01325 Mathematics 4, Spring 2013

Week no. 13

Theory: Wednesday May 15 the lecture (1 hour) will cover Section 11.2 (see also def. 11.1.2 and def. 11.1.4). The lecture will be followed by 3 hours problem session.

Exercises for Wednesday May 15: 10.6, 11.3, Problem 101, Problem 102, Problem 217, Problem 218, Problem 108, 11.4, 1.22

These exercises deal with several topics from the course, not just material related to the lecture. I you have completed the program you can look at Problem K below (you might skip the concrete calculations in (i) and (ii) and just think about (iii) and (iv)). For the sake of repetition before the exam, note that the file Exercises+Exam.pdf contains the exam from 2012 and that none of the problems from this exam have appeared on the weekly programs.

Problem K We recall Taylors theorem. Assume that $f : I \to \mathbb{R}$ and that there exists a constant $C > 0$ such that $|f^n(x)| \leq C$ for all $n \in \mathbb{N}$ and all $x \in I$. Let $x_0 \in I$. Then, for $x \in I$ and $N \in \mathbb{N}$,

$$\left| f(x) - \sum_{n=0}^{N} \frac{f^n(x_0)}{n!}(x - x_0)^n \right| \leq \frac{C}{(N+1)!} |x - x_0|^{N+1}.$$

Now consider the function $f(x) = e^x$, $x \in [0, 50]$.

(i) Find $N \in \mathbb{N}$ such that there exists a polynomial P of degree N such that

$$|f(x) - P(x)| \leq 0.1, \ \forall x \in [0, 50].$$

(ii) Find $N \in \mathbb{N}$ such that there exists a polynomial P of degree N such that

$$|f(x) - P(x)| \leq 0.1, \ \forall x \in [40, 50].$$

(iii) Is it realistic to approximate exponential functions by polynomials over large intervals?

(iv) Can you describe a more convenient approach to approximate exponential functions, or a more general class of functions? Check how your idea works on the function f above.

Regards,
Ole