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A Generalized Fourier Method for Quasi-periodic Oscillations in Nonlinear Circuits

1. INTRODUCTION

Electric circuits containing a saturable inductor exhibit various nonlinear phenomena, such as the coexistence of periodic oscillations, higher-harmonic and sub-harmonic resonances, the appearance of quasi-periodic and chaotic states of oscillations, etc [7]. The well-known tangent (fold) and period-doubling (flip) bifurcations lead to periodic solutions, which can be computed and continued by existing high-performance software like AUTO (E.Doedel), CONTENT (Y.A.Kuznetsov), INSITE (L.Chua) and other codes for nonlinear dynamical systems.

At Andronov-Hopf points – the third type of generic bifurcations – usually quasi-periodic solutions arise. Important numerical tasks are the detection, approximation and analysis of this rather complicated and sensitive type of oscillations. As a basic model we consider systems of periodically forced ordinary differential equations (ODEs) of order \( n \geq 2 \)

\[
\frac{dx}{dt} = f(x, t), \quad f : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}^n
\]

with sufficiently smooth functions \( f \). We assume that this system has a locally unique quasi-periodic solution with \( p \) rationally independent (incommensurate) basic frequencies, where \( 2 \leq p \leq n \). During the last 15 years different numerical methods have been developed for approximating quasi-periodic solutions and invariant tori. The method of invariance equations tries to compute a parametrization of an invariant torus by solving quasi-linear partial differential equations [1]. The drawback of this approach is that an a-priori transformation from Cartesian to radius-angle coordinates \((u, \theta)\) is required, but in most applications such a global parametrization is neither possible nor numerically feasible.

In our new approach we investigate quasi-periodic oscillations by an approximation and continuation of the associated invariant torus with respect to free system parameters. For the invariant torus we derive a simplified invariance equation without using an a-priori coordinate transformation [5]. This equation can be solved by semi-discretization methods where Fourier-Galerkin methods in the case of periodically forced ”weakly nonlinear” ODEs lead to low dimensional autonomous systems which can be treated by standard algorithms. From the mathematical point of view, this method can be considered as a generalization (a) of the harmonic balance approach, (b) of the Van der Pol method for 2nd order ODEs and (c) of the spectral balance method, but now the equations in the frequency domain are also differential equations. Unlike for the averaging method, a small parameter \( \varepsilon \) and the so called ”standard form” are not required for applying our method. Alternatively, we can also use a finite difference approach for solving the invariance equations [6].

As applications to electrical systems we study a ferroresonant circuit of T. Yoshinaga & H. Kawakami [7] and a parametrically forced circuit of E.Philippow & W.Büntig [4]. It will be
demonstrated that quasi-periodic responses can be approximated, which undergo interesting kinds of bifurcations.

2. TRANSFORMATION INTO TORUS EQUATIONS

We consider periodically forced ordinary differential equations of order \( n \geq 2 \)

\[
\frac{dx}{dt} = f(x,t), \quad f : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}^n
\]

which fulfill the following two basic assumptions:

(A\(_1\)) \( f \in C^r(\mathbb{R}^n \times \mathbb{R}) \) with \( r \geq 1 \) sufficiently large, and \( f(x,t+2\pi) = f(x,t) \) for all \((x,t) \in \mathbb{R}^n \times \mathbb{R}\).

(A\(_2\)) System (1) has a locally unique quasi-periodic solution \( x \in C^1(\mathbb{R}) \) with \( p \) rationally independent basic frequencies \( \omega_1 = 1, \omega_2, \ldots, \omega_p \), where \( 2 \leq p \leq n \).

These \( p \) frequencies form a rationally independent frequency basis \( \Omega = (\omega_1, \omega_2, \ldots, \omega_p) \) with \( \omega_1 = 1 \) because of the \( 2\pi \)-periodicity of \( f \). The following methods can be applied to autonomous systems as well, but then the basic frequency \( \omega_1 \) is also unknown and an additional phase condition must be introduced.

The aim of our approach is the reliable numerical approximation of quasi-periodic solutions \( x(t) = u(\Omega t) \) of the original system (1) without using a-priori transformations into radius-angle coordinates \((u, \theta)\). By means of a suitable torus system, it will be possible to analyze quasi-periodic solutions \( x(t, \lambda) \) and their corresponding torus solutions \( u(\theta, \lambda) \) depending on parameters \( \lambda \in \mathbb{R}^m \) by methods for periodic solutions. So we are able to use existing continuation methods and methods of bifurcation analysis. For the quasi-periodic solution \( x \) of (1) we use the representation

\[
x(t) = u(\Omega t) = u(t, \omega_2 t, \ldots, \omega_p t)
\]

with the associated torus function \( u = u(\theta) : \mathbb{T}^p \rightarrow \mathbb{R}^n \). \( \mathbb{T}^p \) is the \( p \)-dimensional standard torus and \( u \) is assumed to be continuously differentiable and \( 2\pi \)-periodic in every variable \( \theta_i \), \( i = 1, 2, \ldots, p \). Inserting this formulation for \( x \) into (1) yields

\[
\omega_1 \frac{\partial u}{\partial \theta_1} (\Omega t) + \sum_{j=2}^{p} \omega_j \frac{\partial u}{\partial \theta_j} (\Omega t) = f(u(\Omega t), t),
\]

which by using the vector-valued function \( g : \mathbb{R} \rightarrow \mathbb{R}^n \)

\[
g(t) = \frac{\partial u}{\partial \theta_1} (\Omega t) + \sum_{j=2}^{p} \omega_j \frac{\partial u}{\partial \theta_j} (\Omega t) - f(u(\Omega t), t),
\]

becomes equivalent to the equation \( g(t) = 0 \), \( t \in \mathbb{R} \). According to the basic assumptions \((A_1)\) and \((A_2)\), the vector-valued function \( g \in C(\mathbb{R}) \) is also quasi-periodic with basic frequencies \( \omega_1 = 1, \omega_2, \ldots, \omega_p \). Its associated torus function \( G : \mathbb{T}^p \rightarrow \mathbb{R}^n \) with \( g(t) = G(\Omega t) = G(t, \omega_2 t, \ldots, \omega_p t) \) is defined by

\[
G(\theta) = \frac{\partial u}{\partial \theta_1} (\theta) + \sum_{j=2}^{p} \omega_j \frac{\partial u}{\partial \theta_j} (\theta) - f(u(\theta), \theta_1).
\]
For $G \in C(\mathbb{T}^p)$ the range of the quasi-periodic function $g(t) = G(\omega t)$ is dense in the range of the torus function $G(\theta)$, $\theta \in \mathbb{T}^p$ (see [3]). With scalar product and norm in $\mathbb{C}^n$

$$\langle x, y \rangle = \sum_{j=1}^{n} x_j \overline{y}_j, \quad |x|^2 = \langle x, x \rangle_2 = \sum_{j=1}^{n} |x_j|^2$$

the identity $\sup_{t \in \mathbb{R}} |g(t)| = \max_{\theta \in \mathbb{T}^p} |G(\theta)|$ holds. As a consequence it follows that $g(t) = 0 \ \forall t \in \mathbb{R}$ if and only if $G(\theta) = 0 \ \forall \theta \in \mathbb{T}^p$, which leads to the invariance equation (the torus system) on $\mathbb{T}^p$

$$\frac{\partial u}{\partial \theta_1}(\theta) + \sum_{j=2}^{n} \omega_j \frac{\partial u}{\partial \theta_j}(\theta) = f(u(\theta), \theta_1).$$

(4)

Any solution $u(\theta)$ of this system yields a quasi-periodic solution $x(t) = u(\Omega t)$ of $g(t) = 0$, $t \in \mathbb{R}$. In our approach, the basic frequencies $\omega_j$ for $j > 1$ are unknowns and can be determined by appropriate extensions to system (4).

For simplicity we consider the 2-dimensional case, but all the ideas can also be generalized to p-tori. In the case $p = 2$ semi-discretization methods with respect to $\theta_1$ may be applied, for example Fourier-Galerkin, finite differences or collocation. System (4) becomes thereby transformed into an autonomous system of ordinary differential equations for functions $u_k(\theta_2) : \mathbb{T}^1 \rightarrow \mathbb{R}^n, k \in \mathbb{Z}$. We obtain an initial value for the frequency $\omega_2$ at an Andronov-Hopf bifurcation point (nonresonant case) of a periodic solution from the argument of the characteristic multiplier with $|m_1| = 1$, when a quasi-periodic solution is born. The Galerkin approach is now discussed in more detail.

3. SEMI-DISCRETIZATION BY FOURIER-GALERKIN METHODS

We define the nonlinear operator $F : H^1 \rightarrow H^0$ as

$$F(u) = \frac{\partial u}{\partial \theta_1} + \omega_2 \frac{\partial u}{\partial \theta_2} - f(u, \theta_1),$$

(5)

where $H^s = H^s(\mathbb{T}^2)$, $s = 0, 1$ are the Sobolev spaces of torus functions $F : \mathbb{T}^2 \rightarrow \mathbb{C}^n$ with generalized derivatives $D^\alpha F \in L_2(\mathbb{T}^2)$ up to order $s$. Especially let $H^0 = L_2(\mathbb{T}^2)$. Scalar product and norm in $H^s$ are defined by

$$(F, G)_s = \sum_{0 \leq |\alpha| \leq s} \int_{\mathbb{T}^2} \langle D^\alpha F(\theta), D^\alpha G(\theta) \rangle \, d\theta_1 \, d\theta_2, \quad ||F||^2_s = (F, F)_s .$$

With (5) we obtain a zero problem which is equivalent to $G(\theta) = 0$, $\theta \in \mathbb{T}^2$, in operator form

$$F(u) = 0, \quad u \in H^s(\mathbb{T}^2).$$

(6)

For simplicity we now replace $t = \theta_1$ because of $\omega_1 = 1$ and $\theta = \theta_2$ with basic frequency $\omega = \omega_2$. Let $\varphi_i(t)$, $i = -N, \ldots, -1, 0, 1, \ldots, N$, be an orthonormal system of a linear subspace of $L_2(\mathbb{T}^1)$ for which

$$\hat{\varphi}_k(t) = \sum_{j=-N}^{N} c_{kj} \varphi_j(t)$$

(7)
holds. In case of the trigonometric functions
\[ \varphi_k(t) = \frac{1}{\sqrt{2\pi}} e^{ikt}, \quad k = -N, -N+1, \ldots, N \]
the constant matrix \( C = (c_{kj}) \) is especially
\[ C = \text{diag}(-iN, \ldots, -2i, -i, 0, i, 2i, \ldots, iN). \]
We discretise \( u(t, \theta) \) by projection onto the subspace \( H_N = \text{span}\{\varphi_k, |k| \leq N\}^n \) with projector \( P_N \) defined by
\[ u^N(t, \theta) = P_N u(t, \theta) = \sum_{k=-N}^{N} u_k(\theta) \cdot \varphi_k(t), \quad (8) \]
where the Fourier coefficients are
\[ u_k(\theta) = \int_{\mathbb{T}^1} u(t, \theta) \overline{\varphi_k(t)} \, dt. \quad (9) \]
Inserting \( u^N \in H_N \) into (5) and applying \( P_N \) to (6) yields the Galerkin or spectral system
\[ P_N F(u^N) = 0, \quad u^N \in H_N. \quad (10) \]
This Galerkin procedure can be defined component-wise by introducing the vector function
\[ \varphi(t) = (\varphi_{-N}(t), \ldots, \varphi_0(t), \varphi_1(t), \ldots, \varphi_N(t))^T \]
and the matrix function with columns \( u_k(\theta) \)
\[ U(\theta) = (u_{-N}(\theta), \ldots, u_{-1}(\theta), u_0(\theta), u_1(\theta), \ldots, u_N(\theta)). \]
Then (8) in the matrix-vector representation
\[ u^N(t, \theta) = U(\theta) \varphi(t), \quad (t, \theta) \in \mathbb{T}^2 \quad (11) \]
can be inserted into (5) by using (7)
\[ F(u^N(t, \theta)) = U(\theta)C \varphi(t) + \omega U'(\theta) \varphi(t) - f(U(\theta) \varphi(t), t). \quad (12) \]
Now we have to expand the nonlinear term \( f(U(\theta) \varphi(t), t) \) of (12) into a Fourier series with \( \theta \)-dependent coefficients \( \Gamma \)
\[ f(U(\theta) \varphi(t), t) = \Gamma(U(\theta)) \varphi(t) + R_N(\theta, t), \]
where \( R_N(\theta, t) \) is the remainder for \( |k| > N \) and the coefficients are \( \Gamma(U(\theta)) = (\gamma_{jk}) \in \mathbb{C}^{n \times (2N+1)} \). Applying the scalar product (9) to (12) in \( L_2(\mathbb{T}^1) \) yields the component-wise representation
\[ \omega \cdot u^j_{il}(\theta) + (U(\theta) \cdot C)_{il} - \gamma_{il}(U(\theta)) = 0, \quad i = 1, \ldots, n, \quad |l| \leq N. \]
In vector notation, the spectral system (Galerkin system) is now
\[ \omega \cdot U'(\theta) + U(\theta) \cdot C = \Gamma(U(\theta)). \quad (13) \]
If we consider the Fourier series for \( N \rightarrow \infty \), then the periodic solutions of spectral system (13) will obviously yield quasi-periodic solutions of the original system (1). The following theorem can be proven (see [5]):
Theorem 1  With basic assumptions \((A_1)\) and \((A_2)\) it holds for \(N \to \infty\):

(i) \(U(\theta)\) is a \(2\pi\)-periodic solution of system (13) if and only if \(u(t) = U(\omega t)\varphi(t)\) is a quasi-periodic solution of the original system (1).

(ii) \(A = (a_{kl}), k = 1 \ldots n, l \in \mathbb{Z}\), is an equilibrium solution (a stationary point) of system (13) with

\[ A \cdot C = \Gamma(A) \]

if and only if \(u(t) = A\varphi(t)\) is a \(2\pi\)-periodic solution of the original system (1).

The Galerkin system (13) is an autonomous system with \(n(2N + 1)\) equations. For harmonically forced ”weakly nonlinear” systems which frequently appear in electrical circuits we already achieve in practice good approximations for small discretisation values \(N = 1, 2, 3\).

Applying the transformation of the independent variable

\[
\theta = \omega \tau \quad \text{with} \quad U(\theta) = U(\omega \tau) = Y(\tau),
\]

to (13) we can eliminate the unknown parameter \(\omega\) and obtain the spectral system

\[
Y'(\tau) = \Gamma(Y(\tau)) - Y(\tau) \cdot C
\]

for periodic solutions \(Y(\tau)\) with unknown period \(T\). This standard problem can now be solved by software tools for periodic oscillations and is an efficient way to compute and continue quasi-periodic solutions. With such an approximation at hand the invariant closed curves \(\gamma_1\) and \(\gamma_2\) of the two Poincaré sections \(P_1\) and \(P_2\) of a quasi-periodic solution can be computed. Using (8)

\[
u^N(t, \theta) = \sum_{k=-N}^{N} u_k(\theta) \cdot \varphi_k(t)
\]

we obtain the approximations of the torus sections:

\[
\gamma_1^N(\theta) = \frac{1}{\sqrt{2\pi}} \sum_{k=-N}^{N} u_k(\theta) \quad \text{for} \quad t = 2\pi m, m \in \mathbb{N}, \theta \in \mathbb{T}^1
\]

\[
\gamma_2^N(t) = \frac{1}{\sqrt{2\pi}} \sum_{k=-N}^{N} u_k(0) \cdot \varphi_k(t) \quad \text{for} \quad \theta = Tm, m \in \mathbb{N}, t \in \mathbb{T}^1.
\]

4. APPLICATION TO A CIRCUIT OF KAWAKAMI AND YOSHINAGA

As an example we study a dynamical system given by H. Kawakami and T. Yoshinaga in [7]. The Duffing-type system of order 3 is given by

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -k_1 x_2 - \frac{1}{8}(x_1^2 + 3x_3^2)x_1 + B \cos t \\
\dot{x}_3 &= -\frac{1}{8}k_2(3x_1^2 + x_3^2)x_3 + B_0,
\end{align*}
\]
As the right hand sides of (19) are polynomials in $x_1, x_2, x_3$, we can use a computer algebra system to generate the spectral system. With Maple we obtain the following 9-dimensional spectral system with symbolic parameters $B_0, B, k_1$ and $k_2$ (The dots denote derivatives with

Figure 1: Solution scenario of system (19)

which describes a resonant electric circuit with two saturable inductors. The period of the Poincaré map is $T = 2\pi$ and we explore the system for the parameter values $B_0 = 0.03$, $B = 0.22$, $k_2 = 0.05$ and $k_1 \in [0.04, 0.15]$.

By numerical integration of the initial value problem, a bifurcation of the $2\pi$-periodic solution into an invariant torus can be observed at $k_1^* \approx 0.1214$. A stable quasi-periodic solution arises for smaller values $k_1 < k_1^*$, which is continued in Figure 1. Obviously a cascade of period doublings with respect to one basic frequency (torus doublings) arises and finally a strange attractor can be seen. The $2\pi$-stroboscopic Poincaré map is displayed by bold dots.

Now we choose a truncated Fourier series in real form of order 1

$$
\begin{align*}
  x_1(t) &= y_1(\omega t) + y_2(\omega t) \sin(t) + y_3(\omega t) \cos(t) \\
  x_2(t) &= y_4(\omega t) + y_5(\omega t) \sin(t) + y_6(\omega t) \cos(t) \\
  x_3(t) &= y_7(\omega t) + y_8(\omega t) \sin(t) + y_9(\omega t) \cos(t)
\end{align*}
$$

with the real functions

$$
Y(\omega t) = \begin{pmatrix} y_1(\omega t) & y_2(\omega t) & y_3(\omega t) \\
                     y_4(\omega t) & y_5(\omega t) & y_6(\omega t) \\
                     y_7(\omega t) & y_8(\omega t) & y_9(\omega t) \end{pmatrix} \quad \text{and} \quad \varphi(t) = \begin{pmatrix} 1 \\
                                                                                                      \sin(t) \\
                                                                                                      \cos(t) \end{pmatrix}.
$$

As the right hand sides of (19) are polynomials in $x_1, x_2, x_3$, we can use a computer algebra system to generate the spectral system. With Maple we obtain the following 9-dimensional spectral system with symbolic parameters $B_0, B, k_1$ and $k_2$ (The dots denote derivatives with
respect to $\theta = \omega t$).

$$
\begin{align*}
\dot{y}_1 &= y_1 \\
\dot{y}_2 &= y_2 + y_3 \\
\dot{y}_3 &= y_4 - y_2 \\
\dot{y}_4 &= -0.1875 y_1 y_8^2 - k_1 y_4 - 0.1875 y_1 y_9^2 - 0.1875 y_1 y_2^2 - 0.375 y_1 y_7^2 \\
&\quad - 0.375 y_2 y_9 y_8 - 0.375 y_3 y_7 y_9 - 0.1875 y_1 y_3^2 - 0.125 y_1^3 \\
\dot{y}_5 &= -0.375 y_2 y_1^2 - 0.375 y_2 y_7^2 - 0.75 y_1 y_7 y_8 - 0.28125 y_2 y_8^2 + y_6 \\
&\quad - 0.09375 y_2 y_3^2 - 0.09375 y_2 y_9^2 - k_1 y_5 - 0.09375 y_3^2 \\
&\quad - 0.1875 y_3 y_9 y_8 \\
\dot{y}_6 &= -0.375 y_3 y_7^2 - 0.375 y_3 y_1^2 - 0.28125 y_3 y_9^2 - 0.09375 y_3 y_8^2 \\
&\quad - 0.1875 y_2 y_9 y_8 + B - y_5 - 0.09375 y_4^3 - k_1 y_6 - 0.75 y_1 y_7 y_9 \\
&\quad - 0.09375 y_3 y_2^2 \\
\dot{y}_7 &= -0.375 k_2 y_9 y_1 y_3 - 0.1875 k_2 y_7 y_2^2 - 0.1875 k_2 y_9^2 y_7 \\
&\quad - 0.1875 k_2 y_7 y_3^2 - 0.375 k_2 y_7 y_1^2 - 0.125 k_2 y_7^3 + B_0 \\
&\quad - 0.1875 k_2 y_8^2 y_7 - 0.375 k_2 y_8 y_1 y_2 \\
\dot{y}_8 &= -0.375 k_2 y_8 y_1^2 - 0.09375 k_2 y_8^3 - 0.75 k_2 y_7 y_1 y_2 + y_9 \\
&\quad - 0.1875 k_2 y_6 y_3 y_2 - 0.09375 k_2 y_8 y_9^2 - 0.09375 k_2 y_8 y_3^2 \\
&\quad - 0.375 k_2 y_8 y_7^2 - 0.28125 k_2 y_8 y_2^2 \\
\dot{y}_9 &= -0.75 k_2 y_7 y_1 y_3 - 0.09375 k_2 y_9^3 - 0.09375 k_2 y_8^2 y_9 \\
&\quad - 0.09375 k_2 y_9 y_2^2 - 0.375 k_2 y_9 y_7^2 - 0.1875 k_2 y_8 y_3 y_2 - y_8 \\
&\quad - 0.375 k_2 y_9 y_1^2 - 0.28125 k_2 y_9 y_3^2.
\end{align*}
$$

This autonomous system can now be analyzed by the continuation and bifurcation code AUTO 97 of E.J. Doedel et al. [2]. Some of the results are displayed in Figures 2 and 3.

![Figure 2: Bifurcation diagram of the spectral system](image)

In Figure 2 the $L_2$-norm of the solution of the spectral system is displayed for parameter $k_1 \in [0.025, 0.20]$. Hopf bifurcations arise at $k_1 \approx 0.1315$ and 0.1281 (labels 2 and 3). The branch arising at label 3 for $k_1 \approx 0.1281$ has been followed. A cascade of period doublings
Figure 3 displays corresponding periodic orbits of the spectral system. Obviously a sequence of period doublings arises. For $k_1 = 0.03$ a phase portrait is given together with its Poincaré map.

An interpretation of the periodic orbits of the spectral system in connection with the quasi-periodic solutions of the original system can be found in Table 1.
5. APPLICATION TO A CIRCUIT OF PHILIPPOW

The 2nd order ODE of E.S.Philippow (see [4]) describes a parametrically forced nonlinear electric circuit

\[
\ddot{x} + \alpha \dot{x}^3 - \beta \dot{x} + (1 + B \sin 2t) x = 0
\]

(21)

with system parameters \( B = 0.1, \alpha = \varepsilon - B, \beta = \frac{\varepsilon}{2} - B \). The period of the Poincaré map is \( T = 2\pi \).

In the following intervals of the parameter \( \varepsilon \) we observed different types of solutions:

<table>
<thead>
<tr>
<th>( \varepsilon )-interval</th>
<th>Solutions types and bifurcations</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0 &lt; \varepsilon &lt; \varepsilon_1 )</td>
<td>zero solution ( x = 0 )</td>
</tr>
<tr>
<td>( \varepsilon_1 = 0.100013665 )</td>
<td>first bifurcation point</td>
</tr>
<tr>
<td>( \varepsilon_1 &lt; \varepsilon &lt; \varepsilon_2 )</td>
<td>zero + 2 periodic solutions</td>
</tr>
<tr>
<td>( \varepsilon_2 = 0.299986338 )</td>
<td>second bifurcation point</td>
</tr>
<tr>
<td>( \varepsilon_2 &lt; \varepsilon &lt; \varepsilon_3 )</td>
<td>zero + 4 periodic solutions (global) fold bifurcation</td>
</tr>
<tr>
<td>( \varepsilon_3 = 1.560818 )</td>
<td></td>
</tr>
<tr>
<td>( \varepsilon &gt; \varepsilon_3 )</td>
<td>zero + 1 quasi-periodic solution</td>
</tr>
</tbody>
</table>

Table 2: Solution types and bifurcations of (21)

A bifurcation diagram of the original system (21) can easily be obtained by AUTO 97. In Figure 4 (left) the norm \( ||x||_{L^2} \) is displayed via \( \varepsilon \in [0, 3] \). The Hopf bifurcations at \( \varepsilon_1 \) and \( \varepsilon_2 \) are detected and the fold at \( \varepsilon_3 \) can be seen. Quasi-periodic orbits, however, cannot be computed and continued.

![Figure 4: Original system and spectral system with \( N = 1 \)](image-url)
The 1st order spectral method \((N = 1)\) yields a Galerkin system with 6 ODEs. Its bifurcation diagram (right) does not reflect the real behavior of the periodic solutions. Therefore we increased the number \(N\) of basic functions. In Figures 5 and 6 the corresponding bifurcation diagrams for \(N = 3\) and \(N = 5\) are displayed with periodic (lines) and quasi-periodic (circles) solutions. The second basic period \(T_2 = 2\pi/\omega_2\) of the quasi-periodic solutions (right figures) tends to infinity at the bifurcation point \(\varepsilon_3 = 1.560818\).

Solutions of the spectral system \hspace{1cm} \text{Period } T_2 = 2\pi/\omega_2

Figure 5: Spectral system with \(N = 3\) (14 equations)

Solutions of the spectral system \hspace{1cm} \text{Period } T_2 = 2\pi/\omega_2

Figure 6: Spectral system with \(N = 5\) (22 equations)

6. CONCLUSION

The Fourier-Galerkin approach has been applied successfully to several nonlinear circuits, where quasi-periodic solutions arise. Unlike other known methods, an a-priori transformation is not needed for this method. It is applicable to sufficiently smooth problems with quasi-periodic p-tori, but it fails at strong resonant tori. In case of 2-tori the spectral system is not algebraic, but a moderate dimensional system of ordinary differential equations, which can be treated by standard software for nonlinear dynamical systems.
References


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