Analysis of Wave Propagation in Structures and Solids

Ph.D. course at the International Doctoral School of Technology and Science
Aalborg University, October 2010

The course concerns the physics and modelling of wave propagation in structures and solids. The theories and formulations covered by the course are applicable within structural and mechanical engineering as well as earthquake and geotechnical engineering. Firstly, an introduction is given to the basic properties of body waves in an elastic continuum. Phenomena such as geometrical and material dissipation, dispersion, impedance, reflection and refraction are discussed. Furthermore, the formulation of theories for wave propagation in one-dimensional structures, including beams and fluid-filled pipes, is considered. Secondly, the finite element method is introduced as a computational tool for the analysis of wave propagation in soil and structures. Time and frequency-domain solutions are considered, and guidelines are given for the treatment of artificial boundaries in numerical models. Thirdly, boundary-element schemes for wave propagation in solids and structures are derived in the time as well as the frequency domain, and an alternative solution for stratified materials, e.g. layered soil, is given in the wavenumber-frequency domain. Finally, lumped-parameter models (LPMs) are introduced as a means of representing the dynamic stiffness of continuous structures and soil in a computationally efficient manner. As an example, the application of LPMs for wind-turbine or machine foundations is illustrated.

Organizers: Lars Andersen, Associate Professor (phone: +45 9940 8455; e-mail: la@civil.aau.dk).
Sergey Sorokin, Professor (phone: +45 9940 9332; e-mail: svs@me.aau.dk).

Lecturers: Lars Andersen, Associate Professor of Civil Engineering, Aalborg University;
Sergey Sorokin, Professor of Mechanical Engineering, Aalborg University;
Michael J. Brennan, Guest Professor, Institute of Sound and Vibration Research (ISVR), University of Southampton, Southampton, United Kingdom.

Workload: 4 ECTS

Time: October 2010

Venue: Aalborg University, Department of Civil Engineering
Sohngaardsholmsvej 57, DK-9000 Aalborg, Denmark
Lecture room to be announced later.

Prerequisites: The participant must have a solid background in continuum mechanics and partial differential equations. Experience with numerical methods and programming is strongly recommended. The participants are expected to read the texts in the literature list before the course. The literature will be available upon registration.

Form: Each part of the course consists of a lecture followed by a workshop. The lectures are given in English.

Evaluation: As part of the course work, the participant must hand in a portfolio containing answers to exercises as well as computer codes elaborated by the participant during and after the course. Some of this work may be carried out in the workshops.

Lecture Plan and Literature List

Day 1 — 7 October 2010

Morning class 09:00 – 12:00

The basics of elastodynamics – Part 1
Applications of elastodynamics. Body waves in an elastic solid: P- and S-waves. Surface waves. Mechanical impedance. Reflection and refraction at interfaces and surfaces. Exercise 1 in [1].

Lecturer: Lars Andersen — **Literature:** [1] Chapter 1

Afternoon class 13:00 – 16:00

The basics of elastodynamics – Part 2
Dispersion and dissipation of waves. Cut-on/cut-off frequencies. Geometrical and material damping. Viscoelastic constitutive models. Introduction to Green's functions for body waves in an elastic solid. Exercise 2 in [1].

Lecturer: Lars Andersen — **Literature:** [1] Chapter 1

Day 2 — 8 October 2010

Morning class 09:00 – 12:00

The finite-element method and wave analysis – Part 1
Basic steps of the finite-element method: Beam, shell and continuum models. Discretization considerations regarding space and time. Finite element analysis of unbounded domains and infinite structures. Exercise 3 in [1].

Lecturer: Michael J. Brennan — **Literature:** [5,6]

Afternoon class 13:00 – 16:00

The finite-element method and wave analysis – Part 2
Basic approach of wave and finite element analysis – examples for rods and beams. Use of the method for the plate-strip to illustrate the method for waveguides. Examples given for a car tyre.

Lecturer: Michael J. Brennan — **Literature: [4]**

Day 3 — 11 October 2010

Morning class 09:00 – 12:00

Wave propagation in one dimensional structures
Dilatation waves, torsion waves and flexural waves in straight rods (elementary theories). Timoshenko beam analysis. Wave propagation in curved and non-uniform structures

Lecturer: Sergey Sorokin — **Literature:** [2] Sections 1.4, 1.14-1.16; [3] Chapters 1-2

Afternoon class 13:00 – 16:00

Boundary integral equations for stationary waves in elastic rods
The boundary integral equation method. Green's function and Somigliana's identity: Axial waves in a rod; flexural waves in a beam. Wave propagation, reflection and transmission in elastic rods. Exercises in [2], 1.11-1.12

Lecturer: Sergey Sorokin — **Literature:** [2] Sections 1.9-1.13, 1.17-1.22

Day 4 — 12 October 2010

Morning class 09:00 – 12:00

Wave propagation in periodic structures
Floquet theory. Stop and pass frequency bands in infinite periodic straight rods: dilatation and flexural waves. Effect of a finite number of 'periodicity cells'.

Lecturer: Sergey Sorokin — **Literature:** [2]

Afternoon class 13:00 – 16:00

Vibro-acoustics of elastic fluid-filled pipes
Thin shell theory and duct acoustics. Heavy fluid loading. Simplified theories. The effect of fluid flow Waves in compound and curved pipes. Periodicity effects.

Lecturer: Sergey Sorokin — **Literature:** [2,3]

Day 5 — 18 October 2010

Morning class 09:00 – 12:00

Boundary-integral equations in elastodynamics
Introduction to boundary integral equations. Dynamic reciprocity theorems in two and three dimensions. Green's functions for displacements and stresses. Somigliana's identity for wave propagation in an elastic solid. Exercise 5 in [1].

Lecturer: Lars Andersen — **Literature:** [1] Chapter 3

Afternoon class 13:00 – 16:00

The boundary-element method for elastodynamic analysis
Direct formulation in the time and frequency domains. Frequency-domain analysis in two and three dimensions. Coupling of boundary elements with finite elements. Boundary element analysis in the time domain.

Lecturer: Lars Andersen — **Literature:** [1] Chapter 4

Day 6 19 October 2010	Morning class 09:00 – 12:00	**The domain-transformation method for stratified elastic solids** Introduction. Response of a layered half-space. Flexibility matrix for a layered half-space. Computational optimisations. Dispersion of waves in a stratum. Applications to earthquake engineering and foundations. Exercise 4 in [1] **Lecturer:** Lars Andersen **Literature:** [1] Chapter 5
	Afternoon class 13:00 – 16:00	**Lumped-parameter models for dynamic structures and solids** Introduction and motivation. Standard and fundamental models. Consistent models: Fitting and implementation in finite-element codes. Application to the analysis of wind-turbine and machine foundations. **Lecturer:** Lars Andersen **Literature:** [1] Chapter 6

[1] Lars Andersen (2006). “Linear Elastodynamic Analysis”, DCE Lecture Notes No. 3. Aalborg University, Department of Civil Engineering, December 2006.

[2] Sergey Sorokin (2007). Basics of the Theory of Wave Propagation in Elastic Structures, Lecture Notes, Aalborg University, Department of Mechanical Engineering.

[3] Sergey Sorokin (2004). An Introduction to the Theory of Wave Propagation in Elastic Cylindrical Shells Filled with an Acoustic Medium, Pre-print of the Centre of Machine Acoustics, Aalborg University, Department of Mechanical Engineering.

[4] F. Fahy and P. Gardonio (2006). “Sound and Structural Vibration: Radiation, Transmission and Response”, Academic Press; 2 edition 2006.

[5] B.R. Mace, D. Duhamel, M.J. Brennan and L. Hinke (2005). “Finite element prediction of wave motion in structural waveguides”. *Journal of the Acoustical Society of America* **117**(5), 2835–2843.

[6] Y. Waki, B.R. Mace, B.R. and M.J. Brennan (2009). “Numerical issues concerning the wave finite element approach for free and forced vibrations of waveguides”. *Journal of Sound and Vibration* **327**(1–2), 92–108.