

# 01234 Differential Geometry with Applications

## Project Exercise: Map Projection Distortion



Figure 1: The Peirce Quincuncial map.

### 1 Wanted

A well known consequence of Gauss' Theorema Egregium is the Proposition 10.1 p. 238 in [P] that "Any map of any region of the Earth's surface must distort distances."

It follows (how?), that no such map preserves both area and angles - no map can be both equiareal and conformal.

The purpose of this project exercise is to construct one or more measures of distortion for maps from regions of the sphere into the plane and illustrate how the measures work in concrete examples like the classical ones hinted at below (and the not-so-classical one in Figure 1):



### 2 Methods involved

If a given map is known to be conformal, then the measure should tell and show how areas change under that map. And vice versa: An equiareal map must change angles between intersecting curves, but how and how much?

Small circles on the sphere are not necessarily mapped into circles in the plane. Why not? This may also be useful for illustrating a measure of (local) distance distortion.

## 2.1 Estimated specific types of work loads

- Theory from [1], [2] and elsewhere: \*\*
- Maple: \*\*\*

## References

- [1] A. Pressley, *Elementary Differential Geometry*, Springer, 2010.
- [2] J. Benítez and N. Thome, *Applications of Differential Geometry to Cartography*, Int. J. Math. Educ. Sci. Technol. 2004, Vol. 35, No. 1, 29–38. Attached as BenTho.pdf
- [3] R. Taylor, R. Baur, and J. Oprea, *Maple Maps*,  
<http://people.clarkson.edu/~chengweb/faculty/taylor/maps/maps1.html>
- [4] Vince Costanzo, *The Maple Maptools Package*,  
<http://www3.villanova.edu/maple/projects/costanzo/carto/index.htm>

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