with

$$v_{ref} = \omega_{orbit} R_P \tag{31}$$

where ω_{orbit} is the orbit frequency and R_P the distance of the payload from the centre of Earth. Further, $\Delta l_P = \Delta l_P(t)$ is the overall stretch of the tether.

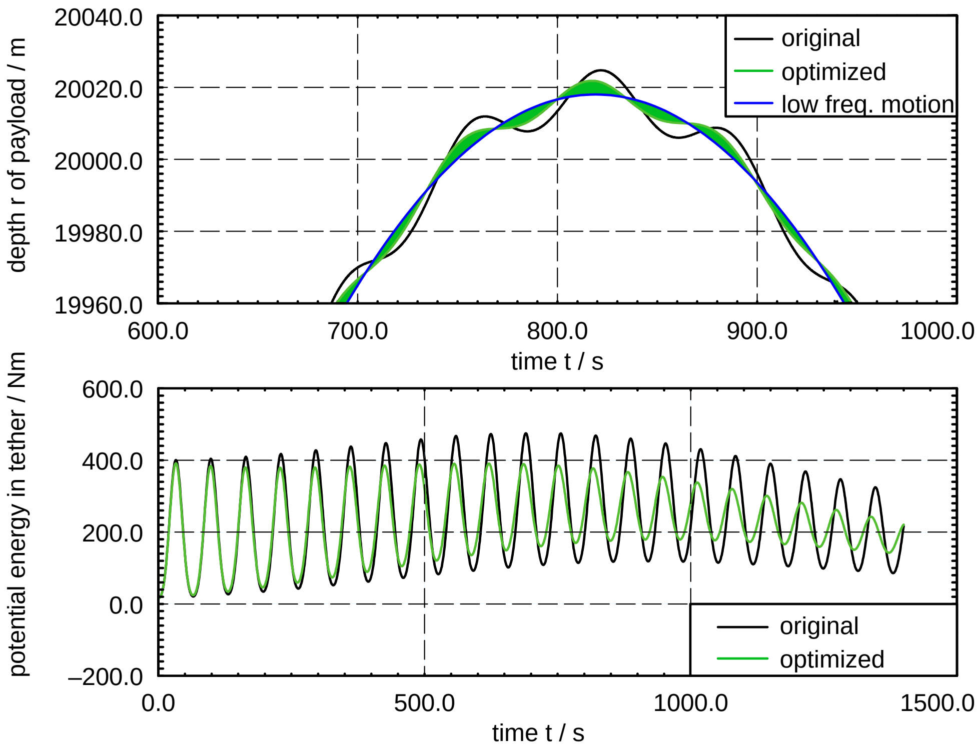

Figure 5: Optimization with respect to longitudinal vibrations

Parameter	initially	optimized	limits
k_1/c_1	0.0	-0.275	-0.5;10.0
k_2/d_1	10.0	101.08	0;1000.0

Table 2: Results of the longitudinal optimization

The results are shown in Figure 5 and Table 2 . The optimized parameters lead to smaller amplitudes of the longitudinal vibrations while the large scale motion is not influenced. This is shown in Figure 5 for the depth r around the tether cut–off point at about 800 s. In this graph, the optimized dynamics are shown with respect to a low frequency motion without structural vibrations. It can further be seen that the energy in the stiffness is dissipated faster than with the initial parameters and, of course, also faster than without active damping. Obviously, the active control of the tether winch is suited for the purpose of providing additional damping to the tethered satellite system and stabilizing the motion in this way, if appropriate parameters are chosen.

4 Pendulum Benchmark

For the optimization of the tethered satellite system in section 3 so far only local controller optimization was considered. Additionally, local structural optimization could be applied but better results are expected by the use of global optimization.

As a simple example to demonstrate the potentials of the global optimization an elastic pendulum is used. By active control of the pendulum suspension its motion is reduced. It is shown that the simultaneous optimization of structural and control parameters yields a better performance than separate optimizations.

4.1 System Model

The system model of the pendulum is shown in Figure 6. The pendulum has two degrees of freedom with the generalized coordinates,

$$\boldsymbol{y} = \begin{bmatrix} \theta \\ r \end{bmatrix} \tag{32}$$

while the additional displacement u is controlled by an actuator.

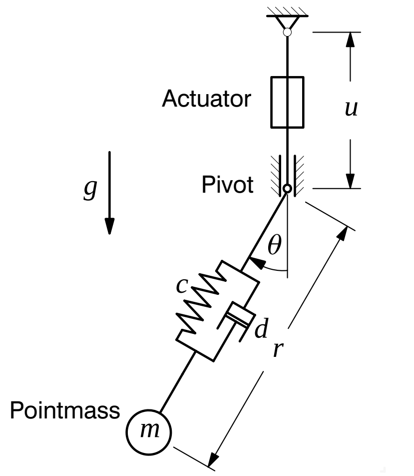

Figure 6: Model of the elastic pendulum

With $\boldsymbol{z} = \dot{\boldsymbol{y}}$, the equations of motion (2) read as

$$\begin{bmatrix} mr^2 & 0 \\ 0 & m \end{bmatrix} \begin{bmatrix} \ddot{\theta} \\ \ddot{r} \end{bmatrix} + \begin{bmatrix} 2m\dot{r}\dot{\theta}r \\ -m\dot{\theta}^2 r \end{bmatrix} = \begin{bmatrix} -mgr\sin\theta + m\ddot{u}r\sin\theta \\ mg\cos\theta - m\ddot{u}\cos\theta - c(r-l_0) - d\dot{r} \end{bmatrix} \tag{33}$$

where g is the gravitational constant and l_0 is the unstretched spring length. To reduce the vibrations of the pendulum the control law is chosen similar to a sliding mode controller as

$$\ddot{u} = -k\theta\dot{\theta} \ . \tag{34}$$

Then, the moment generated by the displacement of the actuator is always opposite to the motion of the pendulum. For the solution of the equations (33) and (34) there are four initial conditions to be specified

$$\theta(0) = \theta_0, \ \dot{\theta}(0) = \dot{\theta}_0, \ r(0) = r_0, \ \dot{r}(0) = \dot{r}_0 \ . \tag{35}$$

4.2 Optimization Problem

The structural parameters of the pendulum, c and d, and the controller gain k are to be optimized with respect to vibration damping. It is assumed that the initial energy W_{ini} of the pendulum is constant, i.e. independent from the design variables. Then, only three initial conditions can be chosen,

$$\theta_0 = \text{const.} \ , \tag{36}$$
$$\dot{\theta}_0 = 0 \ , \tag{37}$$
$$\dot{r}_0 = 0 \ . \tag{38}$$

The fourth initial condition follows from the constant initial energy

$$r_{eq} = \text{const.} \ , \tag{39}$$
$$W_{ini} = \text{const.} \ , \tag{40}$$

where r_{eq} is the pendulum length in the equilibrium position, i.e. the end position and W_{ini} is the initial energy due to gravity and the initial stretch of the spring. If the equilibrium position r_{eq} is given, out of this conditions the unstretched spring length l_0 can be calculated to

$$l_0 = r_{eq} - \frac{mg}{c} \ , \tag{41}$$

and the initial length of the spring is calculated to

$$r_0 = r_{eq} - \Delta r(W_{ini}, l_0, \theta_0) \ . \tag{42}$$

With the initial offset angle $\theta = 30°$, the equilibrium position of $r_{eq} = 1.5$ m, $W_{ini} = 8$ Nm, a pendulum mass of $m = 1$ kg and the original design variables from Table 3 the system behaves as shown in Figure 7.

Parameter	c	d	k
original	20.0	1.0	1.0
lower bound	10.0	0.1	-4.0
upper bound	40.0	10.0	4.0

Table 3: Parameter space of the pendulum benchmark

While the longitudinal vibration is sufficiently damped, the transversal vibration is only weakly damped, because the structural damper can act only via nonlinear

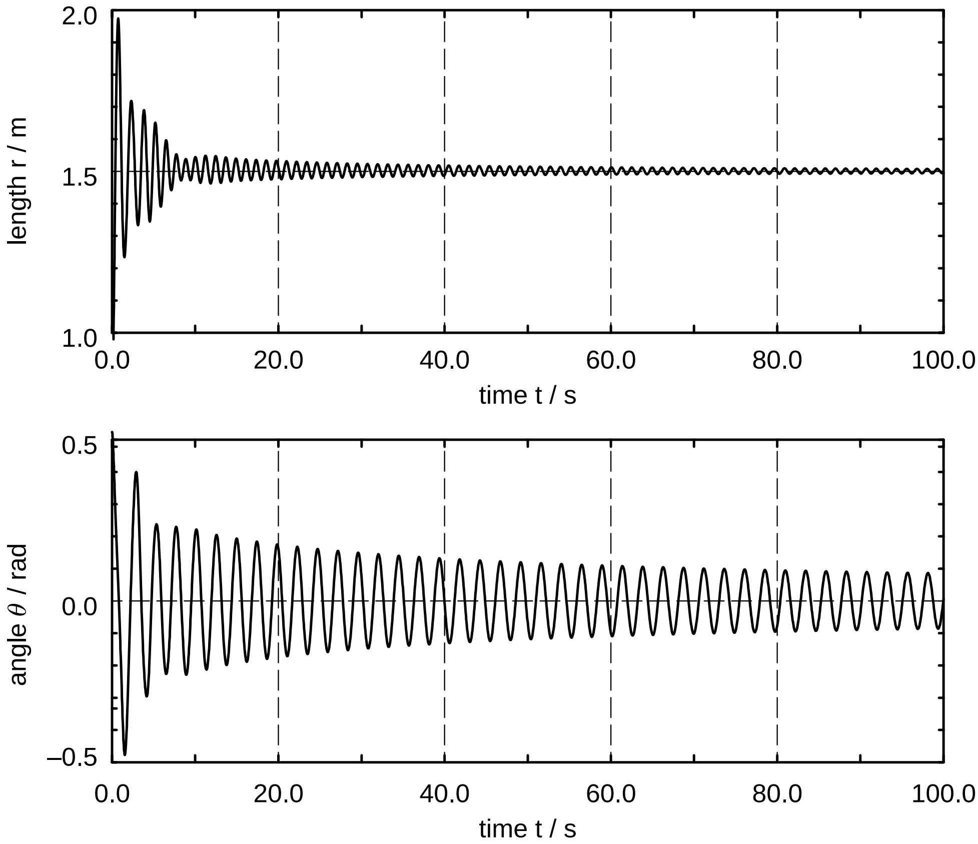

Figure 7: Dynamical behaviour of the non-optimized pendulum

coupling terms. The optimization of the design variables is expected to improve this damping effect. The optimization criterion is therefore formulated as

$$\psi = \int_0^{t_{end}} \theta^2 dt \tag{43}$$

with the final time $t_{end} = 100$ s. The parameter space in which the optimization acts is shown in Table 3.

4.3 Optimization Results

The described optimization problem is solved by three different types of optimizations. Firstly, by controller optimization that only applies to the gain k, secondly by structural optimization that only applies to c and d and thirdly by global optimization that considers simultaneously to the structural parameters c and d and the control gain k.

The results are presented in Table 4 and Figure 8. Obviously the global optimization gives the best damping behaviour and leads to the lowest value of the optimization criterion. This was to be expected, since the adaptation of structural to control parameters and vice versa can best be achieved by the variation of all the design variables. The interesting point however is, that the results of the global optimization lead to design variables completely different to the separated

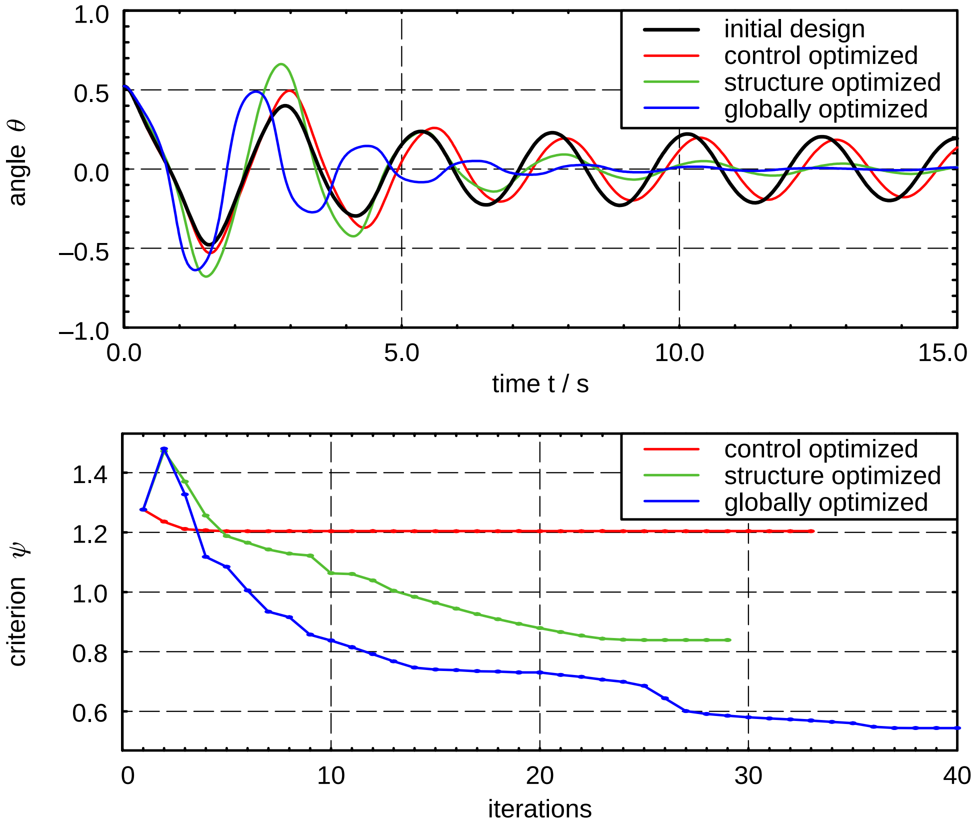

Figure 8: Dynamical behaviour of the optimized pendulum

Parameter	c	d	k
controller optimization	(20.0)	(1.0)	-0.134
structural optimization	25.08	0.387	(1.0)
global optimization	40.0	0.135	4.0

Table 4: Optimization results for the example pendulum

optimizations. This means that by applying both, structural and controller optimization separately, it is not possible to obtain the optimal parameters of the global optimization.

5 Conclusions

In this paper, a method for the damping of structural vibrations by passive and active elements was presented. The parameters of these elements were adapted to each other by optimization techniques using the complete set of nonlinear equations of motion including active controllers.

As an application a space tether system was optimized with respect to structural damping by control of the tether winch. The complex dynamic motion of this system was presented in simulations with different initial conditions including structural vibrations. By an optimization process the control parameters could be improved with respect to the dissipation of energy of longitudinal structural vibrations.

A simple elastic pendulum served as a benchmark for the comparison of three optimization concepts, i.e. local optimization of structural and control parameters, respectively, and global optimization using the joint set of parameters. It was shown that the global optimization clearly results in the best performance with respect to vibration damping. The method is of particular advantage if adaptive structures are considered where passive and active functions cannot be separated or when investigating large, complex motions, where linearization methods cannot be applied, respectively.

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