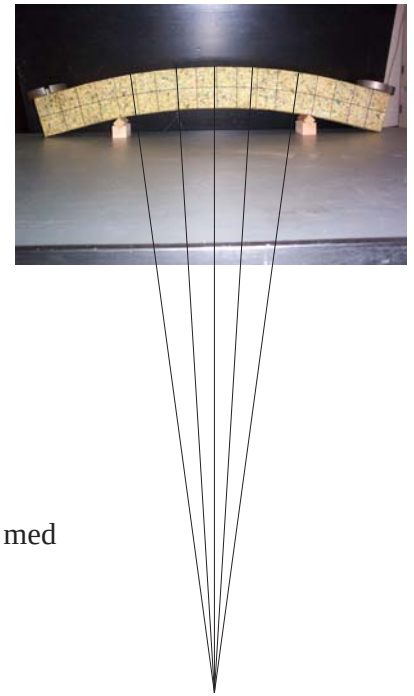
41015
Mekanik og Materialer

Bjælkebøjning
(Kap. 9-9.3+9.6 i Riley)

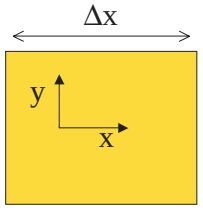
Ole Sigmund

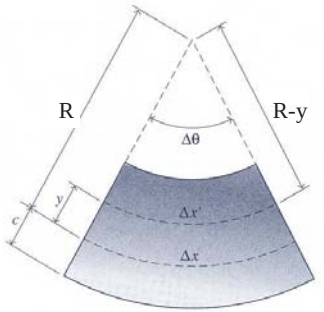
Firepunktsbøjning



Fælles skæringspunkt for snit med samme bøjningsmoment

Bernoullis antagelse: Plane snit vinkelret på middelaksen (neutralaksen) forbliver plane og vinkelret på denne under bøjning.





Geometriske betingelser:

$$\varepsilon_x(y) = \frac{\Delta x' - \Delta x}{\Delta x} = \frac{(R-y)\,\Delta\theta - R\,\Delta\theta}{R\,\Delta\theta}$$

$$\varepsilon_x(y) = -\frac{y}{R}$$

Vigtige formler for bjælkebøjning:

Spænding som funktion af afstand fra neutralakse og krumning

$$\sigma_x(y) = -E\frac{y}{R}$$

Moment som funtion af normalspændingsfordeling

$$M_r = -\int_A y\,\sigma_x(y)\,dA$$

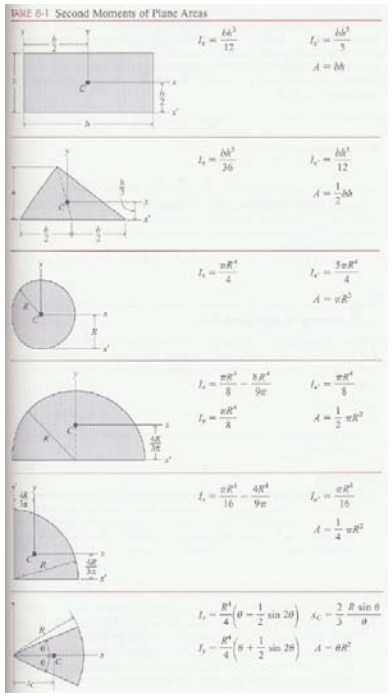
Forhold mellem maksimal spænding og moment

$$|\sigma|_{max} = \frac{|M|_r\,|y|_{max}}{I_x} \quad \text{eller} \quad |M|_r = \frac{I_x}{|y|_{max}}|\sigma|_{max}$$

"Inertimomentet" $I_x = \int_A y^2\,dA$

Inertimomenter:

$$I_x = \int_A y^2 \, dA$$



Regneregler for inertimomenter:

Inertimoment om neutralaksen

$$I_x = \int_A y^2 \, dA$$

Inertimoment om en anden akse

$$I_x' = I_x + y_0^2 A$$

Additionsregel

$$I_{\square} = I_{\square} + I_{\bigcirc}$$

Subtraktionsregel

$$I_{\square} = I_{\square} - I_{\bigcirc}$$

Bjælkens differentialligning:

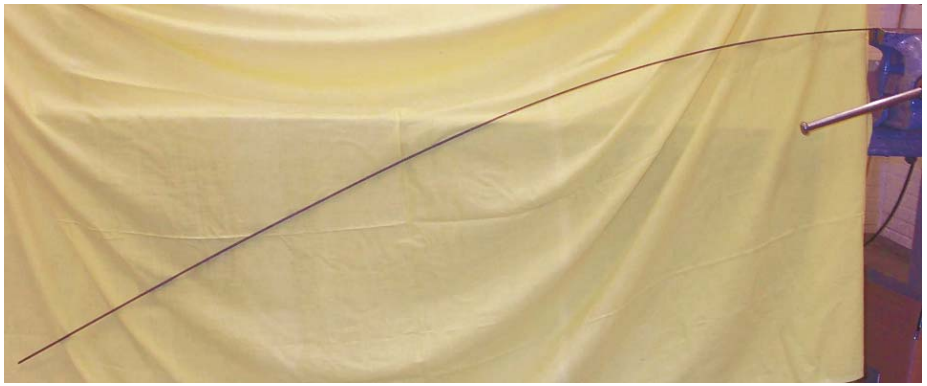
Kombiner $M_r = -\int_A y\,\sigma_x(y)\,dA$ og $\sigma_x(y) = -E\frac{y}{R}$

og få $M_r = \frac{E}{R} I_x$ eller $\boxed{\frac{1}{R} = \frac{M_r}{E\,I_x}}$

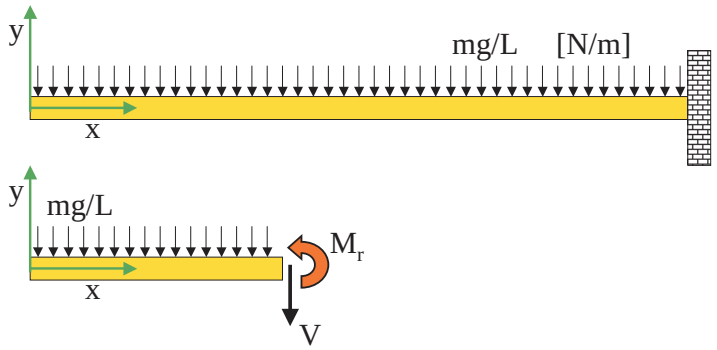
Hvis neutralaksens udbøjning beskrives ved funktionen y(x)
så kan krumningen 1/R approksimeres med d^2y/dx^2 dvs.

Bjælkens differential ligning: $\boxed{\frac{d^2y}{dx^2} = \frac{M(x)}{E\,I_x}} \Leftrightarrow \boxed{E\,I_x \frac{d^2y}{dx^2} = M(x)}$

y: Neutralaksens udbøjning	$\frac{d^2y}{dx^2}$: Neutralaksens krumning
$\frac{dy}{dx} = \Theta$: Neutralaksens hældning	$EI_x \frac{d^2y}{dx^2}$: Momentet



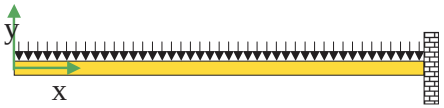
Eksempel: Konsolbjælke



Momentligevægt for delbjælke:

$$M(x) = -\frac{m\,g}{2L}\,x^2$$

Eksempel: Konsolbjælke



Indsæt momentet i bjælkens differential ligning:

$$E\,I_x\,\frac{d^2y}{dx^2} = -\frac{mg}{2L}\,x^2$$

Integrer ligning to gange:

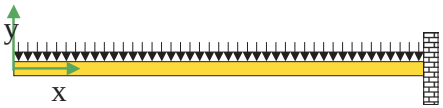
$$E\,I_x\,\frac{dy}{dx} = -\frac{mg}{6L}\,x^3 + c_1$$

$$E\,I_x\;y = -\frac{mg}{24L}\,x^4 + c_1 x + c_2$$

Randbetingelser:

$$y(L) = 0 \quad \text{og} \quad y'(L) = 0$$

Eksempel: Konsolbjælke



Indsæt randbetingelser

$$E\,I_x \cdot 0 = -\frac{mg}{6L}\,L^3 + c_1 \qquad \Rightarrow \quad c_1 = \frac{mg}{6}\,L^2$$

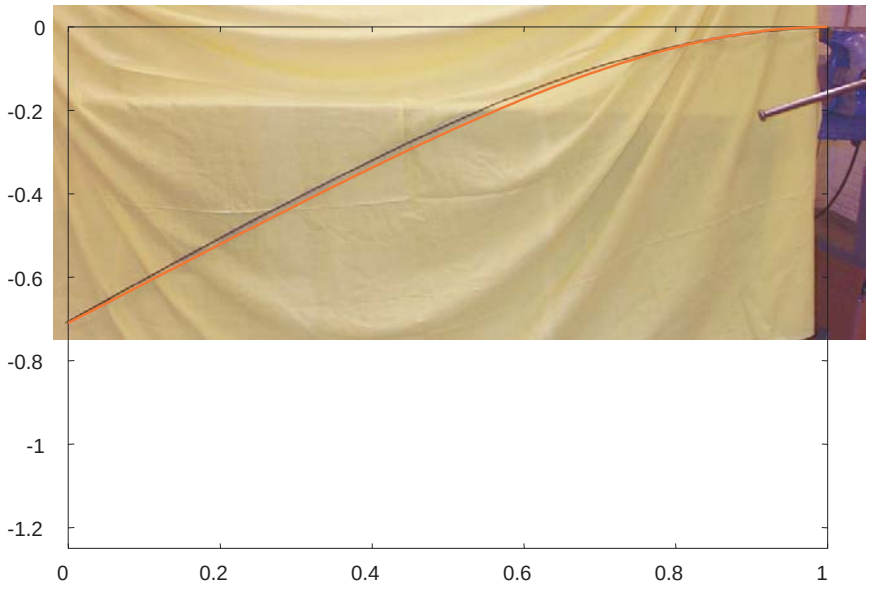
$$E\,I_x \cdot 0 = -\frac{mg}{24L}\,L^4 + \frac{mg}{6L}\,L^3 L + c_2 \qquad \Rightarrow \quad c_2 = -\frac{mg}{8}\,L^3$$

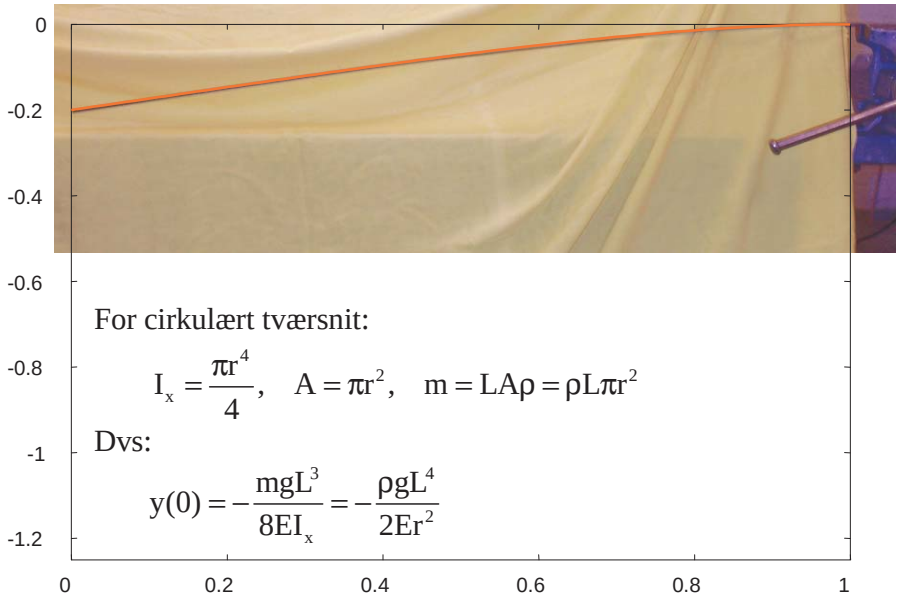
Hermed fåes neutralaksens udbøjning som

$$y(x) = -\frac{mg}{24EI_xL}\left(x^4 - 4xL^3 + 3L^4\right)$$

Udbøjningen i venstre ende (x=0)

$$y(0) = -\frac{mgL^3}{8EI_x}$$





For cirkulært tværsnit:

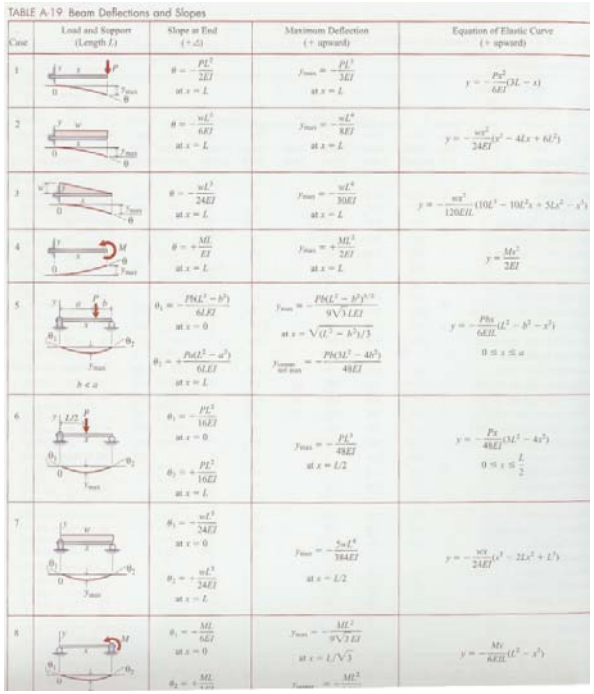
$$I_x = \frac{\pi r^4}{4}, \quad A = \pi r^2, \quad m = LA\rho = \rho L \pi r^2$$

Dvs:

$$y(0) = -\frac{mgL^3}{8EI_x} = -\frac{\rho g L^4}{2Er^2}$$

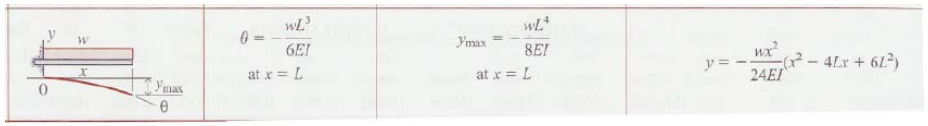
Elementar-tilfælde

(side 706 i Riley)



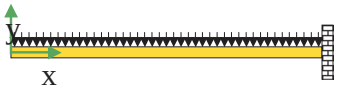
Elementartilfælde

konsolbjælke med fordelt last:



"Vores" beregning havde modsatrettet x-akse:

$$y(0) = -\frac{mgL^3}{8EI_x} \qquad y(x) = -\frac{mg}{24EI_x L}\left(x^4 - 4xL^3 + 3L^4\right)$$



Regneregler for bjælkebøjning:

"Superpositionsprincippet"

Udbøjningen af en bjælke udsat for kombinerede laste findes som summen af udbøjningerne for de enkelte laste