Erratum for Third Order Invariants of Surfaces

In Section 7.3 *Darboux's Classification of Umbilical Points* there is an error. It is not the root lines of the form $c = Px^3+3Qx^2y+3Sxy^2+Ty^3$ that determines the Darboux's classification, but rather the root lines of the Jacobian with x^2+y^2, i.e., the cubic form

$$Qx^3 + (2S-P)x^2y - (2Q-T)xy^2 - Sy^3,$$

see [14]. In order to obtain the correct invariants we simply make the substitutions

$$P \mapsto Q, \quad Q \mapsto \frac{2S-P}{3}, \quad S \mapsto \frac{T-2Q}{3}, \quad \text{and} \quad T \mapsto -S$$

in the discriminant I_7, and in $3I_3 + I_5$. We then obtain

$$I_7 \mapsto \frac{20}{27}I_3^2 - \frac{4}{27}I_3I_5 - \frac{1}{27}I_7 = \frac{20I_0I_3^2 - 4I_3I_5 - I_0^2I_7}{27},$$
$$3I_3 + I_5 \mapsto I_3.$$

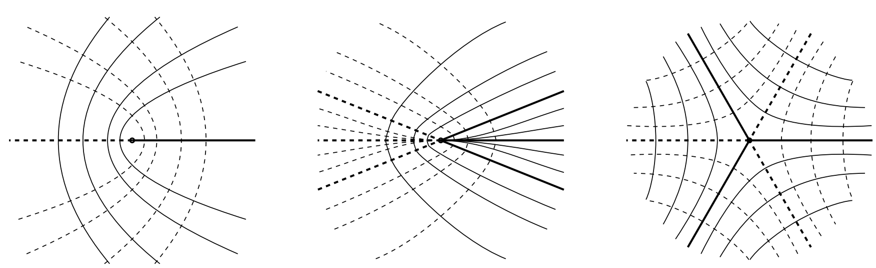
So now we have

$$20I_0I_3^2 - 4I_3I_5 - I_0^2I_7 < 0 \iff \text{the Jacobian has 3 distinct real root lines,}$$
$$20I_0I_3^2 - 4I_3I_5 - I_0^2I_7 > 0 \iff \text{the Jacobian has exactly 1 real root line,}$$

and if $20I_0I_3^2 - 4I_3I_5 - I_0^2I_7 < 0$ then

$$I_3 > 0 \iff \text{The root lines of the Jacobian are contained in a right angle,}$$
$$I_3 < 0 \iff \text{The root lines of the Jacobian aren't contained in a right angle.}$$

This gives the classification in Fig. 1.



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$20I_0I_3^2 - 4I_3I_5 - I_0^2I_7 > 0$	$20I_0I_3^2 - 4I_3I_5 - I_0^2I_7 < 0$	$20I_0I_3^2 - 4I_3I_5 - I_0^2I_7 < 0$
	$I_3 > 0$	$I_3 < 0$

Fig. 1. Curvature lines around an isolated umbilical point.

Acknowledgment. I am indebted to Marc Pouget who found the error and took the time to inform me about it.