Talks at MLC and tools workshop 2023

Talks

MLC and Tools for studying it

Dzimitry Dudko, Michail Lyubich : Minicourse

The backbone of the workshop is a mini-course on MLC and tools to study the problem.

Misha Lyubich: Quadratic-like Renormalization Theory

- Lecture 1: Quadratic-like renormalization and applications: setting the stage.
- Lecture 2: Main tools: Thin-thick decomposition and Covering Lemma.
- Lecture 6 & 7: Proof of a priori bounds for bounded combinatorics.

Abstract of Lectures 1, 2 and 6 & 7. This theory describes the universal small scale geometry of infinitely renormalizable quadratic-like maps. It has numerous consequences: to the local connectivity of the corresponding Julia sets, to the MLC Conjecture, to the problem of Lebesgue measure of Julia sets, to self-similarity features of the dynamical and parameter pictures, etc. In the mini-course we will focus on the case of bounded combinatorics, where the theory has recently assumed a complete form, with the proof of a priori bounds in the satellite case (in a joint work with Dima Dudko)

Dzmitry Dudko: Uniform a priori bounds for neutral renormalization and their applications

- Lecture 3: Covering and Snake-Lair Lemmas.
- Lecture 4: Shift Argument and parabolic fjords.
- Lecture 5: Pseudo-Siegel disks.

Abstract of Lectures 3, 4, 5 and 6. Near-neutral Renormalization controls how attracting periodic cycles evolve into repelling. In the mini-course, we will discuss how bounded-type Siegel disks of neutral quadratic polynomials develop parabolic fjords as combinatorics grow. By appropriately filling-in such fjords, we obtain pseudo-Siegel disks with uniformly bounded qc geometry. This leads, by taking limits, to uniform a priori bounds for all neutral quadratic polynomials; the bounds can be formulated in various forms. The construction is justified in the near-degenerate regime. One of the consequences is the existence of the “Mother Hedgehog” for all neutral quadratic polynomials. Joint work with Misha Lyubich.

Kuntal Banerjee: Image of a Curve under Chaos

Igors Gorbovickis: Critical points of the multipliers

A parameter $c_0\in\mathbb C$ in the family of quadratic polynomials $f_c(z)=z^2+c$ is a *critical point of a period n multiplier*, if the map f_{c_0} has a periodic orbit of period n, whose multiplier, viewed as a locally analytic function of c, has a vanishing derivative at $c=c_0$. Information about the location of the critical points and critical values of the multipliers can be helpful for understanding the geometry of the Mandelbrot set. In this talk we will review some results about the critical points of the multipliers in the quadratic family and discuss some generalizations for the families of higher degree polynomials.

Inou Hiroyuki: Accessible hyperbolic components of the tricorn

It is known that the tricorn, the connectedness locus of the anti-quadratic family, is not locally connected. Every hyperbolic component of odd period has decoration accumulating to the boundary, which becomes more and more complicated as the period increases. Hence it looks natural to expect that if the period is sufficiently large, such a hyperbolic component is not accessible from the complement. Contrary to this intuition, we prove that there exists a hyperbolic component of arbitrarily large odd period. This is joint work with Tomoki Kawahira.

Wolf Jung: Small copies of connectedness loci in parameter spaces

Denote the one-dimensional parameter spaces of cubic polynomials or quadratic rational maps, having a persistently n-periodic critical point, by S_n and V_n. When $n > 1$, these contain a finite number of bitransitive hyperbolic components: for the respective maps f , the two critical points are in different components of the n-cycle of Fatou basins. Around such a hyperbolic component, the parameter space shall contain a small copy of the connectedness locus B of the quartic polynomial families $(z^2 - q^2)^2 + q$ and $(z^2 + q)^2 - q^2$, which are semi-conjugate.
There are two approaches to provide (partial) proofs:
• When the small Julia sets are disjoint, use renormalization;
• For postcritically finite maps, use Thurston theory.
In the case of V_2 , the small Julia sets share all of their boundaries in fact; this is a case of anti-mating, which was discussed by Timorin for the boundary of the bitransitive hyperbolic component.

Tomoki Kawhira: Derivatives of mildly degenerating holomorphic motions of the quadratic Julia sets

We present optimal derivative estimates of the holomorphic motions of the quadratic Julia sets along two kinds of paths in the parameter space:

1. the external rays of the Mandelbrot set that land on semi-hyperbolic parameters; and
2. the internal rays of the Mandelbrot set that land on parabolic parameters.

We also show integrability of the derivatives, Hölder continuity of the semiconjugacies given by the degenerating holomorphic motions, and the derivative of the Hausdorff dimensions (if time permits). This talk is based on joint works with Yi-Chiuan Chen, arXiv:1803.03130 and arXiv:2304.11231.

Pascale Roesch: The parabolic Mandelbrot set

There is a unique dynamics preserving homeomorphism between the Mandelbrot set and the parabolic Mandelbrot set. Moreover for a dense set of parameters in the boundary of M, this homeomorphism is not Hölder for any exponent.

Zhang Runze: Rigidity of bounded type cubic Siegle polynomials

Joint work with M. Yampolsky and J. Yang

We prove the combinatorial rigidity for non-renormalisable cubic Siegle polynomials of bounded type. As a consequence, we show that the closure in a bounded type Siegel slice of the collection of such polynomials is equal to the intersection of the boundary of the main hyperbolic component and the slice. This confirms partially a conjecture by A. Blokh, L. Oversteegen, R. Ptacek and V. Timorin in 2014.

Mitsuhiro Shishikura: Near Parabolic Renormalization

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