

MAT 91112 Opgave E 45

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Vi skal finde det 2. Taylorpolynomium for $f(x) = x \cos x$ udfra $\frac{\pi}{2}$. Dette polynomium er givet ved

$$P_2(x) = f\left(\frac{\pi}{2}\right) + f'\left(\frac{\pi}{2}\right)\left(x - \frac{\pi}{2}\right) + \frac{1}{2}f''\left(\frac{\pi}{2}\right)\left(x - \frac{\pi}{2}\right)^2$$

Vi finder, at

$$\begin{aligned}f'(x) &= \cos x - x \sin x \\f''(x) &= -2 \sin x - x \cos x\end{aligned}$$

Derfor har vi, at $f'\left(\frac{\pi}{2}\right) = -\frac{\pi}{2}$ og $f''\left(\frac{\pi}{2}\right) = -2$. Desuden har vi, at $f\left(\frac{\pi}{2}\right) = 0$.
Altså fås

$$P_2(x) = -\frac{\pi}{2}\left(x - \frac{\pi}{2}\right) - \left(x - \frac{\pi}{2}\right)^2 = x\left(\frac{\pi}{2} - x\right)$$