01234 Differential Geometry with Applications

Project Exercise: Minimal Surfaces

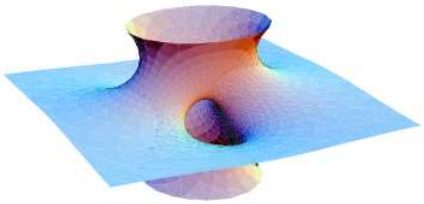

Figure 1: The Costa minimal surface.

1 Wanted

What is a minimal surface? You could choose from the exercises in [1], Chapter 12. Report on your solutions. Give examples. You may also want to look into [2] and [3] for similar, but more elaborate presentations.

2 Methods involved and other elaborations

All basic tools are given in Chapter 12 of [1].

Further possible advanced elaborations could involve studies in the following directions:

Of particular (local) interest is the Weierstrass representation of Costa's minimal surface as shown in Figure 1. This representation is developed in detail in [2] Chapter 32, Section 32.5, on the basis of the Weierstrass functions $\wp(z)$ and $\zeta(z)$, see also [4]. There is an interesting claim in the literature, to the effect that the lines of principal directions (the principal patch) for the Costa surface are (in the notation of the references) those of constant α and β defined in the parameter domain as follows:

$$\alpha + i\,\beta = \int \sqrt{\frac{1}{2} + \frac{\wp^2(z)}{\wp^2(z) - e_1^2}}\, dz$$

A possible project exercise in itself would be to check that claim and produce a figure of Costa's surface using this principal patch and thereby obtain an even 'better' conformal presentation of the surface.

Please Turn ⟶

Yet another independent project exercise is concerned with the Gauss map of minimal surfaces (see [1] Section 7.6). The Gauss map of a minimal surface is conformal. But how much does the image of a complete minimal surface fill up the target, the unit sphere? How many directions are omitted by the Gauss map of the helicoid and of the other classical examples of minimal surfaces?

References

[1] A. Pressley, *Elementary Differential Geometry*, Springer, 2010.

[2] A Gray, *Modern Differential Geometry of Curves and Surfaces with Mathematica*, 2.nd Edition, CRC Press, 1998. Chapters 30–33.

[3] R. Osserman, *A Survey of Minimal Surfaces*, Dover, 1986.

[4] E. Weisstein, Mathworld, http://mathworld.wolfram.com/CostaMinimalSurface.html

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