01325 Mathematics 4, Spring 2013

Week no. 7

Theory: Wednesday March 20 the lecture covers

Section 6.2 (almost done last week), definition 4.3.1, Section 4.7 (only page 79-81) and Section 6.4.

Exercises for the week March 18–22: 1.1 (can you formulate and prove the similar result in Hilbert spaces?), 6.5, 6.15, Problem 219, 2.14 (ii), Problem 106, Problem 107, 4.31 (repeated)

I am confident that this program is enough, but if you want more, you can look at Problem C and D below.

Problem C In this exercise we give the formal definition of the composition of two operators, as used, e.g., in Exercise 3.14 and Exercise 4.17. Let $\mathcal{H}$ denote a Hilbert space and consider two bounded linear operators

$$S, T : \mathcal{H} \to \mathcal{H}.$$

We define the composed operator $ST : \mathcal{H} \to \mathcal{H}$ by

$$ST\mathbf{v} := S(T\mathbf{v}), \ \mathbf{v} \in \mathcal{H}.$$

(i) Show that the operator ST is linear.

(ii) Show that ST is bounded, and that

$$||ST|| \leq ||S|| \, ||T||.$$

Problem D Does $||f|| := \sup\{|f(x)| \mid x \in \mathbb{R}\}$ define a norm on the vector space $C(\mathbb{R})$?

Homework 7, to be turned in no later than April 3: 5.11, 4.28, Problem 105

Hint to exercise 4.28: show first that $\{\delta_k\}_{k=1}^{\infty}$ is an orthonormal system. Then you can either apply def. 4.7.1 directly (using an argument like in Example 3.2.2), or you can use one of the characterizations in Theorem 4.7.2.

Regards,
Ole