Virtual current density in magnetic flow meter

Formulation of a problem in relation to magnetic flow meters

(Claus Nygaard Rasmussen, Siemens Flow Instruments, 24/6-2004)

Background

Siemens Flow Instruments would like to submit this problem for the 51st European Study Group with Industry, held at the Technical University of Denmark, 16-20 August 2004. The problem relates to performance calculation on magnetic flow meters.
A magnetic flow meter measures the velocity of a liquid flowing in a pipe by applying a magnetic field to the flowing liquid and measuring the voltage induced across the liquid as it moves through the magnetic field. This is done by inserting two electrodes into the liquid, one on each side of the pipe. The liquid (typically water) needs to have a minimum conductivity of approximately $\sigma \sim 10^{-4}$ S/m. If the pipe is made of an electrically conducting material then an insulating liner is used to prevent a short circuit between the pipe and the electrodes. The liner length is typically 3 times the diameter of the pipe.
When designing magnetic flow meters it is important to be able to calculate the performance with respect to sensitivity and linearity. That is: The voltage signal generated at a certain flow speed and the variation in signal with flow velocity. This is done by using finite element modelling of the magnetic system, followed by a calculation procedure involving the flow profile of the liquid.

Theory

When a magnetic field is applied to a moving conductor, a voltage is induced across the conductor. It can be shown that the voltage (U) between the two electrodes in a magnetic flow meter is given as:

$$U = \int_V \vec{\upsilon} \cdot \vec{w} dv \quad \text{and} \quad \vec{w} = \vec{B} \times \vec{j} \tag{1}$$

With B [T] being the applied magnetic field, j [m^{-2}] is a virtual current density resulting from an imaginary current flowing between the two electrodes and υ [m/s] is the liquid flow velocity. Figure 1 shows an end-view sketch of a magnetic flow meter.

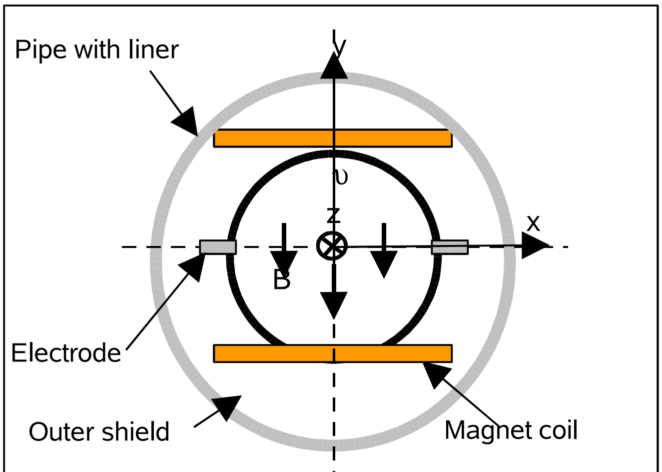

Figure 1: Sketch of a flow meter, end view.

The velocity $\upsilon = [\upsilon_x \; \upsilon_y \; \upsilon_z]$ can be assumed to have υ_x=0 and υ_y=0, meaning that the voltage between electrodes can be expressed as:

$$U = \int_0^R \int_0^{2\cdot\pi} \int_{-\infty}^{\infty} \upsilon_z \cdot (B_x \cdot j_y - B_y \cdot j_x) \cdot r \cdot dz \cdot d\theta \cdot dr \tag{2}$$

With R being the pipe radius. Currently, this calculation is performed by combining the results of two independent finite element calculations. A simulation of the magnetic system combined with a simulation of the electrode system that yields the virtual current.

Analytical expression for virtual current density in magnetic flow meter

Given the relatively simple geometry of the pipe and electrode system, it seems reasonable to assume that it is possible to derive an analytical expression for the virtual current so that a rather time-consuming finite calculation of the virtual current can be avoided.
Figure 2 shows a sketch of the electrode system. Because of symmetry, the calculation only needs to involve 1/8 of the sensor, ¼ of the circle and ½ the length. The calculation volume is show in figure 3. L/2 is half the liner length and D/2 is the pipe radius. The electrode is assumed cylindrical with a radius r_e and it enters the sensor pipe with a length l_e. The Y-Z plane can be set to zero potential (V=0) and the electrode potential is set to V=1V.

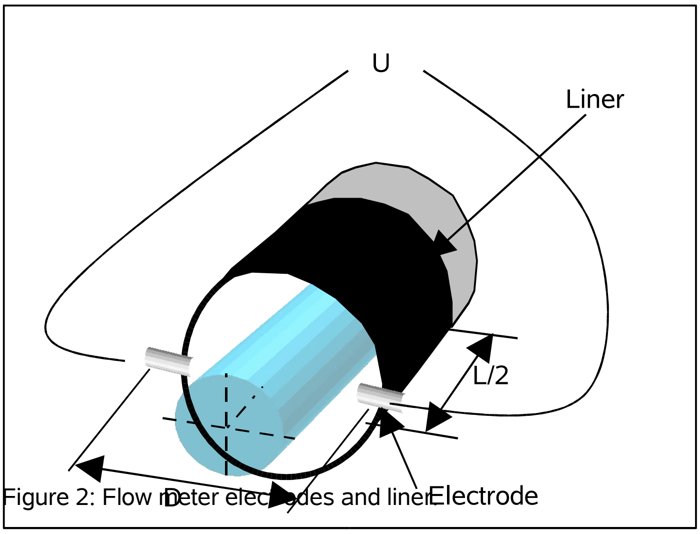

Figure 2: Flow meter electrodes and liner.

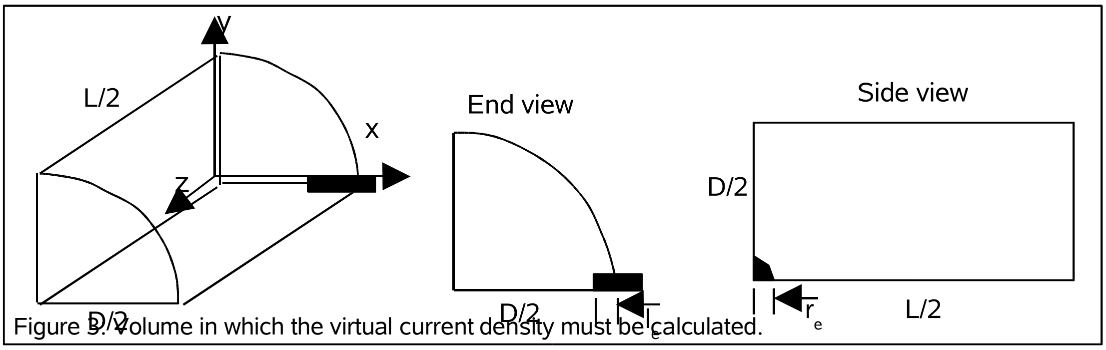

Figure 3: Volume in which the virtual current density must be calculated.

The question is now: Based on the knowledge of tube diameter as well as length and radius of the flow-meter electrodes, is it possible to derive an analytical expression for the virtual current density in the tube? That is: j = [jx jy jz] = f(x,y,z). This could improve speed and probably also accuracy of the flow-meter sensitivity- and linearity calculations. Alternatively, can some rules of scaling be applied so that a finite element calculation on one set of parameters can be transferred to a different set of parameters?

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