FATOU DYNAMICS OF COMPLEX HÉNON MAPS

The following is a loose outline of what we intend to cover. If we don't finish the agenda of a given lecture, we might continue the topic in the next one.

Lecture 1: Introduction to Hénon maps. (Han)

What are Hénon maps, and why we should study them.
Basic concepts: Filtration, $K^{+/-}$, $J^{+/-}$, K and J. Fatou components. Constant Jacobian.
Green's function, equidistribution currents, equilibrium measure, entropy. Basic question: $J = J^*$?
Definition of hyperbolicity.

References: [H], [HOV1], [FS], [BS1] , [Us1]

Lecture 2: Fatou components of hyperbolic Hénon maps. (Eric)

Definition of hyperbolicity. Existence of cone fields; Stable Manifold Theorem.
Abundance of hyperbolic maps: (1) Horseshoes (and the associated symbolic dynamics), (2) perturbations of one-dimensional maps.
Structure of hyperbolic maps: (1) All Fatou components are periodic sinks, (2) $J = J^*$.

References: [BS1], [BSh], [HOV2]

Lecture 3: Classification of periodic components, recurrent case. (Eric)

Dichotomy: either all orbits converge to ∂U, or every orbit is contained in compact subset. Definition of recurrent domains.
Conservative case. Existence through local dynamics. Topological observations.
Classification of Fatou components in the dissipative case.
Visualization of Fatou components.

References: [BS2], [Us2]

Lecture 4: Non-recurrent case. (Han)

Snail lemma.
Smooth limit sets.
Substantial dissipativity and consequences.

References: [LP], [Ue]

Lecture 5: Beyond hyperbolicity: quasi-expansion. (Eric)

Quasi-expansion and a new appearance of normal families. Proper, bounded area condition. Expansion of a (possibly singular) metric.
Real maps of maximal entropy.

References: [BS8], [BSme]

Lecture 6: Beyond hyperbolicity: dominated splitting. (Han)

Results and some proofs. Questions for future research.

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