

MAT 91121-22 Opgave E32

Preben Alsholm
IFAK, DTU

21. november 2003

Der er givet funktionen f ved forskriften

$$f(x, y) = (x + y^2) \cos(x + 3y)$$

Vi skal finde funktionens 2. Taylorpolynomium med udviklingspunkt $(\pi, 0)$.

Vi har

$$\begin{aligned} P_2(x, y) &= f(\pi, 0) + f_x(\pi, 0)(x - \pi) + f_y(\pi, 0)y + \\ &\quad \frac{1}{2} \left(f_{xx}(\pi, 0)(x - \pi)^2 + 2f_{xy}(\pi, 0)(x - \pi)y + f_{yy}(\pi, 0)y^2 \right) \end{aligned}$$

Vi finder

$$\begin{aligned} f_x(x, y) &= \cos(x + 3y) - (x + y^2) \sin(x + 3y) \\ f_y(x, y) &= 2y \cos(x + 3y) - 3(x + y^2) \sin(x + 3y) \\ f_{xx}(x, y) &= -2 \sin(x + 3y) - (x + y^2) \cos(x + 3y) \end{aligned}$$

Herved har vi, at $f(\pi, 0) = -\pi$, $f_x(\pi, 0) = -1$, $f_y(\pi, 0) = 0$, $f_{xx}(\pi, 0) = \pi$. Vi får oplyst, at $f_{xy}(\pi, 0) = 3\pi$ og $f_{yy}(\pi, 0) = 9\pi - 2$. Herved har vi

$$\begin{aligned} P_2(x, y) &= -\pi - (x - \pi) + \frac{1}{2} \left(\pi(x - \pi)^2 + 6\pi(x - \pi)y + (9\pi - 2)y^2 \right) \\ &= -x + \frac{\pi}{2}(x - \pi)^2 + 3\pi(x - \pi)y + \left(\frac{9\pi}{2} - 1 \right) y^2 \end{aligned}$$